Limits on the Time Delay Induced by Slow-Light Propagation

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Motivation: Maximum Slow-Light Time Delay

“Slow light”: group velocities $< 10^{-6} \ c$

Proposed applications: controllable optical delay lines
optical buffers, true time delay for synthetic aperture radar.

Key figure of merit:
normalized time delay $= \frac{\text{total time delay}}{\text{input pulse duration}} \approx \text{information storage capacity of medium}$

Best result to date: delay by 4 pulse lengths (Kasapi et al. 1995)

But data packets used in telecommunications contain $\approx 10^3 \ \text{bits}$

What are the prospects for obtaining slow-light delay lines with $10^3 \ \text{bits capacity}$?
Review of Slow-Light Fundamentals

slow-light medium, \( n_g \gg 1 \)

group velocity: \( v_g = \frac{c}{n_g} \)
group index: \( n_g = n + \omega \frac{dn}{d\omega} \)
group delay: \( T_g = \frac{L}{v_g} = L\frac{n_g}{c} \)

controllable delay: \( T_{\text{del}} = T_g - \frac{L}{c} = \frac{L}{c}(n_g - 1) \)

To make controllable delay as large as possible:

- make \( L \) as large as possible (reduce residual absorption)
- maximize the group index
Generic Model of EIT and CPO Slow-Light Systems

Probe absorption

\[ \alpha(\delta) = \alpha_0 \left(1 - \frac{f}{1 + \frac{\delta^2}{\gamma^2}}\right) \approx \alpha_0 \left[(1 - f) - f \frac{\delta^2}{\gamma^2}\right] \quad \text{where} \quad \delta = \omega - \omega_0 \]

Probe refractive index (by Kramers Kronig)

\[ n(\delta) = n_0 + f \left(\frac{\alpha_0 \lambda}{4\pi}\right) \frac{\delta / \gamma}{1 + \frac{\delta^2}{\gamma^2}} \approx n_0 + f \left(\frac{\alpha_0 \lambda}{4\pi}\right) \frac{\delta}{\gamma} \left(1 - \frac{\delta^2}{\gamma^2}\right) \]

Probe group index

\[ n_g \approx f \left(\frac{\alpha_0 \lambda}{4\pi}\right) \frac{\omega}{\gamma} \left(1 - \frac{3\delta^2}{\gamma^2}\right) \]

Induced delay

\[ T_{\text{del}} \approx \frac{f \alpha_0 L}{2\gamma} \left(1 - \frac{3\delta^2}{\gamma^2}\right) \]

Normalized induced delay \((T_0 = \text{pulse width})\)

\[ \frac{T_{\text{del}}}{T_0} \approx \frac{f \alpha_0 L}{2\gamma T_0} \left(1 - \frac{3\delta^2}{\gamma^2}\right) \]
Limitations to Time Delay

Normalized induced delay

\[ \frac{T_{\text{del}}}{T_0} \approx \frac{f_0 \alpha_0 L}{2 \gamma T_0} \left( 1 - \frac{3 \delta^2}{\gamma^2} \right) \]

**Limitation 1:** Residual absorption limits \( L \); Solution: Eliminate residual absorption

**Limitation 2:** Group velocity dispersion

A short pulse will have a broad spectrum and thus a range of values of \( \delta \). There will thus be a range of time delays, leading to a range of delays and pulse spreading. Insist that pulse not spread by more than a factor of 2. Thus

\[ L_{\text{max}} = 2 \gamma^3 T_0^3 / 3 f_0 \alpha_0 \quad \text{and} \quad \left( \frac{T_{\text{del}}}{T_0} \right)_{\text{max}} = \frac{1}{3} \gamma^2 T_0^2. \]

**Limitation 3:** Spectral reshaping of pulse (more restrictive than limitation 2)

Pulse will narrow in frequency and spread in time from \( T_0 \) to \( T \) where \( T^2 = T_0^2 + f_0 \alpha_0 L / \gamma^2 \).

Thus

\[ L_{\text{max}} = 3 T_0^2 \gamma^2 / (2 f_0 \alpha_0) \quad \text{and} \quad \left( \frac{T_{\text{del}}}{T_0} \right)_{\text{max}} = \frac{3}{2} \gamma T_0. \]

Note that \( \gamma / T_0 \) can be arbitrarily large!
Summary: Fundamental Limitations to Time Delay

- If one can eliminate residual absorption, the maximum relative time delay is
  \[
  \left( \frac{T_{\text{del}}}{T_0} \right)_{\text{max}} = \frac{3}{2} \gamma T_0,
  \]
  which has no upper bound.

- But to achieve this time delay, one needs a large initial (before saturation) optical depth given by
  \[
  \alpha_0 L = \frac{4}{3} \left( \frac{T_{\text{del}}}{T_0} \right)_{\text{max}}^2.
  \]

- For typical telecommunications protocols, the bit rate B is approximately \( T_0^{-1} \) and the required transparency linewidth must exceed the bit rate by the relative delay
  \[
  \gamma = \frac{2}{3} B \left( \frac{T_{\text{del}}}{T_0} \right)_{\text{max}}.
  \]
Numerical Example Showing Large Relative Delay

\[ \alpha_0 L = 7500 \]
\[ 1 - f = 8 \times 10^{-5} \]
\[ \gamma T_0 = 50 \]

Relative time delay \( T_{\text{del}}/T_0 = 75 \).
Specific Example: Electromagnetically Induced Transparency

- The response to the probe field in the presence of the strong coupling field is given by
  \[ \chi^{(1)} = -\frac{\alpha_0 c}{\omega} \frac{[i(\delta - \Delta) - \gamma_{ca}]}{(i\delta - \gamma_{ba})[i(\delta - \Delta) - \gamma_{ca}] + |\Omega_s/2|^2} \]

- The width of the transparency window displays power broadening:
  \[ \gamma = \frac{|\Omega_s/2|^2}{\gamma_{ba}} \]

- The residual absorption can be rendered arbitrarily small \((f \rightarrow 1)\) through use of an intense coupling field.
  \[ f = \frac{|\Omega_s/2|^2}{\gamma_{ca} \gamma_{ba} + |\Omega_s/2|^2} \]

- For \((f \rightarrow 1)\) the normalized delay can be arbitrarily large
  \[ \left( \frac{T_{del}}{T_0} \right)_{\text{max}} = \frac{3}{2} \frac{|\Omega_s/2|^2}{\gamma_{ba}} T_0. \]
We conclude that there are no \textit{fundamental} limitations to the maximum fractional pulse delay [1]. Our model includes gvd and spectral reshaping of pulses.

However, there are serious \textit{practical} limitations, primarily associated with residual absorption.

Another recent study [2] reaches a more pessimistic (although entirely mathematically consistent) conclusion by stressing the severity of residual absorption, especially in the presence of Doppler broadening.

\textit{Our challenge is to minimize residual absorption.}

• Ground state population oscillates at beat frequency $\delta$ (for $\delta < 1/T_1$).

• Population oscillations lead to decreased probe absorption (by explicit calculation), even though broadening is homogeneous.

• Rapid spectral variation of refractive index associated with spectral hole leads to large group index.

• Ultra-slow light ($n_g > 10^6$) observed in ruby and ultra-fast light ($n_g = -4 \times 10^5$) observed in alexandrite by this process.

• Slow and fast light effects occur at room temperature!

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Prospects for Large Fractional Delays Using CPO

\[ \omega + \delta \]
\[ \omega \]

Strong pumping leads to high transparency, large bandwidth, and increased fractional delay.

Boyd et al., Optics Express 2005.
Materials for Large Fractional Delays Using CPO

Material systems under considerations:
Semiconductors (SC), SC heterostructures, dye molecules, atomic vapors.

UC Berkeley group has seen slow light in SC heterostructures (but only at low temperatures) by using an excitonic transition.*

We are presently studying CPO in band-to-band transitions in a SC quantum well structure. We believe that for this system CPO and slow light will persist at room temperature.

DARPA/DSO Project on Applications of Slow Light in Optical Fibers

3 Approaches:

Photonic Crystal Fiber - Utilize EIT effects in gas-filled fiber.

Stimulated Scattering - Raman/Brillouin effect produces gain/delay.

Population Oscillations - Pumped Er-doped fiber with control beam to provide gain/delay.

Our Team:

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Slow Light and Optical Buffers

All-Optical Switch

input ports  switch  output ports

But what happens if two data packets arrive simultaneously?

Use of Optical Buffer for Contention Resolution

Controllable slow light for optical buffering can dramatically increase system performance.
Summary

There are no *fundamental* limitations to the maximum normalized pulse delay.

However, there are serious *practical* limitations, primarily associated with residual absorption.

Exciting possibilities exist for optical buffering and other photonics applications if normalized time delays in the range of 10 – 1000 can be achieved.
Thank you for your attention.

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