Maximum time delay achievable on propagation through a slow-light medium

Robert W. Boyd
Institute of Optics, University of Rochester, Rochester, New York 14627, USA

Daniel J. Gauthier
Department of Physics, Duke University, Durham, North Carolina 27708, USA

Alexander L. Gaeta
School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14853, USA

Alan E. Willner
Department of Electrical and Computer Engineering, University of Southern California, Los Angeles, California 90089, USA
(Received 2 October 2004; published 1 February 2005)

We consider the question of whether there are any fundamental limits to the maximum time delay that can be achieved for a pulse propagating through a slow-light medium. We include in our analysis what we consider to be the dominant competing effects, and we show that in principle they do not lead to a limitation on the maximum achievable time delay. From this result we conclude that, through optimization, one should be able to delay a pulse by very many pulse lengths; the ability to do so can have important implications for the use of slow-light methods for applications in photonics.

DOI: 10.1103/PhysRevA.71.023801 PACS number(s): 42.50.Gy, 42.65.Tg

There is great interest in methods that can control the propagation velocity of light pulses through material systems [1]. Early work in this area demonstrated that extremely slow group velocities ($v_g \ll c$) and even superluminal velocities ($v_g > c$ or v_g negative) can be obtained.

More recently, interest has turned to the use of slow- and fast-light methods for various applications. Many potential applications require that a pulse of light be delayed by one to several times the pulse duration in a tunable and controllable fashion. Specific applications in the field of high-speed all-optical signal processing that might benefit significantly from such controllable optical delay lines include random-access memory, network buffering, data synchronization, and pattern correlation.

However, it has not been clear what physical processes, if any, can lead to a limitation on the total delay that a pulse can experience. Equivalently, it is not clear whether there are any fundamental limitations on the information storage capacity of a slow-light medium. For instance, the maximum fractional time delay reported to date appears to be the value of approximately 4 reported by Kasapi *et al.* [2]; many experimental studies have achieved only considerably shorter time delays. In this paper, we perform a theoretical study of processes that could limit the total time delay. We conclude that, while these processes can impose severe practical limitations, there is no fundamental limit to how large the time delay can become. Similar conclusions were reached by Harris *et al.* [3] for slow light based on electromagnetically induced transparency (EIT); our treatment differs in that we consider EIT under somewhat different conditions and in that we consider additional processes that can produce slow-light propagation.

Let us first note that the time delay (the group delay) that is experienced by an optical pulse in passing through a material system of length L is given by

$$T_g = \frac{L}{v_g} = \frac{Ln_g}{c}, \tag{1}$$

where

$$n_g = n + \omega \frac{dn}{d\omega} \tag{2}$$

is the group index and n is the conventional (phase) refractive index. It is also useful to introduce the material contribution to the group delay $T_{\mathrm{del}} = T_g - L/c$, which is the difference between the group delay and the delay experienced upon propagation through an equivalent distance in vacuum. This quantity is given by

$$T_{\mathrm{del}} = \frac{L}{c}(n_g - 1). \tag{3}$$

Equation (3) demonstrates that the maximal time delay is determined by the value of the group index and by the maximum possible value $L_{\max}$ of the propagation distance L through the material medium. This maximum propagation distance can be limited by physical processes such as absorption and diffraction effects. Absorption effects can be quite appreciable, because it is often necessary to work at or near an absorption resonance to obtain a large value of the group index. However, techniques such as EIT [4] or coherent population oscillations (CPO) [5–7] are often used in slow-light experiments to minimize or even essentially eliminate the effects of material absorption. Diffraction effects can also limit the effective value of $L_{\max}$ to the Rayleigh range of the incident laser beams. However, diffraction effects can be eliminated entirely by working in an optical fiber or other guided-wave structure.

There are other potential limitations to the maximum time delay imposed by the spectral variation of the optical properties of the material system. Since a light pulse necessarily has a nonvanishing spectral width, these effects are intrinsic to the propagation of pulses through a slow-light medium. To treat these effects explicitly, we consider the propagation of a pulse whose frequency is close to that of a transparency window, such as that created by EIT or CPO. For the present, we assume that the shape of transparency window corresponds to a Lorentzian-shaped dip in the frequency dependent absorption coefficient; later in this paper we consider the consequences of non-Lorentzian line shapes. We thus assume that the absorption coefficient of this material can be described by the expression

$$\alpha(\delta) = \alpha_0\left(1 - \frac{f}{1+\delta^2/\gamma^2}\right) \approx \alpha_0\left[(1-f) - f\frac{\delta^2}{\gamma^2}\right], \quad (4)$$

where α_0 is the value of the background absorption, $\delta = \omega - \omega_0$ is the detuning of the optical frequency ω from the resonance frequency ω_0, and γ is the linewidth of the transparency window. In much of the ensuing analysis, we will use the second (approximate) form, which is reasonably reliable for $\delta < \gamma$. In these equations, f is a parameter that describes the depth of the transparency window; complete transparency at line center occurs for $f=1$. We allow this possibility because complete transparency cannot be obtained in many practical situations.

According to the Kramers-Kronig relations, there will be a contribution to the refractive index associated with this absorption feature so that

$$n(\delta) = n_0 + f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\delta/\gamma}{1+\delta^2/\gamma^2} \approx n_0 + f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\delta}{\gamma}\left(1-\frac{\delta^2}{\gamma^2}\right), \quad (5)$$

where n_0 is the background index; under most situations of interest the contribution of n_0 to the group index is very much smaller than that of the second term and will be dropped from the ensuing analysis. From the definition (2) of the group index, we immediately find that

$$n_g \approx f\left(\frac{\alpha_0\lambda}{4\pi}\right)\frac{\omega}{\gamma}\left(1-\frac{3\delta^2}{\gamma^2}\right). \quad (6)$$

We then find that the material delay of Eq. (3) is given by

$$T_{\mathrm{del}} \approx \frac{f\alpha_0 L}{2\gamma}\left(1-\frac{3\delta^2}{\gamma^2}\right) \quad (7)$$

and that the fractional (or normalized) group delay for a pulse of length T_0 is given by

$$\frac{T_{\mathrm{del}}}{T_0} \approx \frac{f\alpha_0 L}{2\gamma T_0}\left(1-\frac{3\delta^2}{\gamma^2}\right). \quad (8)$$

Let us now examine the physical processes that might limit the maximum value of the fractional delay. One such process is group-velocity dispersion. We see from Eq. (8) that the fractional delay will be different for different frequency components of a spectrally broad pulse. A pulse of duration T_0 will have a frequency spread of the order of $1/T_0$. For a pulse centered on the transparency window, the spread in fractional group delay will be the difference in group delays for $\delta=0$ and for $\delta \approx 1/T_0$, and is given by

$$\Delta\left(\frac{T_{\mathrm{del}}}{T_0}\right) \approx \frac{3f}{2}\frac{\alpha_0 L}{\gamma^3 T_0^3}, \quad (9)$$

and represents pulse spreading due to second-order group-velocity dispersion at the center of the transparency window. If we restrict the allowed temporal spread in this quantity to a value of unity (that is, the pulse is allowed to broaden in time by no more than a factor of 2 in passing through the medium), we find that the length of the interaction region is limited to a maximum value of $L_{\max} = 2\gamma^3 T_0^3/3f\alpha_0$.

Through use of Eq. (8), we find that the fractional delay is limited by this process to the value

$$\left(\frac{T_{\mathrm{del}}}{T_0}\right)_{\max} = \frac{1}{3}\gamma^2 T_0^2. \quad (10)$$

We emphasize that there is no limit on how large the quantity γT_0 can become. Indeed, one would usually want the pulse duration T_0 to be long compared to $1/\gamma$ so that the entire spectrum of the pulse fits within the transparency window. Note also that the limit $\gamma T_0 \gg 1$ is consistent with the limit of validity of the approximate form of Eq. (4), which was used in this argument. We also see that there is no formal dependence of Eq. (10) on the fractional transparency f. However, in practical situations the time delay would be limited by strong absorption, unless f is nearly equal to unity.

Another potential limiting process is the spectral reshaping of the incident pulse due to the frequency dependence of the absorption coefficient of the material, as discussed previously by Harris *et al.* [3], Cao *et al.* [8], and Macke and Segard [9]. To treat this effect mathematically, let us assume a Gaussian spectral dependence of the incident pulse such that

$$A(\delta) = A_0 e^{-(1/2)\delta^2 T_0^2}. \quad (11)$$

After propagating through the medium, the pulse spectrum will be given approximately by

$$A(\delta) = A_0 e^{-(1/2)\delta^2 T_0^2} e^{-f\alpha_0(\delta^2/\gamma^2)L} e^{ikL}, \quad (12)$$

where $k = (\omega/c)[n_0 + f(\alpha_0\lambda/4\pi)(\delta/\gamma)]$. Such a pulse will have a duration T given by

$$T^2 = T_0^2 + f\alpha_0 L/\gamma^2. \quad (13)$$

If we argue as above that the propagation distance is limited by the constraint that the pulse length not broaden by more than a factor of 2, we find that $L_{\max} = 3T_0^2\gamma^2/(2f\alpha_0)$. By introducing this value into Eq. (8), we find that the maximum normalized delay is limited to

$$\left(\frac{T_{\mathrm{del}}}{T_0}\right)_{\max} = \frac{3}{2}\gamma T_0. \quad (14)$$

As noted above, the quantity γT_0 is necessarily greater than unity. Thus, Eq. (14) constitutes a more restrictive condition than does Eq. (10). Since the quantity γT_0 possesses no obvious physical upper bound, this treatment shows that long

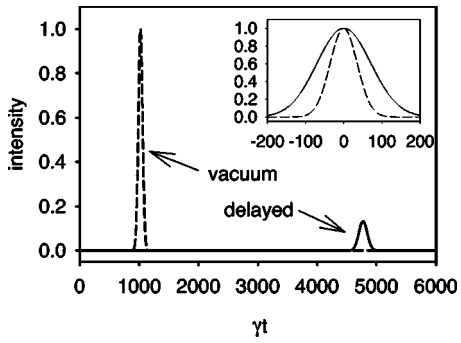

FIG. 1. Numerical simulation demonstrating a large pulse delay in a slow-light medium. The intensity evolution of a Gaussian pulse emerging from the medium for the case of vacuum (dashed line) and a slow-light medium (solid line) with $\alpha_0 L = 7500$, $1-f = 8 \times 10^{-5}$, and $\gamma T_0 = 50$ is shown. The relative time delay is $T_{\mathrm{del}}/T_0 = 75$, as predicted by Eq. (14). The inset shows the vacuum and delayed pulses overlaid so that their peaks coincide; it is seen that the delayed pulse is approximately twice as wide and remains highly symmetric.

time delays and long fractional time delays should be achievable upon propagation through a slow-light medium. Note, however, that to achieve the delay given by Eq. (14), it is necessary that the medium possess a reasonably large optical depth (before saturation) given by $\alpha_0 L = (4/3)(T_{\mathrm{del}}/T_0)^2_{\mathrm{max}}$. From Eq. (14), we can also establish a relationship among the resonance width γ, the bit rate B, and the maximum time delay. In a typical communication system, we have that $B \simeq T_0^{-1}$, so that

$$\gamma = \frac{2}{3} B \left(\frac{T_{\mathrm{del}}}{T_0} \right)_{\mathrm{max}}. \tag{15}$$

Thus, the required resonance width must be larger than the bit rate by a factor of the order of the desired maximum normalized time delay. However, it should be noted that the data signal bandwidth depends on the specific modulation format and coding scheme employed.

To illustrate these points, we show in Fig. 1 the results of a numerical simulation of pulse propagation though a slow-light medium. The simulation was performed by solving numerically the reduced wave equation with optical responses given by Eqs. (4) and (6) using a Fourier transform method. In this example, a pulse is delayed by 75 pulse lengths under realistic laboratory conditions. The pulse undergoes some attenuation and some broadening, but the overall integrity of the pulse is well preserved. The input pulse width is $T_0 = 50\gamma$, the residual absorption is such that $1-f = 8 \times 10^{-5}$, and the interaction path length is such that $\alpha_0 L = 7500$. The residual absorption at line center is thus equal to $(1-f)\alpha_0 L = 0.6$. Based only on the residual absorption, one would expect a transmission of 0.55, whereas the simulation shows a peak transmission of 0.13. The lower peak height in the simulation arises from two factors: (1) the pulse is wider, and thus the peak is lower; and (2) there is some absorption of the tails of the spectrum [10].

Now that we have established some of the basic principles governing the time delay possible upon propagation through slow-light media, we turn next to an analysis of specific processes that can lead to slow propagation of light.

Electromagentically induced transparency. Much of the initial research on slow-light pulse propagation was conducted using an EIT resonance. The transparency window is approximately Lorentzian of the form given by Eq. (4) under appropriate conditions. To make a comparison with our analysis, consider the complex linear susceptibility for the resonance, which is given by [1]

$$\chi^{(1)} = -\frac{\alpha_0 c}{\omega} \frac{[i(\delta-\Delta)-\gamma_{ca}]}{(i\delta-\gamma_{ba})[i(\delta-\Delta)-\gamma_{ca}]+|\Omega_s/2|^2}, \tag{16}$$

where γ_{ba} is the coherence dephasing rate for the electronic transition driven by an intense coupling field of Rabi frequency Ω_s, Δ is the detuning of the coupling field from this transition, and γ_{ca} is the ground-state dephasing rate. For a dilute medium, the refractive index and absorption coefficient are given by $\mathrm{Im}[\chi^{(1)}]/c$. Under the assumptions $\Delta = 0$, $\gamma_{ca} \ll \gamma_{ba}$, $\Omega_s \ll \gamma_{ba}$, and $\delta \ll \gamma_{ca}$, we find

$$f = \frac{|\Omega_s/2|^2}{\gamma_{ca}\gamma_{ba}+|\Omega_s/2|^2}, \quad \gamma = \frac{|\Omega_s/2|^2}{\gamma_{ba}}. \tag{17}$$

Thus, the first of Eqs. (17) demonstrates that it is possible to achieve a high degree of transparency ($f \to 1$) when $|\Omega_s/2|^2 \gg \gamma_{ca}\gamma_{ba}$, in which case the fractional time delay of Eq. (14) becomes

$$\left(\frac{T_{del}}{T_0} \right)_{\mathrm{max}} = \frac{3}{2} \frac{|\Omega_s/2|^2 T_0}{\gamma_{ba}}. \tag{18}$$

Since the EIT resonance is non-Lorentzian, there is the possibility of partially canceling the pulse distortion resulting from frequency dependent absorption, thereby increasing L_{max} and obtaining a time delay greater than that given by Eq. (18). Indeed, we find that it is possible to cancel the lowest-order contribution to the distortion by setting

$$\left| \frac{\Omega_s}{2} \right|^2 = \frac{\gamma_{ca}^3}{\gamma_{ba}}. \tag{19}$$

However, under this condition one finds that $f \simeq \gamma_{ca}^2/\gamma_{ba}^2$, which approaches zero under normal EIT conditions. Therefore, the pulse will experience large absorption, offsetting any benefit of canceling the lowest-order contribution to the pulse distortion.

We note that Harris *et al.* [3] previously investigated the maximum relative time delay for an EIT system. They assumed that residual absorption at the center of the EIT resonance is the limiting effect so that $L_{\mathrm{max}} = [2(1-f)\alpha_0]^{-1}$, resulting in a limited time delay. However, we note that it is possible to achieve very high transparency ($f \to 1$) in an EIT system so that the frequency dependence of the absorption is the dominant source of pulse distortion, as we assumed in our model discussed above.

Coherent population oscillations. Another process that has been used to produce slow light is coherent population oscillations (CPO) [11–14]. CPO lead to transparency windows for which the absorption coefficient has a width of the order of T_1^{-1}, where T_1 is the population relaxation time of

the material system. CPO have been shown to lead to slow-light propagation at room temperature, which is possible because this process is largely insensitive to the dephasing of the atomic coherence. The CPO process was described theoretically by Schwartz and Tan [11]. Treatments closely related to that of the present work include Refs. [14,15].

The experiments on slow propagation in ruby [5] and alexandrite [6] were both conducted in the rate-equation limit, that is, under conditions such that the dephasing rate T_2^{-1} was much larger than both the population relaxation rate T_1^{-1} and the Rabi frequency $\Omega = 2\mu E/\hbar$. In this limit, a simple analytic expression for the shape of the transparency window can be obtained [see Eq. (15) of Ref. [14]]. One finds that

$$\alpha(\delta) = \alpha_0 \left[1 - \frac{I}{\delta^2 T_1^2 + (1+I)^2} \right], \tag{20}$$

where $I = \Omega^2 T_1 T_2$ is the saturation parameter, which can be interpreted as the intensity of the pump laser normalized by the saturation intensity of the material medium. We see that the spectral hole always has a Lorentzian shape. Thus, the model of slow-light propagation developed above is directly applicable. Therefore, by comparison of Eq. (20) with Eq. (4), we find that

$$f = \frac{I}{(1+I)^2}, \quad \gamma = \frac{1}{T_1}(1+I). \tag{21}$$

Since f takes on its maximum value of $1/4$ at $I=1$, complete transparency is never achieved. However, this conclusion is valid ony within the rate-equation limit; outside of this limit complete transparency can be obtained [14]. The fact that complete transparency cannot be obtained can be understood in terms of the result (21) that the linewidth shows power broadening as the factor $(1+I)$. Conventional (non-CPO) power broadening shows a dependence of $(1+I)^{1/2}$.

The group index at line center is found from Eq. (6) to be given by

$$n_g = \frac{1}{2}\alpha_0 c T_1 \frac{I}{(1+I)^3}. \tag{22}$$

Since there is substantial residual absorption for the CPO resonance, we set the maximum propagation distance $L_{\max}$ equal to the inverse of the absorption given by Eq. (20) evaluated at the center of the transparency window. We find that

$$\left(\frac{T_{\mathrm{del}}}{T_0} \right)_{\max} = \frac{T_1}{2T_0} \frac{I}{(1+I)(1+I+I^2)}. \tag{23}$$

The second factor can never be larger than 0.19, which occurs for $I=0.57$. Since T_0 must be greater than approximately T_1 in order for the spectrum of the pulse to fit within the transparency window, this model predicts that the fractional delay cannot exceed approximately 10%, in agreement with the best reported experimental results [5,6].

Summary. We have developed simple physical arguments which suggest that there is no fundamental limit to the fractional time delay experienced by an optical pulse propagating through a slow-light medium. Delays of four pulse widths have already been observed in the case of an EIT slow-light medium [2]. To date, only delays smaller than unity have been observed in CPO media. The analysis presented here suggests that the small fractional delays occur as the result of operating in the rate-equation limit. There is the expectation that much longer delays, which are in principle unlimited in magnitude, can be obtained in CPO media when operating outside of this limit, in that theoretical models predict that complete transparency can be obtained. We also emphasize that limitations imposed by signal attenuation are relevant only to the case of propagation through an absorbing medium. For propagation through a saturable amplifier, strong modification of the group velocity unrestricted by absorption should occur.

We thank Mikhail Lukin, Nick Lepeshkin, and Matthew Bigelow for valuable discussions of the subject matter of this paper. This work was supported by DARPA/DSO.

[1] R. W. Boyd and D. J. Gauthier, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 2002), Vol. 43, pp. 497–530.
[2] A. Kasapi, M. Jain, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **74**, 2447 (1995).
[3] S. E. Harris, J. E. Field, and A. Kasapi, Phys. Rev. A **46**, R29 (1992).
[4] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).
[5] M. S. Bigelow, N. N. Lepeshkin, and R. W. Boyd, Phys. Rev. Lett. **90**, 113903 (2003).
[6] M. S. Bigelow, N. N. Lepeshkin, and R. W. Boyd, Science **301**, 200 (2003).
[7] P.-C. Ku *et al.*, Opt. Lett. **29**, 2291 (2004).
[8] H. Cao, A. Dogariu, and L. J. Wang, IEEE J. Sel. Top. Quantum Electron. **9**, 52 (2003).
[9] B. Macke and B. Segard, Eur. Phys. J. D **23**, 125 (2003).
[10] The conditions assumed for this simulation could be obtained in an EIT system of the sort described by Eq. (16) through the use of the following parameters: $\gamma_{ca} = \gamma_{ba}/(5\times 10^4)$, $\Omega/\gamma_{ba} = 1$, and $\alpha_0 L = 7500$.
[11] S. E. Schwartz and T. Y. Tan, Appl. Phys. Lett. **10**, 4 (1967).
[12] L. W. Hillman, R. W. Boyd, J. Krasinski, and C. R. Stroud Jr., Opt. Commun. **45**, 416 (1983).
[13] M. S. Malcuit, R. W. Boyd, L. W. Hillman, J. Krasinski, and C. R. Stroud Jr., J. Opt. Soc. Am. B **1**, 73 (1984).
[14] R. W. Boyd, M. G. Raymer, P. Narum, and D. J. Harter, Phys. Rev. A **24**, 411 (1981).
[15] R. W. Boyd, *Nonlinear Optics* (Academic, Amsterdam, 2003). See, especially, Eq. (6.6.34).