



## Self-Action Effects in Non-linear Optics

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### Abstract

We provide a brief overview of self action effects in nonlinear optics and examine how quantum noise processes can degrade the stability of laser beam propagation.

### Introduction

Self-focusing was one of the earliest studied non-linear optical processes<sup>1</sup>. Self-focusing can occur if a laser beam of diameter  $D$  containing power  $P > P_{cr}$  where  $P_{cr} = \lambda^2/8n_0 n_2$  propagates through an optical material with a refractive index given by  $n = n_0 + n_2 I$  with  $n_2 > 0$ . For the particular situation in which the laser power  $P$  is exactly equal to the critical power  $P_{cr}$  the effects of diffraction can exactly balance those of nonlinear refraction, and a beam can propagate for a long distance with essentially constant beam diameter. Such a beam is called a self-trapped beam<sup>2</sup> or a spatial soliton<sup>3</sup>.

### Discussion

Another self-action process that can occur is that of small-scale filamentation. Here, small-scale irregularities on a laser wave-front grow as the result of near-forward four-wave mixing processes, eventually producing a highly non-uniform intensity distribution. A classical analysis of this process by Bespalov and Talanov<sup>4</sup> shows that perturbations with transverse wave-vector magnitude  $q$  experience exponential growth with growth rate  $A$  given by

$$A^2 = q^2 / (2k - \chi) (2\chi - q^2 / (2k)) \quad (1)$$

where

$$\chi = n_2 I_0 \omega_0 / c$$

and

$$k = n_0 \omega_0 / c \quad (2)$$

with  $I_0$  representing the incident laser intensity and  $\omega_0$  the laser frequency. Note that maximum gain occurs for perturbations with transverse wave vector  $q_{opt} = (\Omega k - \chi)^{\Omega}$ , in agreement with the model of forward four-wave mixing of Chiao et al.<sup>5,6</sup> Note also that  $A$  becomes imaginary for  $q > 2(k - \chi)^{\Omega} = q_{max}$  implying that perturbations are amplified only if their transverse wave vectors lie in the range  $0 \leq q \leq q_{max}$ .

Filamentation is usually considered to be an undesirable process, and it is useful to develop strategies to prevent its occurrence. Since filamentation can occur as the result of the growth of wave-front perturbations, one strategy for preventing filamentation is to eliminate imperfections from the optical wave-front, for instance by passing the laser beam through a spatial filter. In the ideal situation, in which from a classical point of view the laser wave-front is a perfect plane

wave, the optical wave will still contain fluctuations of a quantum mechanical nature whose origin is the zero-point fluctuations of the electromagnetic field. This situation was recently analyzed by Nagasako et al,<sup>7</sup> who found that the intensity associated with the amplified perturbations after propagating a distance  $z$  into the material could be represented as

$$I_{filament}(z) = I_o^{vac} \exp(2 \Upsilon_o z) \tag{3}$$

where the effective input intensity of the filamentation process associated with vacuum fluctuations is given by

$$I_o^{vac} = 0.05 n_o c / (2\pi) h \omega_o k_o^2 / (4\pi) \Upsilon_o \tag{4}$$

This result can be interpreted by writing  $I_o^{vac}$  as

$$I_o^{vac} = \Delta n(NL, vac) I_o \tag{5}$$

where

$$\Delta n(NL, vac) I_o = n_2 (0.051 \pi n_o) (h \omega_o c / \lambda^3) \tag{6}$$

represents the non-linear change in refractive index of the material system resulting from vacuum fluctuations. In the same manner, the coefficient of  $n_2$  on the right hand side of this equation can be interpreted as the optical intensity associated with zero-point fluctuations.

### Results

On the basis of this model, one can develop a precise prediction for the threshold for the occurrence of filamentation. Over a broad range of conditions, we find that this threshold can be written in terms of the product of the laser intensity with the length of the interaction region, and has the form

$$(IL)_{max} = 15 / (n_2 \omega_o / c) \tag{7}$$

This result shows that for any given material, there is a fundamental limit set by quantum mechanical fluctuation to the intensity-length product of a beam that can propagate stably through the material. A dirty beam can break up sooner, but no beam can propagate farther, with the possible exception of a beam with squeezed quantum fluctuations.

These results are summarized in Table 1, which gives the predicted limit to the *intensity.length* product for a variety of different materials. We see that the limitations imposed by quantum fluctuation become more and more restrictive as the value of  $n_2$  increases.

**Table 1** Value of the *intensity.length* product at the threshold for filamentation for several different materials.

Material	$n_2$ (cm <sup>2</sup> /W)	$(IL)_{max}$ (W /cm)
Vacuum	1 x 10 <sup>-34</sup>	1 x 10 <sup>29</sup>
Air	5 x 10 <sup>-19</sup>	5 x 10 <sup>14</sup>
Fused silica	3 x 10 <sup>-16</sup>	8 x 10 <sup>11</sup>
Carbon-disulfide	3 x 10 <sup>-14</sup>	8 x 10 <sup>9</sup>
Atomic vapour	1 x 10 <sup>-10</sup>	3 x 10 <sup>6</sup>

## Conclusion

In conclusion, we have presented a brief summary of some of the ways in which self-action effects can influence the propagation of a laser beam through a non-linear material. We have presented a careful examination of the process of filamentation, and have found that quantum mechanical fluctuation can play an important role in initiating this process. In detail, we have presented explicit predictions for the maximum intensity that can propagate through a non-linear optical material of specific length without the occurrence of the filamentation process.

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