Slow-Light Fourier Transform Interferometry

Zhimin Shi and Robert W. Boyd
The Institute of Optics, University of Rochester

at FiO 2007 / LS XXIII, San Jose, CA
10:45 am, September 20th, 2007

Paper: FThG2
Session: General Optical Design and Instrumentation II
Motivation

• A desired spectroscopic interferometer
  – Fine resolution / high sensitivity
  – Compact device size
  – High stability
  – Fast measurement
  – High SNR
  – ...

*Slow light can benefit interferometers in all these aspects!*
Introduction of slow light

- Pulse propagation in a medium

Group velocity

\[ v_g = \frac{d\omega}{dk} = \frac{c}{n_g} \]

Group index

\[ n_g = n + \omega \frac{dn}{d\omega} \]

\[ n_g = 2 \]
Realizations of slow light

- Absorption / gain resonances
- Electromagnetically induced transparency
- Coherent population oscillation
- Other nonlinear effects: SBS, SRS, FWM
- Bandedge effects: PhC, fiber gratings, etc.
- Ring resonators
- ...

Atomic Vapor $n_g \approx 1.76 \times 10^7$

Solid system $n_g \approx 5.2 \times 10^6$

Slow-light interferometry

- M-Z Interferometer

\[ T(\omega) = \frac{1}{2} \left( 1 + \cos \Delta \phi \right) = \frac{1}{2} + \frac{1}{2} \cos \left( \frac{L\omega n(\omega)}{c} \right) \]

\[ \frac{d\Delta \phi}{d\omega} = \frac{L}{c} \left( n + \omega \frac{dn}{d\omega} \right) = \frac{Ln_g}{c} \]
Slow-light interferometry

- Wedged shear interferometer

\[ S = \frac{1}{\Lambda} \frac{dy_m}{d\lambda} = \frac{2L_0 n_g}{\lambda^2} \]

- Spectral sensitivity

\[ R = \frac{\lambda}{\Delta \lambda_{\text{min}}} = \frac{\pi L_0 n_g \sqrt{F}}{\lambda} \]

- Resolving Power
Proof-of-principle experiment

- CdSSe single crystal
- Rhodamine 6G dye laser

Fringe movement as $\lambda$ is tuned

wavelength = 587.5 nm

$S = \frac{1}{\Lambda} \frac{dy_m}{d\lambda} = \frac{2L_0 n_g}{\lambda^2}$

Conventional FT interferometer

\[ I_{\text{out}}(\tau_d) - 0.5 I_{\text{in}} = \int_{-\infty}^{\infty} I_{\text{in}}(\nu) e^{i2\pi\nu\tau_d} d\nu \]

Optical path delay time

\[ \tau_d = nL/c \]
Conventional FT interferometer

• Pros:
  – Only need single detector;
  – High SNR (due to multiplexing);
  – Can achieve high spectral resolution.

Applications in biomedical engineering, metrology, astronomy, radiometry, etc.

• Cons:
  – Need a moving arm;
  – Need a large device size, a large # of data, and a long time of measurement.
Theory of slow-light FTI

- Tunable slow-light medium

\[ n(\nu) = n(\nu_0) + \frac{n_g^{(r)}}{\nu_0} (\nu - \nu_0) \]

\[ n_g(\nu) = n + \nu \frac{dn}{d\nu} \approx n(\nu_0) + n_g^{(r)} \]

Both arms are fixed
Theory of slow-light FTI

- lower arm
\[ \phi_2(\nu) = 2\pi \nu n_2 \frac{L_2}{c} \]
- upper arm
\[ \phi_1(\nu) = 2\pi \nu \left[ n(\nu_0) + \frac{n_g^{(r)}}{\nu_0} (\nu - \nu_0) \right] \frac{L}{c} \]
- phase difference when \( n(\nu_0)L = n_2L_2 \)
\[ \Delta \phi(\nu) = 2\pi (\nu - \nu_0) \frac{n_g^{(r)} L}{c} = 2\pi (\nu - \nu_0) \tau_g \]
- FT relation
\[ I_{\text{out}}(\tau_g) - \frac{1}{2}I_{\text{in}} = \frac{1}{2} \int_{-\infty}^{+\infty} I_{\text{in}}(\nu)e^{i2\pi \nu' \tau_g} d\nu \]
## Comparisons

<table>
<thead>
<tr>
<th></th>
<th>FT pair</th>
<th>$\tau_d$ and $\nu$</th>
<th>$\tau_g$ and $\nu'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay</td>
<td>$\tau_d = nL/c$</td>
<td>$\tau_g = n_g^{(r)} L/c$</td>
<td></td>
</tr>
<tr>
<td>Resolution</td>
<td>$\delta\nu_{\text{min}} = \frac{c}{2nL_{\text{max}}}$</td>
<td>$\delta\nu_{\text{min}} = \frac{c}{2n_g^{(r)}\delta\nu_{\text{min}}}$</td>
<td></td>
</tr>
<tr>
<td>Device size</td>
<td>$L_{\text{max}} = \frac{c}{2n\delta\nu_{\text{min}}}$</td>
<td>$L = \frac{c}{2n_g^{(r)}\Delta\nu_{\text{SR}}/\delta\nu_{\text{min}}}$</td>
<td></td>
</tr>
<tr>
<td># of data pts</td>
<td>$\nu_0/\delta\nu_{\text{min}}$</td>
<td>$\Delta\nu_{\text{SR}}/\delta\nu_{\text{min}}$</td>
<td></td>
</tr>
<tr>
<td>Moving arm</td>
<td>needed</td>
<td>not needed</td>
<td></td>
</tr>
</tbody>
</table>
Experiment setup

Schematic diagram of the experiment setup of an FT-interferometer using a rubidium cell as the slow-light medium.

BS: beam splitter; AOM: acoustic optical modulator; PBS: polarization beam splitter; MZM: Mach-Zehnder modulator; AWG: arbitrary waveform generator.
Experimental results

- **Output intensity vs. group delay**


  Double line separated by 80 MHz
  10 cm long rubidium cell
  Temperature from ~25 ºC to ~150 ºC
Experimental results

- Retrieved input spectrum

\[ \Delta \nu \approx 15 \text{ MHz} \]

100 times better than a conventional FTI with the same size
Summary

• A slow-light Fourier transform interferometer

  – Fine resolution ☺ $\sim 1/n_{g,max}$ (15 MHz)
  – Compact device size ☺ $\sim 1/n_{g,max}$ (10 cm)
  – High stability ☺ No moving parts
  – Fast measurement ☺ $\sim \nu_0/\Delta \nu_{SR}$ reduction in data
  – High SNR ☺ Single detector
Acknowledgement

• Ryan M. Camacho, Praveen K. Vudyasetu, and Prof. John C. Howell at Univ. of Rochester

• Research Group of Nonlinear Optics at Univ. of Rochester. www.optics.rochester.edu/~boyd

• Funding Agencies

Thank you for your attention!