“Off-the-books” (or “missing”) SS/DOF – a supplement for Chapter 7 of your textbook.

The columns in the experimental designs shown in your textbook provide a convenient way to do ANOM and ANOVA calculations. In particular, they let you determine the average values for each level, the sum of squares, and the degrees of freedom for whatever (factor, interaction, or “error”) is present in the column.

However, in a few cases, some of the “information” (degrees of freedom and the corresponding sum of squares) generated by an experiment do not appear in column form. This occurs most notably for the 18TC screening array (equivalent to a Taguchi L18 array), which contains mixed 2 and 3-level columns. It also occurs when you reduce the number of levels in an existing column (using the “virtual level” technique described in Chapter 7). In these cases, the “missing information” can be obtained by subtracting the sum of the values obtained from the columns from the total calculated for the experiment as a whole. (Note: If you are using a software package, this “missing” information will normally automatically be included in your error estimate.)

Following are four examples. The first three are based on papers drawn from the literature. The fourth is a simplified example showing the required calculations in more detail.

Example 1: An L18 design.

To illustrate, consider the L18 design used by Lin et al. in a recent study (“Optimization of machining parameters in magnetic force assisted EDM based on Taguchi method”, YC Lin, YF Chen, DA Wang, and HS Lee, Journal of Materials Processing Technology, 209 (2009), 3374-3383). Among other things, these authors studied the effect of one 2-level factor and five 3-level factors on the material removal rate (MRR) in an electro-discharge machining operation using a Taguchi S/N ratio. The six factors were loaded into the first six columns, leaving the last two columns empty.

A) To analyze this experiment we start by conducting an ANOM calculation to determine for each column in the array the average values corresponding to each of the levels (m_{-1}, m_0, m_{+1}). [See “ANOM algebra” on page 40 of your textbook.] We then use these values to determine the sum of squares for each column using these values. [Chapter 3, equation 7 for the 2-level column, Chapter 7, equation 2 for the 3-level columns.] Finally, we know the DOF for each column is one less than the number of levels in the column [page 134 in your textbook]. The table below shows the SS and DOF for each of the 8 columns in the array.

Column	Assignment	SS	DOF
1	P	1292.78	1
2	I_P	3759.47	2
3	I_H	23.24	2
4	τ_P	228.89	2
5	V	8.19	2
6	S_V	10.48	2

7	empty → error	47.12	2
8	empty→ error	3.44	2
Sum of all columns		5373.61	15

B) Next calculate the Total SS and DOF values for the experiment. [See "Calculations for the Total", equations (8) and (9) on page 44 of the textbook.] These values are shown in the table below.

From your textbook:

"The total degrees of freedom and SS entered in the ANOVA table are the totals without the contribution of the overall average. Since one degree of freedom is used in the calculation of the overall average, the remaining degrees of freedom for entry into the total row in the ANOVA table is:

$$(\text{Total})\ \text{dof} = \text{n} - 1 \quad (8)$$

The total SS for entry into the ANOVA table is given by:

$$(\text{Total})\ \text{SS} = \sum_{i=1}^{n} (\alpha_i - m^*)^2 \quad (9)"$$

C) The "missing information" (DOF and SS not represented in the form of a column) is just the Total minus the sum of the values form all of the columns, as illustrated in the table below:

	SS	DOF
Total	5492.12	17
Sum of all columns (1-8)	5373.61	15
"Missing" (not in columns)	118.49	2

Now, you can see where all of the information is by combining the data from the two tables:

Column	Assignment	SS	DOF
1	P	1292.78	1
2	I_P	3759.47	2
3	I_H	23.24	2
4	τ_P	228.89	2

5	V	8.19	2
6	S_V	10.48	2
7	empty → error	47.12	2
8	empty→ error	3.44	2
Not in a column	error	118.49	2
Total		5492.12	17

D) Finally, to produce the ANOVA table, we add those values that we will be used for the error estimate (in this case columns 7 & 8 and the "missing" information). The ANOVA table below shows the result.

Source	SS	DOF	MS	F	F_{cr} (95%)
P	1292.78	1	1292.78	45.9	5.99
I_P	3759.47	2	1879.74	66.7	5.14
I_H	23.24	2	11.62	0.4	5.14
τ_P	228.89	2	114.44	4.1	5.14
V	8.19	2	4.09	0.1	5.14
S_V	10.48	2	5.24	0.2	5.14
error	169.05	6	28.18	-	-
Total	5492.12	17			

Example 2: An L18 design used for the specified interaction strategy

One unusual feature of the 18TC (Taguchi L18) design is that it is possible to calculate one interaction effect. The "missing information" (information not represented as a column in the array) corresponds to the interaction between the first two columns.

To illustrate how the interaction may be determined, we will use the same information as in Example 1. In the original study, the authors assumed that all of the interactions were zero and, therefore, included the "not in a column" SS/DOF in their error estimate. However, let us assume that the authors instead decided to study the interaction between the first two columns (in this case (P x I_p). The only difference in our analysis would be that these SS/DOF would appear in the final ANOVA table as a separate row (P x I_p) rather than be added to the error estimate. The ANOVA table below shows the result. {Note that the F_{cr} values change because there are fewer DOF in the error estimate.}

Source	SS	DOF	MS	F	F_{cr} (95%)
P	1292.78	1	1292.78	102.3	7.71
I_P	3759.47	2	1879.74	148.7	6.94

I_H	23.24	2	11.62	0.9	6.94
τ_P	228.89	2	114.44	9.1	6.94
V	8.19	2	4.09	0.3	6.94
S_V	10.48	2	5.24	0.4	6.94
P x I_P	118.49	2	54.25	4.3	6.94
error	50.56	4	12.64	-	-
Total	5492.12	17			

Example 3: Virtual levels

When the "virtual level" technique is used to reduce the number of levels in a column, the DOF (and SS) for the column are reduced. These DOF/SS are then no longer represented in column form, but they can be obtained using the same technique (i.e. subtracting the sum of the columns from the total). To illustrate, consider an extreme case from the literature (Application of Taguchi methods to surface mount processes" , GR Bandurek, J Disney, and A Bendell, Quality and Reliability Engineering International, 4 (1988), 171-181.)

This design starts with a 16TC array (fifteen 2-level columns) and then combines columns to produce five 4-level columns. [Note: The authors of the study actually started with a Taguchi-type design in which this had already been done.] A virtual level type technique was then used to produce an array with two 2-level and three 3-level columns.

A) Perform an ANOM and use the results to calculate the SS for each column. Also determine the DOF for each column (= number of levels – 1). This gives:

Column	Assignment	SS	DOF
1	A (2-level)	13.60	1
2	B (3-level)	7.33	2
3	C (3-level)	0.32	2
4	D (3-level)	0.85	2
5	E (2-level)	0.03	1
Sum of all columns		22.13	8

B) Next calculate the Total SS and DOF values for the experiment. [See "Calculations for the Total", equations (8) and (9) on page 44 of the textbook.] These values are shown in the table below.

C) The “missing information” (DOF and SS not represented in the form of a column) is just the Total minus the sum of the values form all of the columns, as illustrated in the table below:

	SS	DOF
Total	28.31	15
Sum of all columns (1-5)	22.13	8
“Missing” (not in columns)	6.18	7

Now, you can see where all of the information is by combining the data from the two tables:

Column	Assignment	SS	DOF
1	A (2-level)	13.60	1
2	B (3-level)	7.33	2
3	C (3-level)	0.32	2
4	D (3-level)	0.85	2
5	E (2-level)	0.03	1
Not in a column	error	6.18	7
Total		28.31	15

D) Finally, the ANOVA table below shows the result.

Source	SS	DOF	MS	F	F_{cr} (95%)
A (2-level)	13.60	1	13.60	15.4	5.59
B (3-level)	7.33	2	3.67	4.2	4.74
C (3-level)	0.32	2	0.16	0.2	4.74
D (3-level)	0.85	2	0.43	0.5	4.74
E (2-level)	0.03	1	0.03	0.03	5.59
error	6.18	7	0.88	-	-
Total	28.31	15	-	-	-

Example 4: A simple numerical example illustrating the calculations in detail.

To demonstrate the individual calculations, we will use a small design with easy integer values of the characteristic response. Assume that we use a 9TC design to test three 3-level factors (A, B, C) and one 2-level factor (D). We modify the last column in this design to accommodate the 2-level factor using the virtual level technique. This design is shown below, along with the characteristic response for each treatment condition.

TC	A	B	C	D	α_i
1	-1	-1	-1	-1	2
2	-1	0	+1	+1	4
3	-1	+1	0	+1	6
4	0	-1	+1	+1	8
5	0	0	0	-1	10
6	0	+1	-1	+1	12
7	+1	-1	0	+1	23
8	+1	0	-1	+1	34
9	+1	+1	+1	-1	27

A) Calculations based on the columns:

$$\mathrm{m}^* = \frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \alpha_{\mathrm{i}}}{n} = \frac{2+4+6+8+10+12+23+34+27}{9} = 14$$

Column 1 (Factor A)

$$m_{-1}^{A} = \frac{\alpha_1 + \alpha_2 + \alpha_3}{n_{-1}^{A}} = \frac{2+4+6}{3} = 4$$

$$m_{0}^{A} = \frac{\alpha_4 + \alpha_5 + \alpha_6}{n_{0}^{A}} = \frac{8+10+12}{3} = 10$$

$$m_{+1}^{A} = \frac{\alpha_7 + \alpha_8 + \alpha_9}{n_{+1}^{A}} = \frac{23+34+27}{3} = 28$$

$$\mathrm{SS} = \mathrm{n}_{-1}^{\mathrm{A}}\left(m_{-1}^{A} - \mathrm{m}^*\right)^2 + \mathrm{n}_{0}^{\mathrm{A}}\left(m_{0}^{A} - \mathrm{m}^*\right)^2 + \mathrm{n}_{+1}^{\mathrm{A}}\left(m_{+1}^{A} - \mathrm{m}^*\right)^2$$
$$= 3(4-14)^2 + 3(10-14)^2 + 3(28-14)^2 = 300 + 48 + 588 = 936$$

$$\mathrm{DOF} = \#\ \text{of levels} - 1 = 3 - 1 = 2$$

Columns 2 and 3 (Factors B and C):

Analogous to column 1 calculation, above.

Column 4 (Factor D)

$$m_{-1}^{D} = \frac{\alpha_1 + \alpha_5 + \alpha_9}{n_{-1}^{D}} = \frac{2 + 10 + 27}{3} = 13$$

$$m_{+1}^{D} = \frac{\alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \alpha_8}{n_{+1}^{D}} = \frac{4 + 6 + 8 + 12 + 23 + 34}{6} = 14.5$$

$$\mathrm{SS} = \mathrm{n}_{-1}^{\mathrm{D}}\left(m_{-1}^{D} - \mathrm{m}^*\right)^2 + \mathrm{n}_{+1}^{\mathrm{D}}\left(m_{+1}^{D} - \mathrm{m}^*\right)^2$$
$$= 3(13 - 14)^2 + 6(14.5 - 14)^2 = 3 + 1.5 = 4.5$$

$$\mathrm{DOF} = \#\,\mathrm{of\ levels} - 1 = 2 - 1 = 1$$

Column	Assignment	SS	DOF
1	A	936	2
2	B	42	2
3	C	18	2
5	D	4.5	1
Sum of all columns		1000.5	7

Sum of columns:

SS = 936+42+18+4.5 = 1000.5

DOF = 2+2+2+1 = 7

B) Next calculate the Total SS and DOF values for the experiment.

$$(\mathrm{Total})\,\mathrm{dof} = \mathrm{n} - 1 = 9 - 1 = 8$$

$$(\mathrm{Total})\,\mathrm{SS} = \sum_{\mathrm{i}=1}^{\mathrm{n}} (\alpha_{\mathrm{i}} - \mathrm{m}^*)^2 =$$

$$(2-14)^2 + (4-14)^2 + (6-14)^2 + (8-14)^2 + (10-14)^2 + (12-14)^2 + (23-14)^2 + (34-14)^2 + (27-14)^2 =$$
$$144 + 100 + 64 + 36 + 16 + 4 + 81 + 400 + 169 = 1014$$

C) The “missing information” (DOF and SS not represented in the form of a column) is just the Total minus the sum of the values form all of the columns:

SS = SS (Total) – SS (sum of columns) = 1014 – 1000.5 = 13.5

DOF = DOF (Total) – DOF (sum of columns) = 8 – 7 = 1

	SS	DOF
Total	1014.0	8
Sum of all columns (1-4)	1000.5	7
"Missing" (not in columns)	13.5	1

D) Finally, to produce the ANOVA table, we add those values that we will be used for the error estimate (in this case it is just the "missing" information). The ANOVA table below shows the result.

Source	SS	DOF	MS	F	F_{cr} (90%)
A	936	2	468	34.7	18.51
B	42	2	21	1.6	18.51
C	18	2	9	0.7	18.51
D	4.5	1	4.5	0.3	161.45
error	13.5	1	13.5	-	-
Total	1014	8	-	-	-