The thermal pulse decay technique for measuring ultrasonic absorption coefficients

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(Received 5 April 1983; accepted for publication 8 August 1983)

The classic method for measuring absorption coefficients of materials requires a determination of the temperature rise within a medium during ultrasonic irradiation. Serious errors may result from a viscous heating effect around the required thermocouple probe, and from heat lost by conduction to surrounding, cooler regions. A thermal pulse-decay technique is proposed and evaluated which allows for the separation, in time and space, of the viscous heating artifact and the true absorptive heating of a medium. In addition, the pulse-decay method explicitly accounts for heat conduction and the ultrasonic beam patterns used in experiments. The inherent advantages of the new technique allow for an extension of absorption measurements to smaller beamwidths and higher frequencies and intensities than were previously possible.

PACS numbers: 43.80.Ev, 43.80.Cs, 43.35.Yb, 43.35.Cg

LIST OF SYMBOLS

ρ	density, g/cm^3
c	specific heat, J/g°C
T	temperature, °C
t	time, seconds
τ	time, seconds
k	thermal diffusivity, cm^2/s
K	thermal conductivity, W/cm°C
q	volumetric heat generation, W/cm^3
α	amplitude absorption coefficient, np/cm
μ	intensity absorption coefficient, np/cm
β	Gaussian variance, cm^2
I	ultrasonic intensity, W/cm^2
x,y,z	axes of Cartesian coordinates
r,θ,z	axes of cylindrical coordinates

INTRODUCTION

The ultrasonic absorption coefficient of tissues represents the rate at which energy contained in a propagating acoustic wave is converted to heat. The magnitude and frequency dependence of the absorption coefficients is, therefore, an important factor in therapeutic and diagnostic applications of ultrasound. Until recently, the only method available for determining the absorption coefficient at a point in tissue was the "transient-thermoelectric" technique which will be referred to in this paper as the "rate-of-heating" method. In this method, the temperature increase of a material during continuous wave, monochromatic insonation at known intensity is measured by an embedded thermocouple probe.[1,2] The proportionality between the ultrasonic intensity and the rate-of-heating yields the magnitude of the absorption coefficient at the operating frequency. Other measurement techniques, such as radiation force insertion loss, measure the attenuation coefficient of a medium. The attenuation coefficient is always greater than or equal to the absorption coefficient since the former includes losses due to absorption, reflections, and scattering within a tissue.[3]

In the classic papers describing the rate-of-heating method, Fry and Fry[1,2] considered errors associated with the presence of the thermocouple probe, such as the conduction of heat along the wires, and the variation of temperature within the wire. Also considered were errors associated with heating the medium, such as the dependence of the quantity $\mu/\rho c$ on temperature, and the conduction of heat to surrounding regions which are cooler because of the characteristic intensity falloff from the center of the ultrasonic beam. A later paper[4] furthered the analysis viscous heating phenomenon which occurs at the wire–medium interface and can create a large temperature artifact. Goss, Cobb, and Frizzell[4] also analyzed the effects of lateral beamwidth on rate-of-heating measurements taken 0.5 s after commencement of cw insonation. The principal difficulty associated with the rate-of-heating method is that the measurement must be made during a temporal "window," where the effects of viscous heating (error decreasing with time) and the effects of heat loss by conduction (error increasing with time) are minimized. The guidelines that emerge for making rate-of-heating measurements which are accurate to within $\pm 10\%$, at 0.5 s following the onset of insonation, are as follows.[1,4] To reduce the viscous heating artifact, the use of a thermocouple wire 75 μm or less is usually required. Furthermore, the use of half-power beamwidths of greater than 3 mm is recommended to minimize the heat loss by conduction to surrounding regions.

In practice these guidelines restrict experimental capabilities, particularly in high-frequency, high-intensity work where small focal regions are commonly used. To counter these restrictions, the pulse-decay technique was developed.[3,5] The viscous heating artifact is minimized in this technique by the ability to isolate both spatially and temporally the true absorptive heating of the material. Also, very small focal regions may be used since the pulse-decay technique explicitly accounts for the intensity distribution and the conduction of heat in the medium. This approach provides an experimental alternative to the rate-of-heating method. The rationale and advantages of the pulse-decay technique are described in this report.

I. THEORY

The thermal pulse-decay method begins with the observation that, for focused beams with numerical apertures (the ratio of focal length to transducer diameter) greater than unity, the focal intensity falloff in the radial (transverse) direction is many times greater than the rate of intensity fall off in the longitudinal axis (axis of insonation).[6] Also, the radial intensity distribution of the mainlobe can be adequately described as having a Gaussian shape, as shown in Fig. 1. We will therefore assume that the intensity distribution from a focused beam can be approximated as constant along the axis of insonation, and Gaussian in the radial direction. Using cylindrical corrdinates, with the z axis aligned with the axis of insonation, the intensity distribution is described as a function of radius r by

$$I(r) = I_{\max} e^{-r^2/\beta}, \quad (1)$$

where the parameter β is a measure of the spread of the focal region which can be determined by a least squares error curve fit applied to any suitable intensity profile. A diagram of the orientation of the ultrasonic beam and coordinate system is shown in Fig. 2.

Let us assume that the distribution given by Eq. (1) is produced in an absorbing medium initially at uniform temperature, for a period of time Δt which is short compared to the time required for significant conduction effects to take place. This occurs for durations such that the dimensionless parameter $kt/\beta < 0.01$.[7,8]

At the end of Δt seconds, a temperature increase is produced by the absorption of ultrasound

$$T(r) = T_{\max} e^{-r^2/\beta}, \quad (2)$$

where the proportionality between intensity and temperature elevation includes the absorption coefficient

$$T_{\max} = (\mu \Delta t/\rho c) I_{\max}. \quad (3)$$

Assuming $\Delta t, \rho, c$, and $I_{\max}$ are known, then measurement of $T_{\max}$ yields the value of μ through Eq. (3). Unfortunately, the viscous heating effect around the thermocouple creates a reading in excess of the temperature rise created by absorption alone, and direct measurement of $T_{\max}$ is not possible.

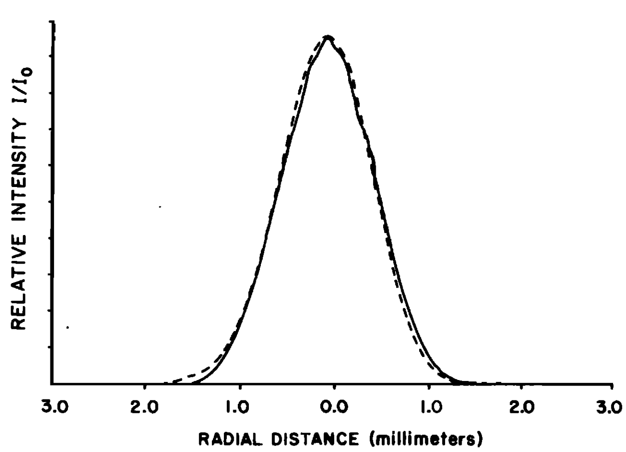

FIG. 1. Intensity of a 2.7-MHz focused beam. Solid line—measured at 0.1-mm increments. Dotted line—least squares error Gaussian curve fit.

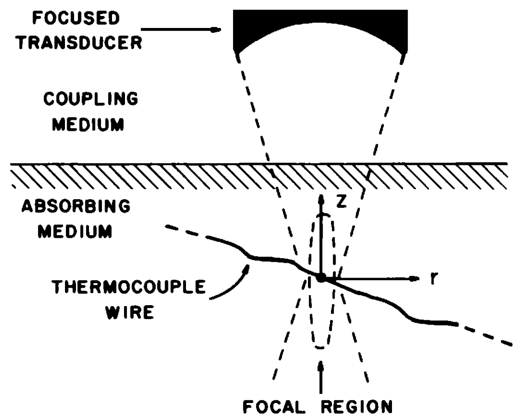

FIG. 2. The positioning of a focused beam over an embedded thermojunction for absorption measurements.

Since the effects of viscous heating are localized to the thermocouple wires, the temperature elevation caused by viscous heating drops off rapidly after insonation, and the thermojunction then reads the decay of temperature which resulted from absorption alone. From these data and a model for the temperature decay, the value of $T_{\max}$, and therefore μ, can be recovered.

The mathematical solution required is the temperature decay in a homogeneous conducting medium following an initial temperature distribution given by Eq. (1). The solution must satisfy the energy conservation equation for a conducting medium:

$$\rho c \frac{\delta T}{\delta t} = K \nabla^2 T + q, \quad (4)$$

with the volumetric heat generation q, equal to zero in the absence of ultrasonic heating. We begin by examining the solution for an infinitely long, instantaneous line source of strength Q, occurring at time $\tau = 0$. With the line source extending along the z axis of cylindrical coordinates, the temperature for times $\tau > 0$ are described by Ref. 6:

$$T(r,\tau) = (Q/4\pi k \tau) e^{-r^2/4k\tau}. \quad (5)$$

Note that for any time $\tau > 0$, the shape of the curve is Gaussian. In fact, this will be identical to the initial conditions following an ultrasonic pulse given by Eq. (2), provided the magnitude and "spread" of the curves are matched. This occurs for some τ_0 and Q_0 such that

$$\tau_0 = \beta/4k, \quad (6)$$

and

$$Q_0 = \pi T_{\max} \beta. \quad (7)$$

Once matched, the subsequent temperature histories will be identical. Thus, we shift the time axis of the line source, and let the variable t represent time following the ultrasonic pulse. Then

$$t = \tau - \tau_0, \quad (8)$$

or, using Eq. (6) and rearranging:

$$\tau = t + \beta/4k. \quad (9)$$

Now, substituting Eqs. (7) and (9) into Eq. (5), we have the desired solution for the temperature following the ultrasonic pulse:

$$T(r,t) = \{T_{\max}/[(4k/\beta)t + 1]\}e^{-r^2/(4kt+\beta)}. \quad (10)$$

At time $t = 0$, Eq. (10) reduces to the initial condition given by Eq. (2). In the commonly used case of a focal region centered on the thermojunction, the distance r is zero and Eq. (10) reduces to

$$T(t) = T_{\max}/(4kt/\beta + 1). \quad (11)$$

This physical situation is pictured in Fig. 2, where the thermocouple and focal region depths are exaggerated for clarity.

II. METHODS

In practice, a focal region is centered on an embedded 51-μm thermojunction by mounting the ultrasonic transducer onto a three-axis positioning device and moving it in a search pattern while pulsing at low power. A maximum temperature pulse is recorded when the peak intensity is aligned with the thermojunction. An intensity profile is then obtained by recording the magnitude of temperature spikes while the focal region is moved in lateral increments across the thermojunction.[6,8] The parameter β is determined from a Gaussian curve fit of the intensity distribution. When the sample temperature has regained equilibrium, a single pulse of ultrasound (usually no longer than 0.1-s duration) creates the initial temperature distribution given by Eq. (2). The temperature history is recorded, and a least squares error comparison is performed on segments of data, against the theoretical temperature decay.

Approximately 1 s is typically required after insonation for viscous heating effects to dissipate. Following this, the thermojunction records the decay of temperature caused by absorption effects alone. A typical temperature history and least squares error curve fit using Eq. (11) is shown in Fig. 3. Once the value of $T_{\max}$ is recovered, the absorption coefficient can be calculated using Eq. (3).

III. RESULTS AND DISCUSSION

A. Comparison with the rate-of-heating method

Both the pulse–decay and rate-of-heating techniques were used to determine the absorption coefficients of soft polyethylene and samples of beef liver, muscle, and kidney cortex, at frequencies between 0.6 and 2.7 MHz. The two techniques were found to agree on measured values within a normal data scatter of $\pm 5\%$, using focal regions with half intensity distances of 3 mm or greater. However, when smaller focal regions were used, conduction losses created significant drop off in the rate-of-heating measurements. In contrast, the thermal pulse-decay technique experimentally produced stable results using half power focal widths of down to 0.9 mm, since this approach explicitly accounts for heat conduction and beam geometry.

Figure 4 shows the results of both rate-of-heating, and pulse-decay measurements on beef liver samples which were frozen, then thawed and kept at 20 °C in degassed saline for the procedures. The absorption coefficient was measured at three frequencies, and at each frequency various lenses were used to control the degree of focusing, and therefore the beamwidth. Measured absorption coefficients were found to be stable with respect to beamwidth using the pulse-decay technique. The rate-of-heating method is sensitive to conduction losses, and measured values therefore drop as the focal area decreases in size. The abscissa of Fig. 4 is scaled in inverse cm for comparison with Ref. 4.

At frequencies approaching or exceeding 10 MHz, nonlinear finite amplitude effects can occur at the intensities required to generate heat in soft tissues.[9] However, focusing can be used to create a high frequency, high intensity region while avoiding the onset of nonlinear shock waves.[10] Since the pulse-decay technique can be used with small focal regions, it thereby enables the measurement of absorption coefficients at higher frequencies than are possible with the rate-of-heating method.

In comparing the absorption measurement techniques,

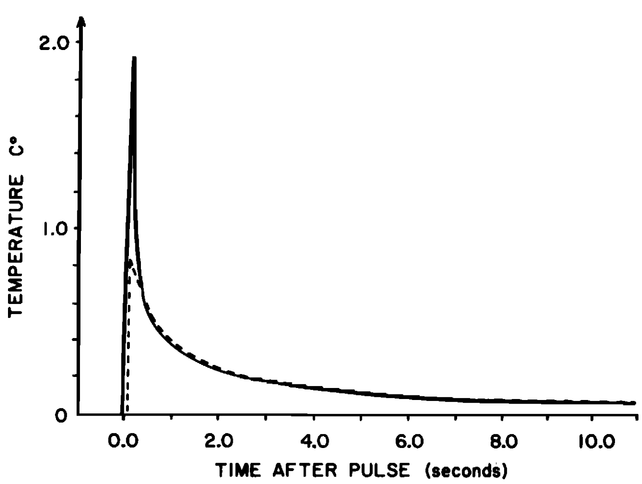

FIG. 3. The temperature history following a 0.08-s ultrasonic pulse. Solid line—measured temperature. Dotted line—curve fit of theory (neglecting viscous heating component) to data in the interval of 2 to 10 s. By the extrapolation back to time $t = 0$, the heating caused by absorption alone is determined.

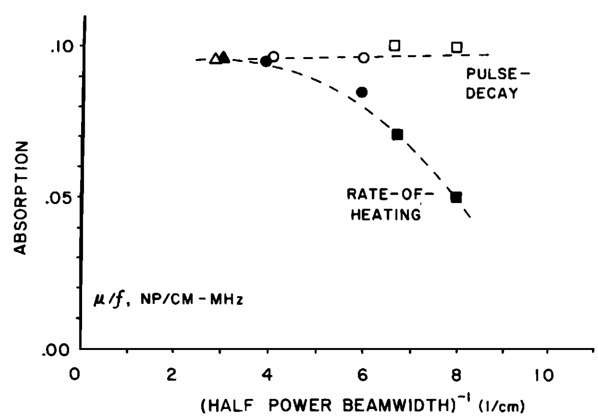

FIG. 4. The absorption coefficient of beef liver samples as a function of beamwidth using the pulse-decay and rate-of-heating methods. The rate-of-heating method does not account for heat loss by conduction and produces low estimates of absorption when small beamwidths are used. Triangles—0.9 MHz. Circles—1.8 MHz. Squares—2.7 MHz.

we also note that the pulse-decay technique requires knowledge of the tissue thermal diffusivity k, as well as the quantity ρc, whereas rate-of-heating measurements require only the latter quantity. This is not a serious disadvantage, since the thermal properties of most soft tissues are relatively close to those of water.[11]

B. Use of the pulse-decay technique with narrow beamwidths

When beamwidths on the order of 1 mm or smaller are used in centered pulse-decay measurements, two effects become important. These are the conduction of heat along the thermocouple wires, and the role of the viscous heating effect in the observed temperature history. The magnitude of heat conducted by the wires during direct insonation can be estimated by following the analyses of Fry and Fry.[1] An estimation of the viscous heating effect can be obtained by assuming that the excess heating around the thermocouple wire creates an initial temperature elevation which is long in the direction of the wire, and narrow but Gaussian shaped across the wire axis into the surrounding medium. This approach poses the temperature decay as resulting from the superposition of two initial Gaussian line sources; one of large radial dimensions (created by the absorptive heating of tissues) and a smaller perpendicular source created by viscous heating effects.

This model neglects the fact that the viscous heating phenomenon occurs only along a finite length of wire within the focal region. The model also neglects the presence of the wire during the temperature decay, and therefore produces an upper bound on the contribution of viscous heating effects to the observed temperature decay.

To proceed, we denote the temperature increase caused by absorption alone with the subscript α. Then, using Eq. (11), the temperature decay of this component at the focal point is given by

$$T_\alpha(t) = \beta_\alpha T_{\alpha\max}/(4kt+\beta_\alpha). \tag{12}$$

Similarly, the temperature decay of the component caused by viscous heating effects is given by

$$T_v(t) = \beta_v T_{v\max}/(4kt+\beta_v), \tag{13}$$

where β_v is defined by the spatial extent of the excess heating, considered to be on the order of the thermocouple wire radius (squared). A measured temperature decay will exhibit the sum of both absorptive and viscous heating effects [Eqs. (12) and (13)]. An important practical consideration is the ratio of these two components as a function of time since it is desirable to use data which is dominated by decay of heat caused by absorption. Accordingly, from Eqs. (12) and (13) the ratio is

$$\frac{T_v(t)}{T_\alpha(t)} = \frac{T_{v\max}}{T_{\alpha\max}}\frac{\beta_v}{\beta_\alpha}\frac{(4kt+\beta_\alpha)}{(4kt+\beta_v)}. \tag{14}$$

An important feature of Eq. (14) is that, for large time t, the ratio asymptotically approaches the value

$$\frac{T_v}{T_\alpha} \cong \frac{T_{v\max}}{T_{\alpha\max}}\frac{\beta_v}{\beta_\alpha}. \tag{15}$$

The ratio of $T_{v\max}/T_{\alpha\max}$ can easily approach ten or higher, as evidenced by pulse-decay curves obtained in soft tissues at low megahertz frequencies. A beamwidth much larger than the thermocouple diameter is then required so that the ratio (β_v/β_α) and hence, Eq. (15) become small over a reasonable interval for data collection.

To compare the decay of these two components, Eq. (14) was plotted using reasonable parameter values. Specifically, the initial temperature ratio was taken to be

$$T_{v\max}/T_{\alpha\max} = 10. \tag{16}$$

Also, thermal diffusivity of soft tissue was used:

$$k = 1.5\times10^{-3}\ \text{cm}^2/\text{s}. \tag{17}$$

The spread β_v of the viscous heating component was taken to be on the order of a 2-mil-diam (51-μm) thermocouple wire (squared). Thus,

$$\beta_v = 6.5\times10^{-6}\ \text{cm}^2. \tag{18}$$

Plots of the decay ratio are shown in Fig. 5, using the above conditions and assuming

$$\beta_\alpha = N^2\beta_v. \tag{19}$$

Thus, the ultrasonic beamwidth is N times the width of the thermocouple wire. From Fig. 5, it is evident that the parameter N must be on the order of 20 or more to ensure that the data recorded between 1 and 10 s is indicative of heating caused by absorption alone. With the parameters used in these calculations, this corresponds to the use of an ultrasonic beamwidth on the order of 1 mm or greater, for use with a 2-mil (51-μm) thermocouple wire. Smaller focal regions require the approach considered in the next section.

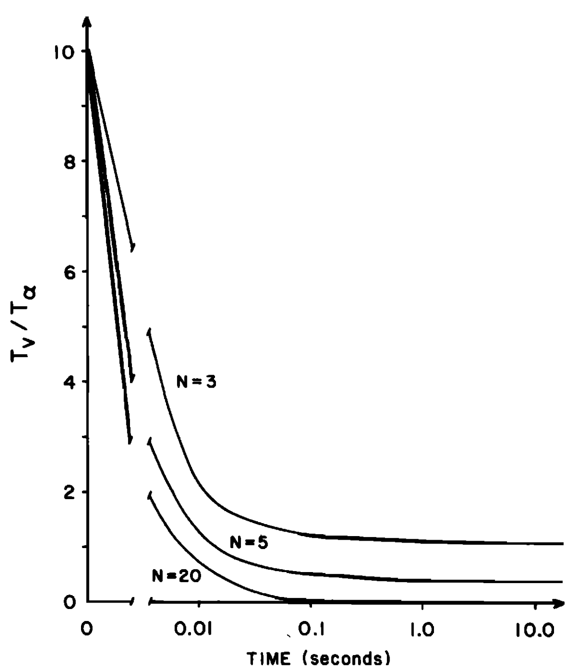

FIG. 5. The ratio of elevated temperatures caused by viscous heating effects, to heating from absorption, using typical parameters. N is the ratio of beam width to thermocouple diameter.

C. Off-axis applications of the pulse decay technique

A significant advantage of the pulse-decay technique is the ability to separate the thermocouple and the region of highest intensity by a lateral distance r. The problems discussed in the previous section are thereby alleviated because, when off-axis measurements are made, the thermocouple wire is at all times subjected to a greatly reduced temperature gradient and ultrasonic intensity. This diminishes measurement errors due to conduction along the thermocouple wires as well as viscous heating at the wire–medium boundary.

Equation (10) gives the temperature history for off-axis measurements. If the separation distance r is increased from zero while holding all other parameters constant, a family of curves is obtained which have the appearance of either a primary temperature decay (for $r^2 \ll \beta$), or a secondary temperature rise (for $r^2 \gg \beta$).

To illustrate the use of off-axis measurements, the temperature histories following pulsed ultrasonic heating were recorded with the focal region positioned at four discrete locations. The measured temperature curves are given in Fig. 6. For these experiments, a focused 1-MHz beam was used with a half-power beamwidth of 0.30 cm ($\beta = 3.24 \times 10^{-2}$ cm^2). A 3-mil (76-μm,)-diam thermocouple was embedded at 3-mm depth in an absorbing rubber potting compound to record temperatures. The heating pulse duration was 0.06 s, and the center of the focal region was moved in a lateral direction to 0.02, 0.17, 0.27, and 0.37 cm from the thermojunction, with pulse-decay measurements taken at each location. Figure 6 shows the experimental data compared with theory. The theoretical curves were obtained by using Eq. (10), with the value of T_{max} obtained from a curve fit of $r = 0.0$ cm data (not shown but closely overlapping the $r = 0.02$ cm curve) between 8 and 10 s.

The nearly centered curve ($r = 0.02$ cm) is initially dominated by viscous heating with the peak temperature rising off the scale of Fig. 6. The precise value of the peak temperature at the thermocouple surface, including the viscous heating effect, is difficult to assess due to the low-pass filtering effects of the thermocouple response and signal amplifiers. In any case, the viscous heating component becomes less significant as the thermocouple wire is positioned in regions of lower intensity. As the separation distance increases, the initial amplitude of the temperature curves decreases and the secondary temperature rise becomes the important characteristic. The overlap of theoretical and experimental curves shows that it is possible to obtain the same value of T_{max} (and therefore absorption) from each of the four experiments. Thus, when conditions rule out the use of centered beam experiments, the off-axis temperature curves provide an alternative means of determining the heat produced by absorption. One caution in using this approach is that the peak temperatures measured off axis can be a factor of 5 to 10 below the peak temperature generated at the center of the focal region. Some caution must therefore be used if thermal damage to a tissue or material is a possibility. However, as shown in Fig. 6, it is possible to make useful measurements with peak temperature elevations of 2 °C or less with a measuring system employing very modest gain.

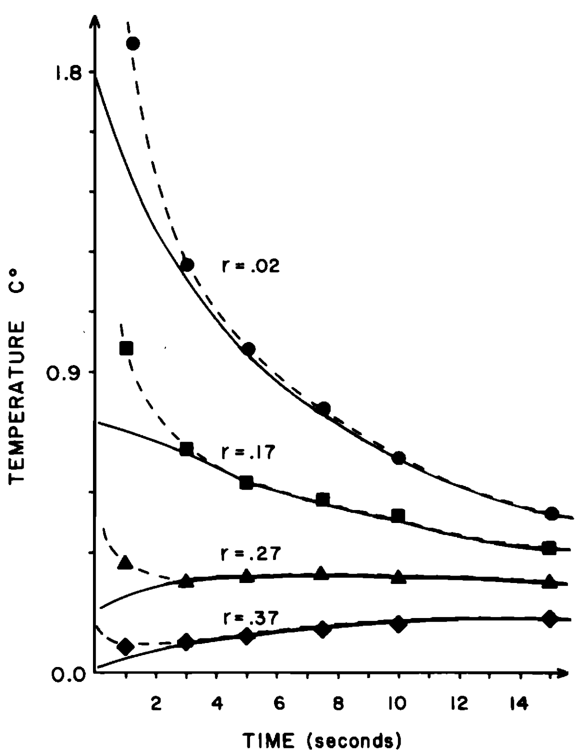

FIG. 6. Temperature histories in an absorbing medium, following a 0.06-s pulse of ultrasound, using the off-axis pulse-delay method. Dotted lines—experimental results. Solid lines—theory, neglecting the viscous heating component. r = the lateral displacement of the focal region relative to the thermojunction, in cm.

D. Intensity variation in the axial direction

The derivation of the pulse-decay technique included the assumptions of infinite tissue length and uniform intensity along the axis of insonation. These conditions will not be met experimentally, and conduction of heat in the axial direction, not accounted for in the original derivation, will create errors in parameter estimation. As a first approximation for analyzing these effects, we will consider intensity variations along the axis of insonation. For focused beams with numerical apertures on the order of, or greater than unity, the focal intensity distribution in the axial direction can be shown to have the same general shape as in the radial dimension, albeit on a scale difference of 10 to 1 or greater.[6,8] Accordingly, let us model a focal intensity distribution, centered on the origin of a cylindrical coordinate system with the z axis aligned with the axis of insonation, as being Gaussian in both axial and radial directions. This gives a good description of the intensity profile around the region of maximum intensity in a nonattenuating medium such as water or saline. Attenuation in a medium will act on this distribution, but we will proceed with the following description as a first approximation. The intensity distribution in a medium is modeled as

$$I(r,z) = I_{max} e^{-r^2/\beta_r} e^{-z^2/\beta_z}, \tag{20}$$

where, because of scale differences described above,

$$\beta_z / \beta_r > 100. \tag{21}$$

If this intensity distribution is created in a medium for a period of time Δt which is short compared to the time required for conduction effects to take place, then the temperature distribution at the end of the pulse is given by

$$T_I(r,z) = T_{\max} e^{-r^2/\beta_r} e^{-z^2/\beta_z}, \tag{22}$$

where the relation between $I_{\max}$ and $T_{\max}$ is given by Eq. (3).

Using this as an initial temperature distribution in a medium, the resulting temperature history at the origin is given by[6]

$$T(r=0,z=0,t>0) = \frac{1}{8(\pi kt)^{3/2}} \int_{-\infty}^{\infty} dz' \int_0^{\infty} r'dr' \int_0^{2\pi} d\theta \, T_I(r',z',t=0) \times e^{-(z'^2+r'^2)/4kt}, \tag{23}$$

which is derived by applying the principle of superposition to the initial temperature distribution, using the solution for an instantaneous point source. By substituting Eq. (22) into the integrand and evaluating, we find that

$$T(r=0,z=0,t) = [T_0/(4kt/\beta_r + 1)][\beta_z/(\beta_z + 4kt)]^{1/2}. \tag{24}$$

This solution is similar to that derived for the "infinite length" case, with the addition of a correction term involving β_z. As a practical example of the importance of this factor, we take the intensity distribution measured in water using a 6-cm-diam aperture beam at 2.7 MHz, focused by a lens having a focal length of 12 cm. The intensity profile has a half-intensity width of approximately one millimeter in the radial direction, and 15 mm along the axis of insonation. Least squares error curve fits yield values of $\beta_r = 0.0053$ cm^2, and $\beta_z = 1.2$ cm^2. To find the magnitude of error in neglecting variations in the axial dimension, we compare the pulse-decay temperatures in a tissue with $k = 1.5 \times 10^{-3}$ cm^2/s, using the value of β_r and β_z given above. At 10 s after the pulse, the calculated temperature using Eq. (24) (axial variation included) is 4% lower than that predicted using Eq. (11) (uniform axial distribution assumed). Since this error is small, Eq. (11) is used in actual computations for intensity distributions of the size described in this example.

More complicated expressions are required to account for the flow of heat at the interface between the absorbing medium and the coupling fluid, and the variation in beam intensity with depth due to absorption. A paper now in preparation will further consider these effects and provide analytical expressions incorporating the more realistic geometry. However, in soft bio-materials with a thermojunction located between 2- and 3-mm tissue depth, these effects are typically negligible for measurements taken over a duration of 1–10 s following the ultrasonic pulse. It is the responsibility of the experimenter to ensure conditions of radial heat flow so that the pulse decay assumptions and resulting equations are valid. This rules out, for example, the application of the pulse-decay technique to isolated tissue samples with dimensions smaller than the beamwidth parameter β, unless extra precautions are taken. In general, the flexibility of the centered or off-axis pulse-decay technique allows for measurements of absorption coefficients over a wide range of experimental conditions.

IV. CONCLUSION

The pulse-decay technique provides an experimental alternative to the rate-of-heating method for measuring ultrasonic absorption coefficients. The pulse-decay technique explicitly accounts for heat conduction in a material as well as the beam geometry, and therefore allows the use of smaller focal regions in absorption measurements. The advantage of the new technique is that it allows for the separation, in time and space, of the heating caused by absorption and the heating caused by viscous effects at the thermocouple–material interface. These features reduce the sources of errors which dominate the rate-of-heating method.

Guidelines for absorption measurements can be summarized as follows. When using broad, unfocused beams the rate-of-heating method probably provides the most straightforward means for measuring absorption. When focused beams with a half-power beamwidth of between 1 and 3 mm are used, the centered pulse-decay method is required because heat conduction effects preclude rate-of-heating measurements. If the ultrasonic beamwidth is smaller than 1 mm, the off-axis pulse-decay measurement is the only means of accurately determining the heating caused by absorption alone. This new technique allows tightly focused beams to be used in absorption measurements, which extends our ability to work with high-frequency, high-power ultrasound.

ACKNOWLEDGMENTS

This work could not have been completed without the generous assistance of Professor P. P. Lele at M.I.T.; and Professor E. L. Carstensen at Rochester. The research was partially funded by grants #CA16111 at M.I.T., and #GM09933 at Rochester. Portions of this report were included in the author's Ph.D. thesis, Department of Electrical Engineering, M.I.T., 1981.

[1] W. J. Fry and R. B. Fry, "Determination of absolute sound levels and acoustic absorption coefficients by thermocouple probes—theory," J. Acoust. Soc. Am. **26**, 294–310 (1954).

[2] W. J. Fry and R. B. Fry, "Determination of absolute sound levels and acoustic absorption coefficients by thermocouple probes—experiment," J. Acoust. Soc. Am. **26**, 311–317 (1954).

[3] K. J. Parker, "Ultrasonic attenuation and absorption in liver tissue," Ultrasound Med. Biol. **8** (1983) (in press).

[4] S. A. Goss, J. W. Cobb, and L. A. Frizzell, "Effect of beam width and thermocouple size on the measurement of ultrasonic absorption using the thermoelectric technique," IEEE Ultrasonics Symp. Proc., 206–211 (1977).

[5] K. J. Parker and P. P. Lele, "The thermal pulse-decay method for determining ultrasound absorption coefficients," IEEE Ultrasonics Symp. Proc. 754–756 (1982).

[6] P. P. Lele, "A simple method for production of trackless focal lesions with focused ultrasound; physical factors," J. Physiol. **160**, 494–511 (1962).

[7] H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids* (Clarendon, Oxford, England, 1947).

[8] T. C. Robinson and P. P. Lele, "An analysis of lesion development in the brain and in plastics by high intensity focused ultrasound at low megahertz frequencies," J. Acoust. Soc. Am. **51**, 1333–51 (1972).

[9] T. G. Muir and E. L. Carstensen, "Prediction of nonlinear acoustic effects at biomedical frequencies," Ultrasound Med. Biol. **6**, 345–357 (1980).

[10] E. L. Carstensen, W. K. Law, N. D. McKay, and T. G. Muir, "Demonstration of nonlinear acoustical effects at biomedical frequencies and intensities," Ultrasound Med. Biol. **6**, 359–368 (1980).

[11] H. Bowman, E. Cravalho, and M. Woods, "Theory, measurement and application of thermal properties of biomaterials," in *Annual Review of Biophysics and Bioengineering* Vol. **4**, edited by L. J. Mullins (Annual Reviews, Palo Alto, CA, 1975).