

Response to “The physics of ghost imaging—nonlocal interference or local intensity fluctuation correlation?”

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Shih has objected to the semiclassical explanation for lensless pseudothermal ghost imaging—whose measurement configuration is shown schematically in Fig. 8 of [1]—that is reviewed by Shapiro and Boyd. In Shih’s view, the only valid explanation for lensless pseudothermal ghost imaging is nonlocal two-photon interference, i.e., a purely quantum phenomenon. Shapiro and Boyd also develop that explanation for the lensless pseudothermal ghost image, where they show it to be equivalent, in its quantitative predictions, to the speckle-correlation interpretation arising from semiclassical photodetection theory. Thus, Sect. 1 of [1] concludes: “That the semiclassical and quantum theories yield identical measurements statistics in this case means that pseudothermal ghost-imaging experiments *cannot* distinguish between these two interpretations, even though we know that light is intrinsically quantum mechanical.”

Shih begins by observing that Shapiro and Boyd fail to compare lensless pseudothermal ghost imaging of Scarcelli et al. [2], with the lens-based pseudothermal ghost

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imaging of Gatti et al. [3,4];¹ according to Shih's Klyshko-picture development, the former is a quantum phenomenon, namely nonlocal two-photon interference, whereas the latter is due to classical speckle-to-speckle correlation. With the exception of its reporting of the turbulence-limited ghost imaging behavior seen by Dixon et al. [5], Shapiro and Boyd eschew lens-based ghost imaging, because one goal of their paper is to resolve the controversy about the physical interpretation of lensless pseudothermal ghost imaging for vacuum propagation, see, for example, [2] and [6], which appeared in the *same* issue of Phys. Rev. A. Thus the main bone of contention between Shih's comment and the Shapiro and Boyd paper—and the only point to which we need respond—is whether the lensless pseudothermal ghost image can be explained *quantitatively* via semiclassical photodetection theory, i.e., classical electromagnetic waves and shot-noise limited photodetection. In this regard, Shih's lamenting Shapiro and Boyd's failure to contrast lensed and lensless configurations for pseudothermal ghost-image formation can be seen to be irrelevant. He correctly explains the former as being due to the presence of correlated speckles on the object and reference planes that were obtained from imaging the source's speckles, so we need not repeat that argument. However, [1] shows that in the latter (lensless) case, correlated speckles cast on the object and reference planes by vacuum propagation from the pseudothermal source also provide a semiclassical interpretation for ghost-image formation in that configuration, indeed one that is quantitatively equivalent to Shih's nonlocal two-photon interference explanation from quantum theory.

In support of his assertion regarding the failure of semiclassical photodetection theory to account for formation of the lensless pseudothermal ghost image, Shih presents an argument that the pseudothermal source in that configuration does *not* project a speckle pattern in the imager's object and reference measurement planes that is "point-to-point" correlated with the source-plane speckle pattern. From this statement he concludes that the semiclassical theory cannot give rise to the ghost image.² We feel that Shih's conclusion misses the point. It is irrelevant that the speckle patterns in the object and reference planes are not correlated with the one in the source plane. What is relevant is that the speckle pattern in the object plane is correlated with the one in the reference plane, because this correlation is what creates the pseudothermal ghost image. In this response we shall also show that Shih's classical analysis is at odds with well-accepted results from coherence theory. Before doing so, however, we

¹ In both lensless and lensed pseudothermal ghost imaging, a laser beam is rendered spatially incoherent by passage through a rotating ground-glass diffuser and is then separated into two identical beams by a 50–50 beam splitter. In the lensless configuration, the outputs from the beam splitter propagate over identical L - m -long paths until they reach the object and the reference planes, respectively. In the lensed configuration, however, the speckle pattern at the output of the ground glass is imaged onto the object and reference planes. See Fig. 1 of Shih's comment.

² In an aside, Shih notes that the nonlocal two-photon interference theory correctly predicts the turbulence-free nature of the lensless pseudothermal ghost image, as seen in the experiments from [7]. We have shown [8], however, that both those experiments, and the accompanying theory, were limited to the regime in which the source size is smaller than the source-plane turbulence coherence length, wherein the semiclassical theories of Cheng [9] and Hardy and Shapiro [10] show that the lensless pseudothermal ghost image is free from any turbulence-induced loss of spatial resolution. Moreover, the theory we used in [8] is nonlocal two-photon interference.

feel it is important to reprise the metalessons from Shapiro and Boyd, because they deeply undercut the overall theme of Shih's comment.

1 The metalessons from Shapiro and Boyd

The following five metalessons from [1] are repeated here.

1.1 All optical imaging is quantum

As stated in Sect 2.2 of [1], "Light is intrinsically quantum mechanical, and high-sensitivity photodetection is a quantum measurement capable of revealing nonclassical features in its illumination. Therefore, *all* optical imaging phenomena are fundamentally quantum mechanical."

1.2 Semiclassical and quantum photodetection may yield identical statistics

From Sect. 2.2 of [1] we also have: "it has long been known [11–13] that quantitatively identical measurement statistics result in all three basic photodetection paradigms—direct detection, homodyne detection, and heterodyne detection—when the illumination's quantum state is a coherent state or a classically-random mixture of coherent states and the semiclassical theory of photodetection is employed, i.e., the field is treated classically and the discreteness of the electron charge in the detector leads to shot-noise generation."

1.3 Classical-state light and classical versus quantum imaging need careful definitions

Because of the equivalence between the quantitative predictions of semiclassical and quantum photodetection theories when the illumination is a coherent state or a random mixture of coherent states, Sect. 2.1 of [1] introduces the following *definition* of a classical state: "a quantum state is classical if its quantum photodetection statistics are identical to those obtained from semiclassical photodetection using an appropriate random process." Sect. 2.2 of [1] then adds the following *definitions* of classical and quantum imaging: "in all that follows we shall say that a ghost-imaging configuration is 'classical' if its measurement statistics are correctly described by the use of semiclassical photodetection theory, and that the configuration is 'quantum' if a correct description of its measurement statistics *requires* the use of quantum photodetection theory."

1.4 Pseudothermal light is classical-state light

An ideal laser produces coherent-state light. Linear propagation of laser light through vacuum, the atmosphere, optical fiber, through lenses, beam splitters, ground-glass

diffusers, etc., *all* remain within the realm of classical-state light. Thus Sect. 2.1 of [1] points out that: “Pseudothermal light is well modeled by a zero-mean Gaussian state whose phase-sensitive covariance vanishes, but whose phase-insensitive (normally-ordered) covariance function is nonzero. All such states are classical [14].”

1.5 Quantum mutual coherence propagates through vacuum like classical mutual coherence

Vacuum propagation of classical mutual coherence follows from the Huygens-Fresnel principle via standard random process techniques for propagation of correlation functions through linear systems. Section 2.3 of [1] explains that: “It turns out that the *same* quasimonochromatic Huygens-Fresnel principle governs the propagation of a quasimonochromatic, paraxial, positive-frequency scalar field operator... [11, 12, 15].” Thus vacuum propagation of quantum mutual coherence is identical in form—random process theory applied to the Huygens-Fresnel principle—to that for classical mutual coherence.

Shih does not directly address *any* of the preceding metalessons. Instead, he confines his attention to ghost imaging with pseudothermal light, but it is important to realize—as will be explained below—that his comment implicitly contradicts one or more of Shapiro and Boyd’s metalessons. As such, his arguments must withstand much closer scrutiny than just their application to pseudothermal ghost imaging. We shall make the case against their broader validity after we study their more narrow application to lensless pseudothermal ghost-image formation.

2 Pseudothermal light produces correlated object and reference plane speckles

We are now ready to examine—and refute—Shih’s assertion that lensless ghost imaging has no semiclassical explanation because there is no “point-to-point” correlation between the intensity fluctuations in the source and measurement (object and reference) planes. We agree that there is no “point-to-point” correlation between the intensity fluctuations in the source and measurement planes in the lensless configuration, but we note that this statement has no bearing on whether semiclassical theory can account for formation of the lensless ghost image. This is because—as shown in [1] and summarized below—it is the identical nature of the speckle patterns cast by the pseudothermal source on the object and reference planes that leads to the ghost image. Shih evidently believes that there are no such measurement-plane speckles because he states “From Eqs. (4) and (5) it is easy to find: (1) The incoherent superposition of a large number of sub-fields, which may take all possible random phases, produces constantly distributed intensities on the observation plane.” This statement, however, is at odds with well known results from the theory of partial coherence. Consider illuminating a rotating ground-glass diffuser with a strong, continuous-wave laser beam, such that the field transmitted through the diffuser is sufficiently bright that the correspondence principle allows us to employ classical electromagnetism for its description, *and* to neglect all noise sources in its photodetection. Beran and Parrent [16], Born and Wolf [17], Wolf [18], and Goodman [19] all teach that a

quasimonochromatic spatially-incoherent source that is nominally at a beam waist with center wavelength λ , coherence radius ρ_0 , and intensity radius $a_0 \gg \rho_0$ will project a field with coherence radius $\lambda L/a_0$ and intensity radius $\lambda L/\rho_0$ after propagation to a measurement plane over an L -m-long vacuum path. A photodetector placed in that measurement plane whose response time is shorter than the field’s coherence time will produce a photocurrent that is proportional to the light power illuminating the detector’s active region. Furthermore, in keeping with the Central Limit Theorem applied to the complex envelope of the light exiting the diffuser and reaching the detector [19], this photocurrent will have exponentially-distributed fluctuations when the detector’s active region is smaller than the field’s coherence area. Moreover, two such detectors, separated by more than $\lambda L/a_0$, will see independent fluctuations of this type, i.e, the speckles projected by the pseudothermal source. More than 40 years ago, the Bose-Einstein photon-counting (low-light level) signature of pseudothermal light was observed in the experiments of Estes, Narducci, and Tuft [20], in accord with the Mandel rule [21] from semiclassical photodetection theory *and* the Bose-Einstein photon-number statistics from quantum theory for a single-mode thermal state.

To make abundantly clear the incorrectness of Shih’s claim regarding “constantly distributed” intensities in the measurement planes—and the conclusion he draws therefrom that the lack of “point-to-point” intensity-fluctuation correlations between the source plane and the measurement planes precludes a semiclassical explanation of the lensless pseudothermal ghost image—we will expand upon the coherence theory that we reviewed in the preceding paragraph. We start with the following simple calculation. Suppose that the instantaneous complex-field observed at a point detector some distance away from a quasimonochromatic spatially-incoherent source can be taken to be

$$E = \sum_{n=1}^N A e^{i\phi_n}, \tag{1}$$

where $A > 0$ is the common, deterministic amplitude of $N \gg 1$ sub-sources whose phases, $\{\phi_n\}$, are statistically independent, identically distributed random variables that are uniformly distributed on $[0\ 2\pi]$, and we have neglected time dependence because we will be concerned with measurements made by a detector whose response time is shorter than the coherence time (reciprocal bandwidth) of the quasimonochromatic source. It follows that the intensity associated with this complex field has mean (ensemble average over the sub-source’s phase statistics)

$$\langle I \rangle = \langle |E|^2 \rangle = \left\langle \sum_{n=1}^N \sum_{m=1}^N A^2 e^{i(\phi_n - \phi_m)} \right\rangle = NA^2, \tag{2}$$

because the $n \neq m$ cross-terms average to zero. We also have that the mean-squared intensity satisfies

$$\langle I^2 \rangle = \left\langle \sum_{n=1}^N \sum_{m=1}^N \sum_{j=1}^N \sum_{k=1}^N A^4 e^{i(\phi_n - \phi_m + \phi_j - \phi_k)} \right\rangle = (2N^2 - N)A^4, \tag{3}$$

because phase randomness implies that only terms with $n = m$ and $j = k$ and terms with $n = k$ and $m = j$ do not average to zero. So, the instantaneous intensity at a point detector *does* fluctuate. In particular, its variance is

$$\text{Var}(I) = \langle I^2 \rangle - \langle I \rangle^2 = N(N-1)A^4 \approx N^2 A^4 = \langle I \rangle^2, \quad (4)$$

where the approximation holds because $N \gg 1$. That the variance of I equals the square of its mean is consistent with the Central Limit Theorem result that I becomes exponentially distributed for $N \gg 1$.

Having established the origin of the exponentially-distributed intensity fluctuations in the lensless ghost-imager's measurement planes, let us present a simple result demonstrating that the measurement-plane intensity will have a finite spatial correlation length, i.e., that it will exhibit speckles. Suppose that a space-limited (to a diameter- D circular pupil), quasimonochromatic (center wavelength λ) $z = 0$ plane spatially-incoherent source with complex field $E_0(\boldsymbol{\rho})$, as a function of its transverse coordinate vector $\boldsymbol{\rho} = (x, y)$, has mutual coherence function³

$$\langle E_0(\boldsymbol{\rho}_1)E_0^*(\boldsymbol{\rho}_2) \rangle = \begin{cases} \lambda^2 I_0 \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2), & \text{for } |\boldsymbol{\rho}_1|, |\boldsymbol{\rho}_2| \leq D/2 \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

From the van Cittert–Zernike theorem [17] we have that $E_L(\boldsymbol{\rho}')$, the complex field resulting from vacuum propagation of this source-plane field to $z = L$, has mutual coherence function

$$\langle E_L(\boldsymbol{\rho}'_1)E_L^*(\boldsymbol{\rho}'_2) \rangle = e^{ik(|\boldsymbol{\rho}'_1|^2 - |\boldsymbol{\rho}'_2|^2)/2L} \frac{\pi D^2 I_0}{4L^2} \frac{J_1(\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/\lambda L)}{\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/2\lambda L}. \quad (6)$$

Combining this result with the Central Limit Theorem's implication that $E_L(\boldsymbol{\rho}')$ is a zero-mean, isotropic, complex-valued Gaussian random process yields the following mean and covariance function for $I_L(\boldsymbol{\rho}') \equiv |E_L(\boldsymbol{\rho}')|^2$:

$$\langle I_L(\boldsymbol{\rho}') \rangle = \frac{\pi D^2 I_0}{4L^2}, \quad (7)$$

and

$$\langle \Delta I_L(\boldsymbol{\rho}'_1) \Delta I_L(\boldsymbol{\rho}'_2) \rangle = \langle I_L(\boldsymbol{\rho}'_1) \rangle \langle I_L(\boldsymbol{\rho}'_2) \rangle \left(\frac{J_1(\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/\lambda L)}{\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/2\lambda L} \right)^2, \quad (8)$$

where $\Delta I_L(\boldsymbol{\rho}') \equiv I_L(\boldsymbol{\rho}') - \langle I_L(\boldsymbol{\rho}') \rangle$ is the $z = L$ plane intensity-fluctuation at transverse coordinate $\boldsymbol{\rho}'$, whose covariance function follows from the *same* Gaussian moment-factoring formula that Shih uses implicitly in his Eq. (6). The preceding

³ Here again we are suppressing time dependence because we are implicitly assuming a detector whose response time is shorter than the reciprocal source-bandwidth.

covariance expression reproduces the $\text{Var}[I_L(\boldsymbol{\rho}')] = \langle I_L(\boldsymbol{\rho}') \rangle^2$ formula that we developed earlier via a simple random-phase argument. It also shows that the $z = L$ plane intensity has a spatial coherence length—measured to the first zero of its covariance function—equal to $1.22\lambda L/D$, in accord with the qualitative summary of coherence theory that we gave earlier in this section.

In lensless pseudothermal ghost imaging in the absence of turbulence, a 50–50 beam splitter is used at the source plane and the signal and reference fields result from propagation of the output fields from that beam splitter over identical L -m-long paths. Taking $E_{S_0}(\boldsymbol{\rho})$ and $E_{R_0}(\boldsymbol{\rho})$ to be the complex fields emerging from the beam splitter, in terms of their respective transverse coordinates, we have that $E_{S_0}(\boldsymbol{\rho}) = E_{R_0}(\boldsymbol{\rho}) = E_0(\boldsymbol{\rho})/\sqrt{2}$, so that the signal and reference fields, $E_{S_L}(\boldsymbol{\rho}')$ and $E_{R_L}(\boldsymbol{\rho}')$, in the measurement planes have cross-coherence function⁴

$$\langle E_{S_L}(\boldsymbol{\rho}'_1)E_{R_L}^*(\boldsymbol{\rho}'_2) \rangle = e^{ik(|\boldsymbol{\rho}'_1|^2 - |\boldsymbol{\rho}'_2|^2)/2L} \frac{\pi D^2 I_0}{8L^2} \frac{J_1(\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/\lambda L)}{\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/2\lambda L}, \tag{9}$$

which leads (via Gaussian moment-factoring) to

$$\langle \Delta I_{S_L}(\boldsymbol{\rho}'_1) \Delta I_{R_L}(\boldsymbol{\rho}'_2) \rangle = \langle I_{S_L}(\boldsymbol{\rho}'_1) \rangle \langle I_{R_L}(\boldsymbol{\rho}'_2) \rangle \left(\frac{J_1(\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/\lambda L)}{\pi D|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|/2\lambda L} \right)^2 \tag{10}$$

for the cross-covariance of their intensities. It is this covariance behavior that gives rise, in the semiclassical theory, to formation of the lensless pseudothermal ghost image. Whatever speckles may exist in the source plane—and whatever their correlation may or may not be with the signal and reference arm speckles—have no bearing on the semiclassical explanation for formation of the lensless pseudothermal ghost image. That can be seen explicitly in Eqs. (64)–(72) from [1], which clearly demonstrate that a classical-field, incoherent sum of sub-sources representation leads to *both* the non-local two-photon interference *and* the intensity-fluctuation correlation interpretations of the lensless pseudothermal ghost image, something that Shih argues is impossible.

3 Conclusions

The preceding section presented a compelling case for dismissing the core of Shih’s comment, namely that the pseudothermal source does not cast speckles in the lensless ghost imager’s object and reference planes whose cross correlation gives rise to the lensless pseudothermal ghost image. In closing, let us return to the metalessons from Shapiro and Boyd, and how Shih’s comment is at odds with one or more of them. Shih’s assertion that semiclassical photodetection theory *cannot* explain lensless pseudothermal ghost imaging—as well as his similar claim in [22] with respect to that paper’s anticorrelation experiments, and the like implication in [23] for that paper’s simulated

⁴ Shih, above his Eq. (8), argues that terms like our $\langle E_{S_L}(\boldsymbol{\rho}'_1)E_{R_L}^*(\boldsymbol{\rho}'_2) \rangle$ cannot represent “locally-measured intensity fluctuations.” Yet Gaussian moment-factoring applied to his Eq. (8) will yield his $G_{oi}^{(1)}G_{io}^{(1)}$. Nevertheless, he asserts that it will *not* do so, in contradiction to the theory of partial coherence.

Bell-state experiments—contradicts one or more of three well-accepted results that are metalessons from Shapiro and Boyd. In particular, Shih's comment implies the truth of at least one of the following metalesson-contradicting statements: (1) pseudo-thermal sources produce light beams that are *not* random mixtures of coherent states; (2) the classical theory of coherence propagation through vacuum, and its extension to quantized fields, does *not* apply to pseudothermal light; (3) semiclassical and quantum photodetection can *disagree* in their statistical predictions for illumination whose quantum state is a random mixture of coherent states. Were any of these statements proven by Shih, they would represent a great scientific upheaval in quantum and/or statistical optics.

Our work in [24] provided a quantitative semiclassical explanation for the anti-correlation experiments from [22]. Similarly, our work in [25] gave a quantitative semiclassical explanation for the simulated Bell-state experiments from [23]. These papers, taken together with our present response, clearly show that Shih has *not* made a successful case for any of points (1) through (3) of the preceding paragraph. Thus we find his criticism of the semiclassical analysis of lensless pseudothermal ghost imaging in [1]—whose predictions for the image's spatial resolution and image contrast Shapiro and Boyd show to be identical to those obtained from quantum theory, including Shih's preferred nonlocal two-photon interference explanation—to be without merit.

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