# Bichromatic local oscillator for detection of two-mode squeezed states of light

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We present a new technique for the detection of two-mode squeezed states of light that allows for a simple characterization of these quantum states even for arbitrary frequency separation between the modes. The proposed technique is based on the use of a bichromatic field as the local oscillator in a balanced heterodyne measurement scheme. By the proper selection of the frequencies of the bichromatic field, it is possible to arbitrarily select the frequency around which the squeezing information is located, thus making it possible to use a low-bandwidth detection system and to move away from any excess noise present in the system. 

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## 1. INTRODUCTION

Two-mode squeezed states (TMSSs) play an important role in quantum information science. They provide a platform for the actual laboratory realization of the continuous-variable entanglement that was the basis of the famous Einstein–Podolsky–Rosen (EPR) paradox.[1] They also provide a platform for a form of quantum communication that was recently proposed,[2] as well as a number of other applications such as continuous-variable teleportation,[3,4] quantum key distribution,[5–7] and verification of EPR correlations.[8,9] The presence of quantum correlations between the modes of a TMSS has made this state a basic tool for quantum information with continuous variables, as discussed in the recent review paper by Braunstein and van Loock.[10]

In spite of the important role of these states in quantum information, their use in the laboratory has been restricted to relatively small frequency separations between the two modes by limitations in the frequency response of available ultralow noise detectors and electronics. In this paper we will describe a new method of heterodyne detection that will largely eliminate this limitation and allow low-noise heterodyne detection at arbitrarily large frequency separation between the two modes. There have been a number of papers[11–13] focused on improving either the temporal or spatial character of the local oscillator (LO) used in heterodyne detection in order to optimize the degree of squeezing measured. However, no attention has been given to the necessary requirements of the detection system needed for such measurements.

The usual detection scheme used for the characterization of a TMSS is based on balanced heterodyne measurements. It requires the use of a LO with a frequency equal to the mean of the frequencies of the two modes of the squeezed field. As a result, the squeezing information is located around the beat-note frequency between the LO and either of the field modes that constitute the squeezed state. In general, the frequency separation between the modes of a TMSS can be arbitrarily large, thus the detection of this quantum state becomes complicated when the frequency is of the order of a few gigahertz and impossible when it reaches tens of gigahertz.

In this paper we present a simple scheme based on a bichromatic local oscillator (BLO) that makes it possible to detect through the use of low-frequency detectors a TMSS regardless of the frequency separation between its modes. Characterization of any TMSS then becomes possible, for example, by using the simple low-frequency design of Gray *et al.*[14] The possibility of using a LO composed of multiple frequencies has been previously proposed[15] and implemented.[16] However, the previous techniques require the use of a cavity to separate the sidebands of the squeezed state and are not concerned with the possibility of detecting a TMSS with arbitrary frequency separation between its modes. As will be shown, by the proper selection of frequencies of the bichromatic field it is possible to arbitrarily select the frequency around which the squeezing information is located, thus making it possible to use a low-bandwidth detection system to characterize a TMSS source. Since the measurement frequency can be arbitrarily selected, it is also possible to move away from any excess noise present in the system. We will show that this technique introduces no excess noise beyond that of the usual technique and that as long as the measurement frequency is less than the squeezing bandwidth the excess noise is not significant.

The rest of the paper is organized as follows: In Section 2 we give a general overview of the basic theory of heterodyne detection for the characterization of a TMSS. Then, in Section 3 we introduce the concept of the BLO and show the advantages and limitations of such a detection technique.

## 2. BALANCED HETERODYNE DETECTION

The most commonly used technique for the detection of a TMSS is balanced heterodyne detection. In general, this technique consists of combining the squeezed field being

 

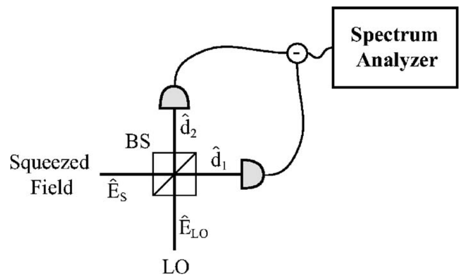


Fig. 1. Balanced heterodyne detection scheme used for the characterization of squeezed light. LO, local oscillator; BS, beam splitter.

measured with a strong LO and detecting each of the resulting fields with a photodetector, as shown in Fig. 1. Combining the two detector signals we obtain the difference signal and analyze the noise in this signal with a spectrum analyzer.

The fields after the beam splitter are given by

$$\hat{d}_1 = t\hat{E}_S + r\hat{E}_{LO},$$

$$\hat{d}_2 = r\hat{E}_S + t\hat{E}_{LO}, \tag{1}$$

where $\hat{E}_S$ and $\hat{E}_{LO}$ are the positive frequency parts of the squeezed and local oscillator fields, and $t$ and $r$ are the transmissivity and reflectivity of the beam splitter. In general, $t$ and $r$ satisfy the relations $|t|^2+|r|^2=1$ and $t^*r=i|rt|$. To have a balanced detection scheme, the beam splitter must satisfy the condition $|t|=|r|=1/\sqrt{2}$, such that the difference signal from the balanced heterodyne detection takes the form

$$\hat{I}_{12} = \hat{d}_1^\dagger\hat{d}_1 - \hat{d}_2^\dagger\hat{d}_2 = i(\hat{E}_S^\dagger\hat{E}_{LO} - \hat{E}_{LO}^\dagger\hat{E}_S). \tag{2}$$

As can be seen from this equation, only the interference terms are left for the balanced case.

For a TMSS, the field takes the form[17]

$$\hat{E}_S = \hat{a}_+ e^{-i(\omega_+ t - \phi_+)} + \hat{a}_- e^{-i(\omega_- t - \phi_-)}, \tag{3}$$

where we have explicitly introduced the phases for the modes of the TMSS, $\phi_+$ and $\phi_-$; the quadratures are defined according to

$$\hat{X} = \frac{1}{2\sqrt{2}}(\hat{a}_+ + \hat{a}_+^\dagger + \hat{a}_- + \hat{a}_-^\dagger),$$

$$\hat{Y} = \frac{1}{i2\sqrt{2}}(\hat{a}_+ - \hat{a}_+^\dagger + \hat{a}_- - \hat{a}_-^\dagger). \tag{4}$$

With these definitions and the properties of the TMSS, the variance of the quadratures can be shown to be given by[18]

$$\langle(\Delta\hat{X})^2\rangle = \frac{1}{4}\left(e^{-2s}\cos^2\frac{\theta}{2} + e^{2s}\sin^2\frac{\theta}{2}\right),$$

$$\langle(\Delta\hat{Y})^2\rangle = \frac{1}{4}\left(e^{-2s}\sin^2\frac{\theta}{2} + e^{2s}\cos^2\frac{\theta}{2}\right), \tag{5}$$

where $s$ is the degree of squeezing and $\theta$ is the squeezing angle. As a result, the variance along the major and minor axes of the squeezing ellipse is given by

$$\langle(\Delta\hat{X})^2\rangle_{\min} = \tfrac{1}{4}e^{-2s},$$

$$\langle(\Delta\hat{Y})^2\rangle_{\max} = \tfrac{1}{4}e^{2s}. \tag{6}$$

In the standard heterodyne technique, the LO is taken to be of the form

$$\hat{E}_{LO} = \hat{b}e^{-i\omega_L t}, \tag{7}$$

and is assumed to be in a coherent state, such that $\langle\hat{b}\rangle = |\beta|e^{i\phi_{LO}}$. To obtain a measurement that is time independent, the frequency of the LO has to be selected between the frequencies of the two modes of the squeezed state, as shown in Fig. 2, that is $\omega_L=(\omega_+ + \omega_-)/2$.

In the ideal case, the variance of the difference signal, Eq. (2), is proportional to the noise in the quadratures of the measured field, thus giving a direct measure of the noise properties of the squeezed field. That is, in the limit that the LO is much stronger than the squeezed state, the variance of the measured signal has the form[18]

$$\langle(\Delta\hat{I}_{12})^2\rangle = 2|\beta|^2\left[e^{2s}\cos^2\left(\chi - \frac{\theta}{2}\right) + e^{-2s}\sin^2\left(\chi - \frac{\theta}{2}\right)\right], \tag{8}$$

where $\chi = \phi_{LO} - (\phi_+ + \phi_-)/2$ is the relative phase difference between the LO and the squeezed field. As can be seen from Eqs. (5) and (8), the measured quadrature variance can be selected by changing the phase of the LO, $\phi_{LO}$. Apart from giving a signal that is proportional to the noise of the squeezed field, the balanced heterodyne detection has the additional advantages of amplifying the measured signal by the strength of the LO, as can be seen in Eq. (8), and of eliminating both the quantum and excess noise contributions of the LO.

Once the heterodyne measurement is performed, the squeezing information is centered around the beat-note frequency between the LO and squeezed field. For the case of a TMSS, the frequency of the LO has to be selected such that $\omega_L=(\omega_+ + \omega_-)/2$. As a result, the squeezing information will be centered around the beat-note frequency $\delta=(\omega_+ - \omega_-)/2$. This technique is especially useful when the frequencies of the modes of the squeezed state are close together. However, in general $\delta$ can be arbitrarily

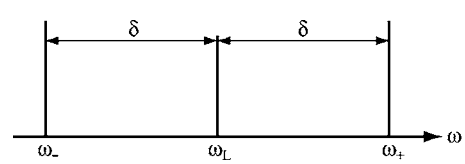


Fig. 2. Frequency components involved in a heterodyne measurement of a TMSS. The frequencies of the two modes of the squeezed state are given by $\omega_-$ and $\omega_+$ while the frequency of the LO is represented by $\omega_L$. To get a measurement that is independent of time, the frequency of the LO has to be selected between the frequencies of the two modes of the squeezed state.

large, limiting this detection technique to cases in which the beat note is of a few gigahertz at the most.

## 3. BICHROMATIC LOCAL OSCILLATOR

As seen in Section 2, one of the main disadvantages of the standard balanced heterodyne technique is that the squeezing information is located around the beat-note frequency $\delta$. Unless the two modes are quite close together, this frequency will be large, making it difficult, and in some cases impossible, to characterize the TMSS. To overcome this limitation, we propose the use of a bichromatic field as the LO in the balanced heterodyne technique described in Section 2. As we will see, by the proper selection of frequencies of the bichromatic field it is possible to perform exactly the same measurement as with the standard technique while making it possible to use a low-bandwidth detection system for characterizing an arbitrary TMSS.

In this new scheme, the local oscillator is taken to be a bichromatic field of the form

$$\hat{E}_{LO} = \hat{b}_1 e^{-i\omega_{L1}t} + \hat{b}_2 e^{-i\omega_{L2}t}, \tag{9}$$

such that the frequency of each of the local oscillators is taken close to one of the modes of the squeezed field, as shown in Fig. 3. That is, $\Delta_1 = \omega_{L1} - \omega_-$ and $\Delta_2 = \omega_{L2} - \omega_+ \ll \Delta = \omega_+ - \omega_-$. In most cases, such a BLO can easily be generated from the laser fields used for the generation of the TMSS with the help of either acousto-optic or electro-optic modulators. However, as will be shown, in principle it is possible to use different laser fields to generate the BLO. In this case it is necessary to actively stabilize the phase between the mode of the squeezed field and the corresponding one from the BLO. Both fields of the BLO are taken to be in coherent states, such that $\langle \hat{b}_1 \rangle = \beta_1$ and $\langle \hat{b}_2 \rangle = \beta_2$. Using this form for the LO, the variance of the measured signal is now given by

$$\begin{aligned}\langle (\Delta \hat{I}_{12})^2 \rangle = &-\{ \langle (\Delta \hat{E}_S)^2 \rangle (\beta_1^2 e^{-i2\omega_{L1}t} + \beta_2^2 e^{-i2\omega_{L2}t} \\ &+ 2\beta_1\beta_2 e^{-i(\omega_{L1}+\omega_{L2})t}) - (\langle \hat{E}_S^\dagger \hat{E}_S \rangle - \langle \hat{E}_S \rangle \langle \hat{E}_S^\dagger \rangle)(|\beta_1|^2 \\ &+ |\beta_2|^2 + \beta_1 \beta_2^* e^{-i(\omega_{L1}-\omega_{L2})t} + \beta_1^* \beta_2 e^{i(\omega_{L1}-\omega_{L2})t}) \\ &+ \text{H.c.}\} + 2\langle \hat{E}_S^\dagger \hat{E}_S \rangle,\end{aligned} \tag{10}$$

where the last term on the right-hand side results from the quantization of the BLO and, as will be seen, can be neglected if the BLO is taken to be sufficiently strong.

For the general case of the BLO shown in Fig. 3, it is necessary to include the image band for each of the modes of the squeezed state.[19,20] Because it is not possible to distinguish between the positive and negative frequency beat-note signals when looking at the heterodyne signal, it is necessary to take into account frequencies that lie symmetrically on either side of the LO, as shown in Fig. 3. The mode opposite to the squeezed field is known as the image band. As a result, the field that is measured is not the TMSS given by Eq. (3); instead it takes the form

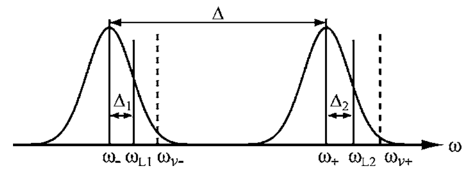


Fig. 3. Frequency components for the characterization of a TMSS using a BLO. The frequencies of the modes of the LO are chosen close to each of the modes of the squeezed field. It is necessary to include in the analysis the influence of the image bands associated with each mode of the squeezed state. The image bands are represented by the dashed lines. The curves centered around the modes of the TMSS represent the squeezing spectrum of the state. The image bands will be correlated as long as they are within the squeezing spectrum.

$$\begin{aligned}\hat{E}_S = &\hat{a}_+ e^{-i(\omega_+ t - \phi_+)} + \hat{a}_{v+} e^{-i(\omega_{v+} - \phi_+)t} + \hat{a}_- e^{-i(\omega_- t - \phi_-)} \\ &+ \hat{a}_{v-} e^{-i(\omega_{v-} - \phi_-)t},\end{aligned} \tag{11}$$

where modes $\hat{a}_{v+}$ and $\hat{a}_{v-}$ are the image bands. The frequency of these modes is such that $\omega_{L1} - \omega_- = \omega_{v-} - \omega_{L1}$ and $\omega_{L2} - \omega_+ = \omega_{v+} - \omega_{L2}$.

The image bands will always correspond to vacuum modes for the idealized case in which each of the modes of the TMSS is treated as a $\delta$ function. However, in a more realistic case, there will be a squeezing bandwidth associated with the squeezed state. In a TMSS quantum correlations are generated between the noise sidebands of the two modes.[21] Due to energy conservation, it is the sidebands on opposite sides of the carrier frequencies, $\omega_-$ and $\omega_+$, that become correlated. This implies that as long as the image bands, $\hat{a}_{v+}$ and $\hat{a}_{v-}$, are within the squeezing spectrum, they will be correlated. We take this correlation into account by assuming that the image bands are also in a TMSS with a degree of squeezing, $s$, that changes with frequency, as shown schematically by the curves centered around $\omega_-$ and $\omega_+$ in Fig. 3. In addition, we assume the squeezing phase to be constant throughout the squeezing spectrum.

The BLO technique becomes useful when $\Delta \gg \Delta_1, \Delta_2$. In this case the bandwidth of the detection system can be designed such that the terms with frequency of the order of $\Delta$ can be neglected. Under this approximation the variance of the difference signal can be shown to be given by

$$\begin{aligned}\langle (\Delta \hat{I}_{12})^2 \rangle = &(|\beta_1|^2 + |\beta_2|^2)(4\sinh^2 s_o + 4\sinh^2 s_v + 4) \\ &+ [4\beta_1\beta_2 e^{-i(\Delta_1+\Delta_2)t} e^{-i(\theta+\phi_+ + \phi_-)} \sinh s_o \cosh s_o \\ &+ 4\beta_1\beta_2 e^{i(\Delta_1+\Delta_2)t} e^{-i(\theta+\phi_{v+}+\phi_{v-})} \sinh s_v \cosh s_v \\ &+ \text{c.c.}],\end{aligned} \tag{12}$$

where $s_o$ and $s_v$ refer to the squeezing parameter at the frequency of the carrier and image band modes, respectively. To make the measurement time independent, it is necessary to select the frequency of the fields of the BLO such that $\Delta_1 = -\Delta_2$. By making $\beta_1 = |\beta_1| e^{i\phi_1}$ and $\beta_2 = |\beta_2| e^{i\phi_2}$ and assuming that $|\beta_1| = |\beta_2| \equiv |\beta|$, we can simplify Eq. (12) to the form

$$\langle(\Delta\hat{I}_{12})^2\rangle = 4|\beta|^2\left[(e^{2s_o}+e^{2s_v})\cos^2\left(\frac{\chi_1+\chi_2-\theta}{2}\right) + (e^{-2s_o}+e^{-2s_v})\sin^2\left(\frac{\chi_1+\chi_2-\theta_o}{2}\right) + \frac{\langle\hat{E}_s^\dagger\hat{E}_s\rangle}{2|\beta|^2}\right], \quad (13)$$

where $\chi_1=\phi_1-\phi_-$ and $\chi_2=\phi_2-\phi_+$ are the relative phase differences between each of the modes of the BLO and the corresponding mode of the TMSS. As described above, the last term in Eq. (13) is due to the quantization of the BLO. This additional term is a phase independent noise term that can limit the minimum amount of squeezing that can be measured. However, it can be neglected when $|\beta|^2 \gg \langle\hat{E}_s^\dagger\hat{E}_s\rangle/2$, that is, the intensity of the BLO is much greater than the intensity of the TMSS being measured. This is usually the case for balanced heterodyne detection.

For the case in which $\Delta_1=\Delta_2=0$ we have that $s_o=s_v$, so that, except for a scaling factor, the result is exactly the same as the one obtained with the usual balanced heterodyne technique [Eq. (8)]. However, the squeezing information is now centered around dc regardless of the frequency separation between the modes of the TMSS, making it possible to use a low-bandwidth detection system.

As is the case when measuring a single-mode squeezed state, it is possible to shift the frequency of the LO to change the measurement frequency and move away from the $1/f$ noise or any technical noise present in the detection system. To see the effect that changing the measurement frequency has on the amount of squeezing that is measured with the BLO technique, we assume the squeezing spectrum to be a Gaussian of the form

$$s(\Omega) = s_o \exp\left(-\frac{\Omega^2}{2\sigma^2}\right), \quad (14)$$

where $\Omega$ is the frequency with respect to the carrier and $\sigma$ is the rms squeezing bandwidth. For this case the amount of squeezing that is measured as a function of the measurement frequency, $|\Delta_{1,2}|$, is shown in Fig. 4. Analogous to the case of single-mode squeezing, the amount of squeezing that is measured is reduced as the measurement frequency is increased. In the limit in which the measurement frequency is much larger than the squeezing bandwidth the image band will be in the vacuum state and will lead to additional noise,[20] such that in the limit of infinite squeezing, $s\rightarrow\infty$, only 3 dB of squeezing will be measured. However, this is generally not the case, as a significant reduction in the measured amount of squeezing does not occur until the measurement frequency gets close to the squeezing bandwidth.

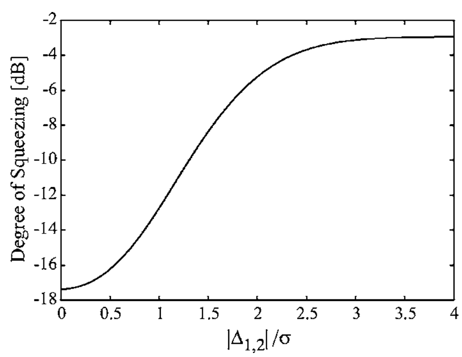


Fig. 4. Effect of changing the measurement frequency on the amount of squeezing measured with the BLO technique. As long as the measurement frequency is less than the squeezing bandwidth $\sigma$ there will not be a significant amount of excess noise.

If the condition $\Delta_1=-\Delta_2$ is not satisfied, then the variance of the measured signal will not be time independent, as can be seen from Eq. (12), and will oscillate between the noise level of the two conjugate quadratures at a frequency $|\Delta_1|-|\Delta_2|$. This is exactly the same situation as the one that occurs when the frequency of the LO is not set exactly between the two modes of the TMSS when using the usual balanced heterodyne technique. As long as the measurement time is small compared to $1/(|\Delta_1|-|\Delta_2|)$ there will be no significant effect from the mismatch in frequencies of the BLO.

One of the things to note from Eq. (13) is that the variance of the quadrature that is measured actually depends on the relative phase difference between each of the modes of the BLO and the corresponding mode of the TMSS, $\chi_1$ and $\chi_2$. As a result, the beat note from the heterodyne detection will contain information regarding these relative phase differences. It is thus possible, in the case in which the required frequencies for the BLO cannot be obtained from the fields used to generate the TMSS, to use different lasers to generate the frequencies for the BLO and to use this information to actively stabilize the relative phases when performing the measurement. The possibility of phase locking the LO and the squeezed field when making a homodyne or heterodyne measurement has been shown for the case of the bright squeezed state by Chelkowski *et al.*[22] and for the vacuum squeezed state by McKenzie *et al.*[23]

Up to now we have considered only the case in which both fields of the BLO have exactly the same amplitude. In practice this is not always possible, so it is necessary to consider the situation in which the amplitudes are not properly matched. To consider this case, the amplitudes of the two fields of the BLO are taken to be $|\beta_1|=|\beta|$ and $|\beta_2|=|\beta|+\delta\beta$, such that Eq. (12) now takes the form

$$\langle(\Delta\hat{I}_{12})^2\rangle = 4|\beta|^2\left\{\left(1+\frac{\delta\beta}{|\beta|}\right)\left[(e^{2s_o}+e^{-2s_v})\cos^2\left(\frac{\chi_1+\chi_2-\theta}{2}\right) + (e^{-2s_o}+e^{-2s_v})\sin^2\left(\frac{\chi_1+\chi_2-\theta}{2}\right)\right] + \frac{1}{2}\left(\frac{\delta\beta}{|\beta|}\right)^2(\cosh 2s_o + \cosh 2s_v)\right\}. \quad (15)$$

As can be seen from Eq. (15), the imbalance in amplitudes leads to an extra source of noise. However, this extra noise term is phase independent and of second order in $\delta\beta/|\beta|$, so that its contribution can easily be made negligible. Thus, to first order, the imbalance in amplitudes has no effect on the measurement other than an overall scaling factor.

The main advantage gained by using a BLO is that, independently of the frequency separation between the modes of the squeezed field, it is still possible to characterize a TMSS with the use of a low-frequency shot-noise limited detector. Another property of using a BLO, as can be seen from Eq. (13), is that it is possible to select the measured quadrature variance by changing the phase of either one of the modes of the BLO, which might have some application in the detection of correlations between the different quadratures of the field.

## 4. CONCLUSION

We have shown that by using a bichromatic local oscillator in a heterodyne detection scheme it is possible to use a low-frequency detection system for the characterization of a TMSS, independently of the frequency separation between the two modes of the squeezed state. This allows for the use of a simple detection system for the characterization of any TMSS source. Analogous to the case of single-mode squeezing, it is possible to arbitrarily select the detection frequency at the expense of extra noise. However, this excess noise does not become significant until the measurement frequency is of the order of the squeezing bandwidth. This freedom to select the desired detection frequency makes it possible to move away from the $1/f$ noise or any technical noise present in the detection system. In principle, it is possible to extend this idea to the general case of multimode squeezed states.

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