Spatial evolution of laser beam profiles in an SBS amplifier

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We have performed an experimental and theoretical analysis of the modification of the transverse structure of a probe beam on propagation through an SBS amplifier. The theoretical analysis is based on performing a mode decomposition of the incident fields and determining how each mode amplitude is modified as a result of the nonlinear coupling. For the case of input beams with Gaussian intensity profiles and in the limit of negligible pump depletion, we find that the input probe profile is preserved during amplification only when the probe spot size is smaller than that of the pump in the focal region.

I. Introduction

Stimulated Brillouin scattering (SBS) is a nonlinear optical process that has proved useful for optical phase conjugation and laser beam amplification. The theory of SBS in the limit of infinite plane waves has been well understood for some time.[1-3] However, the theory of SBS for the case of focused Gaussian beams is much more complicated, and relatively few investigations of SBS in this limit have been performed.[4,5] We present the results of our theoretical and experimental study of the effects of the transverse mode structure of the interacting optical beams on the performance characteristics of an SBS amplifier. Our theoretical procedure entails performing a mode decomposition of the pump (laser) and probe (Stokes) fields and deriving differential equations that describe how the mode amplitudes evolve spatially due to the nonlinear coupling between the two fields. We choose expansion functions that are solutions to the homogeneous wave equation, and hence diffraction effects are automatically incorporated into our formalism. The resulting differential equations are solved numerically but with far less computational effort than would be involved in performing a direct numerical integration of the 3-D inhomogeneous wave equations for the field amplitudes. A similar procedure has recently been used to describe the process of stimulated Raman scattering.[6]

Our theoretical analysis shows that due to the nonlinear coupling between the modes of the laser and Stokes fields, the transmitted fields will in general have transverse field distributions which are different from those of the input fields. Hence, only in special conditions will the input probe beam be amplified without distortion. To illustrate these general principles, we have treated in detail the case in which the pump beam is a fundamental Gaussian mode possessing cylindrical symmetry, which is unaffected by its interaction with the probe beam. In addition, we have performed experimental measurements which confirm many of our theoretical predictions.

II. Theory

To model the propagation of a laser beam within an SBS amplifier, we use the standard technique of mode decomposition.[7] We consider an SBS amplifier with a forward traveling probe beam and a backward traveling pump beam whose amplitudes we denote $A(\mathbf{r},z)$ and $B(\mathbf{r},z)$, respectively, where the radial vector $\mathbf{r}$ is constrained to be perpendicular the system z axis. We assume that the laser is oscillating in a single longitudinal mode. The total electric field within the medium can then be written as

$$E(\mathbf{r},z;t) = A(\mathbf{r},z)\exp[i(k_1 z - \omega_1 t)] + B(\mathbf{r},z)\exp[i(-k_s z - \omega_s t)], \tag{1}$$

where $\omega_l = k_l c/n$, $\omega_s = k_s c/n$, and $\omega_l - \omega_s = \Omega$ is the Brillouin frequency shift. Next, we assume that the Brillouin frequency shift Ω is small compared with the laser frequency, which allows us to decompose the complex amplitudes of the laser and Stokes beams into an orthonormal basis:

$$A(\mathbf{r},z) = \textstyle\sum_n a_n(z)B_n(\mathbf{r},z); \qquad B(\mathbf{r},z) = \sum_n b_n(z)B_n(\mathbf{r},z). \tag{2}$$

Here a_n and b_n denote the amplitudes of the nth mode of the pump and Stokes beams, respectively, while B_n is the nth member of the basis set. We require that the total electric field obey the driven wave equation

$$\left(\nabla^2 + \frac{\epsilon^{(1)}}{c^2}\frac{\partial^2}{\partial t^2}\right)E(r,z;t) = \frac{-4\pi}{c^2}\frac{\partial^2 P^{nl}}{\partial t^2}, \tag{3}$$

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Received 6 July 1988.
0003-6935/89/010092-05$02.00/0.

where the phase matched contributions to the nonlinear polarization are given as

$$P^{nl} = 6\chi_{sbs}^{(3)}ABB^* \exp(+ikz) + 6\chi_{sbs}^{(3)}BAA^* \exp(-ikz). \quad (4)$$

Then, by taking the field amplitudes to be slowly varying, we derive coupled-amplitude equations describing the evolution of the mode amplitudes:

$$\frac{da_n}{dz} = \kappa\sum_{ijp} b_i b_j^* a_n G_{ijpn}, \quad (5a)$$

$$\frac{db_n}{dz} = -\kappa\sum_{ijp} a_i a_j^* b_n G_{ijpn}, \quad (5b)$$

where the coupling coefficient κ is defined by

$$\kappa = \frac{24\chi_{sbs}^{(3)}\pi\omega^2}{c}, \quad (6)$$

and the overlap integral G_{ijpn} is given as

$$G_{ijpn} = \int d^2r dz B_i B_j^* B_p B_n^*. \quad (7)$$

In the case of cylindrically symmetric beams, a natural choice for the basis set is the normalized Laguerre-Gaussian functions, which we define in the usual manner[8] by

$$B_n(r,z) = \frac{1}{w^2(z)}\left(\frac{2}{\pi}\right)^{1/2} L_n\left[\frac{2r^2}{w^2(z)}\right]\exp\left[-\frac{r^2}{w^2(z)}\right]$$
$$\times \exp\left\{i\left[\frac{kr^2}{2R} - (2n+1)\tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\}, \quad (8)$$

where L_n is the nth-order Laguerre polynomial. In this expression the spotsize $w(z)$, radius of curvature $R(z)$, and confocal parameter z_0 are defined by

$$w^2(z) = w_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right], \quad (9a)$$

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right], \quad (9b)$$

$$z_0 = \frac{\pi\omega_0^2}{\lambda}. \quad (9c)$$

If we assume that the pump beam is undepleted and has a Gaussian transverse profile, we can simplify Eqs. (5) into a single coupled-amplitude equation describing the spatial evolution of the amplitude of the probe mode:

$$\frac{da_n}{dz} = \kappa\sum_m a_m G_{mn}. \quad (10)$$

Here the overlap integrals for the individual probe modes are defined in terms of the pump basis through

$$G_{mn} = \int d^3r B_m B_n^* \exp(-r^2/w_p^2). \quad (11)$$

To integrate numerically Eq. (10) we need to specify the initial amplitudes of the Stokes wave at the input face of the SBS medium. If we assume a medium of length L, extending from $z = -(L/2)$ to $z = L/2$, these amplitudes can be obtained by projecting the input Stokes wave onto the pump basis at the input plane $z = -(L/2)$:

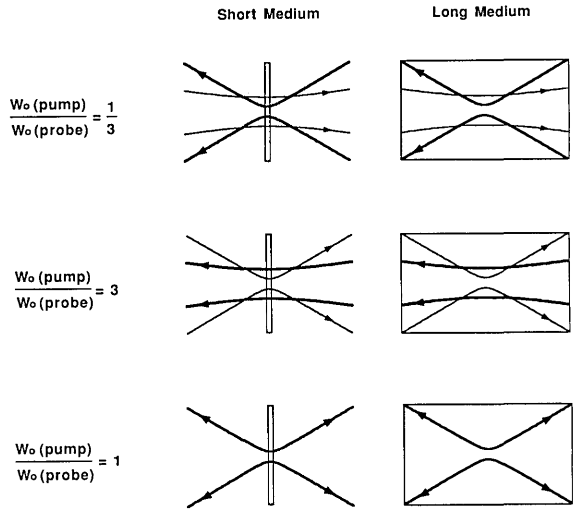

Fig. 1. Six geometries studied theoretically. The thick curves show the backward traveling pump beam, while the thin curves show the forward traveling probe beam. Here the short medium corresponds to $L = 0.01Z_0$ and the long medium to $L = 10Z_0$, where Z_0 is the confocal parameter of the pump beam. Pump beam to probe beam spot size ratios of 1:3, 3:1, and 1:1 are studied.

$$a_n(-L/2) = \int d^2r B_n^*(r,-L/2)A(r,-L/2). \quad (12)$$

In the following numerical analysis we consider the six distinct geometries illustrated in Fig. 1. We treat the cases in which the injected probe beam spot size is smaller than that of the pump in the focal plane by a ratio of 3:1, where the probe beam spot size is larger by that same factor and where the probe and pump beam spot sizes are equal. For each of these cases we consider the spatial evolution of the probe wave for two different lengths of the SBS medium.

Figures 2–4 show the results of numerically integrating Eq. (10). For the case of a probe beam spot size that is smaller than that of the pump beam in the focus (Fig. 2), we see that the probe essentially retains its original beam parameters on amplification for both long and short media. These results suggest that for beam amplification applications it is desirable to use a collimated pump wave. Since most of the amplification occurs in the focal region, the narrow probe beam will experience a uniform gain due to the central portion of the wide pump.

However, if the pump beam is much narrower in the focal region, as shown in Fig. 3, only the central portion of the probe beam will be amplified, tending to narrow the probe in the focal region. Due to diffraction, this process increases the beam divergence angle, causing a much larger spot size in the far field. In fact, since the gain curve resembles the transerse profile of the pump, we see the probe beam begins to take on the beam characteristics of the pump in the limit of high gain.

In a mode-matched case (Fig. 4), in which the pump and probe inputs have the same spot size, we find that the effects of amplification are to widen the probe in the far field. This geometry produces an output beam with characteristics unlike those of either the probe or the pump. This result can be explained by examining

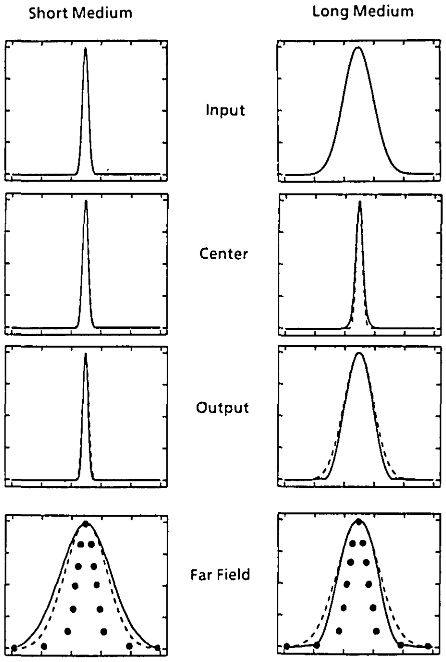

Fig. 2. Transverse profile of the probe beam at four different positions within and beyond the amplifier cell for a pump to probe spot size ratio of 3:1. The solid curves correspond to the case of total power gain equal to 100, while the dashed curves correspond to a power gain of 1.1. The peak amplitudes have been normalized to the same value for each plot. In addition, the pump beam profile in the far field is shown by the dotted curve.

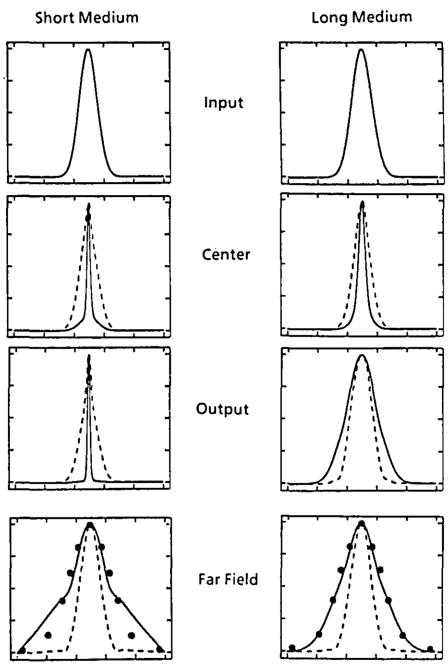

Fig. 3. Same as Fig. 2 for a pump to probe spot size ratio of 1:3.

the input beam and gain profile. Since the input Gaussian is amplified by a Gaussian gain profile, the beam is narrowed in the focal region, since squaring a Gaussian distribution produces a narrower Gaussian

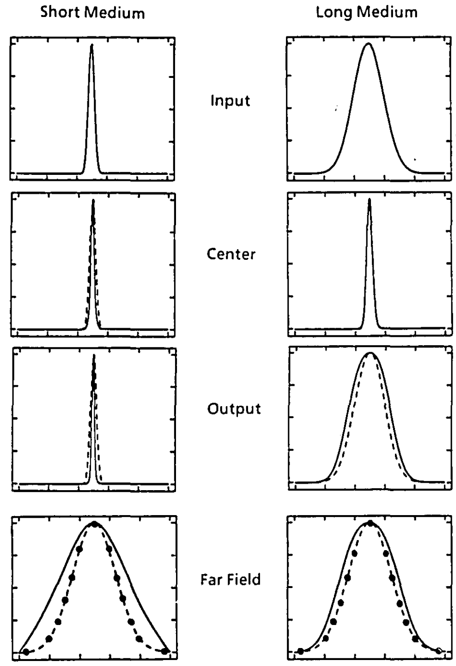

Fig. 4. Same as Fig. 2 for a pump to probe spot size ratio of 1:1 (mode-matched).

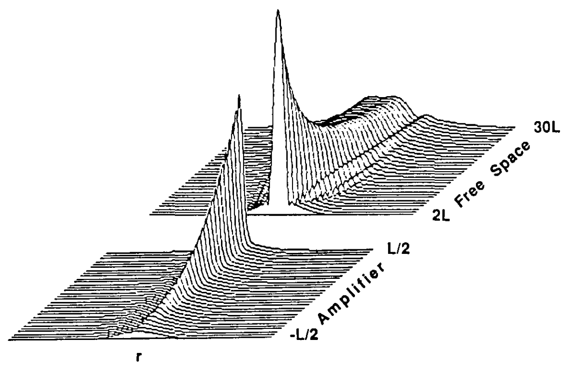

Fig. 5. Spatial evolution of probe-beam transverse profile on propagation through the SBS amplifier $[-(L/2)$ to $(L/2)]$ and its subsequent propagation through free space ($2L$ to $30L$) for a pump to probe spot size ratio of 1:3.

distribution. As with the previous geometry, the narrowing in the near field increases the beam divergence angle due to diffraction.

Figure 5 shows the spatial evolution of the probe beam profile for the case of a narrow pump beam in a short medium with high gain. We show the probe beam's spatial profile both within the amplifier and in free space beyond the amplifying region. Here the diffraction-gain coupling as well as the near-field effects are evident. Within the amplifier we see the probe taking on the characteristics of a narrow pump as has been previously discussed. On propagation through free space the amplified probe exhibits the broadening in the far field as well as a spatial modulation in the near field. This spatial ringing is caused by the diffraction of the probe beam from the effective aperture created by the gain profile.

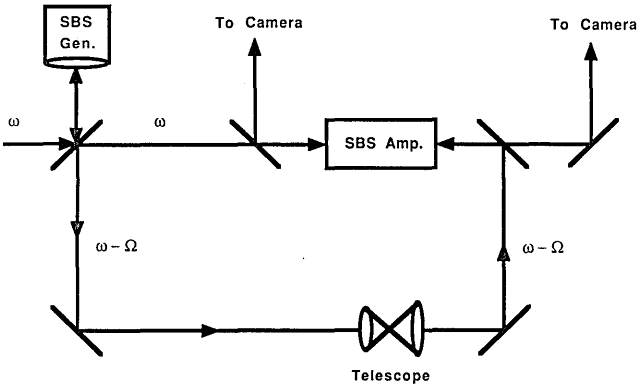

Fig. 6. Experimental setup.

III. Experiment

The setup shown in Fig. 6 was used to examine these effects experimentally. The SBS amplifier consisted of an 8-cm long cell filled with carbon disulfide. A frequency-doubled Q-switched Nd:YAG laser with a pulse duration of 12 ns and pulse energy of 50 mJ was used to generate the input beams. The backward pump beam was injected into the amplifier at the laser frequency ω. A probe beam at the Stokes-shifted frequency was generated in an additional CS_2 SBS cell. The intensity of the pump beam was adjusted to create a total power gain of 100 to match the conditions studied numerically. The transverse profiles of each beam were examined by a TV camera whose output was sampled with a frame grabber for subsequent analysis by a laboratory microcomputer. To examine the effects of gain, the probe beam was recorded in the camera plane after passing through the cell with the pump beam blocked and unblocked ($G = 1$, $G = 100$). The beam parameters of the input probe beam were varied through use of a telescope as shown.

Two different geometries were studied experimentally, the wide probe and mode-matched probe, in each case for a short medium. These configurations were chosen because they were expected to show most dramatically the effects of gain on the probe beam profile. Figures 7 and 8 show the results of the investigation. In both cases we see good qualitative agreement with the results predicted by theory. In Fig. 7, the wide probe exhibits spatial ringing on amplification, (compare with Fig. 5). Here pump depletion effects may explain the reduced amplitude and flat top of the central peak in relation to the rings. Figure 8 shows that the mode-matched geometry exhibits a significant broadening of the transverse profile in the far field. Due to the relative spot sizes, this geometry required a higher laser power to reach the appropriate gain requirements. The noise in these plots was caused by the computer receiving rf noise generated from the laser amplifier flashlamps.

IV. Conclusions

We have studied the performance of an SBS amplifier with various input pump and probe beams. For the six geometries examined theoretically, we come to the

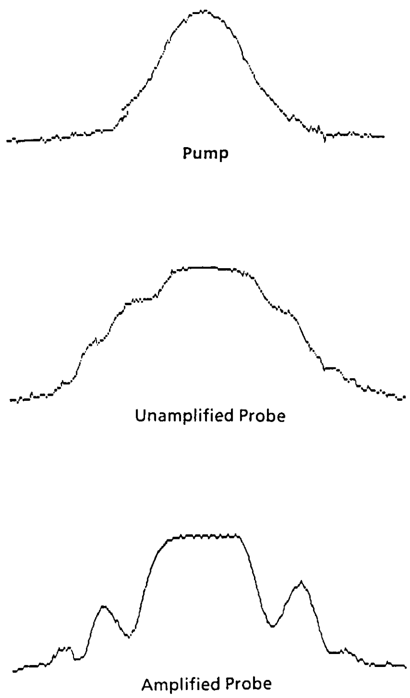

Fig. 7. Experimental results for a pump to probe beam spot size ratio of 1:3. All the profiles were recorded at the camera plane, approximately one confocal parameter away from the SBS amplifier. The unamplified probe plot was taken with the pump blocked, while the amplified probe plot was with the pump on. Note the strong spatial ringing in the amplified probe beam.

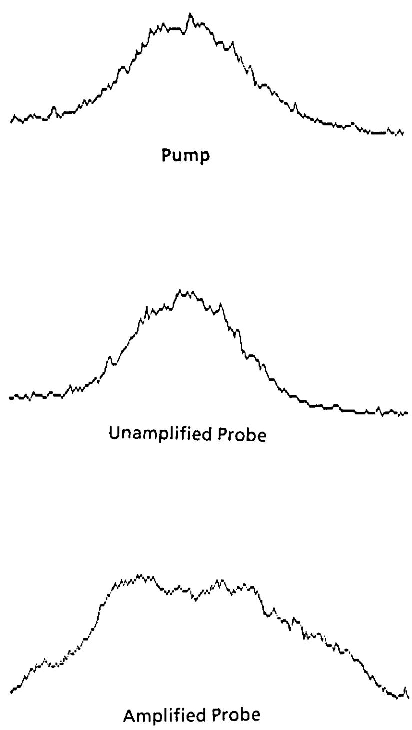

Fig. 8. Same as Fig. 7 for the pump to probe spot size ratio of 1:1. Here the camera plane is approximately five confocal parameters away from the SBS amplifier. The amplified probe beam's characteristics are different from those of the pump or input probe.

following conclusions. First, for the case in which the pump beam is wider than the probe beam in the focal region, the transverse mode structure of the probe is essentially preserved. Hence this geometry is well suited for beam amplification applications where it is desired to preserve the initial mode structure. However, if the pump beam is narrower than the probe beam, not only will the probe be amplified but energy will be redistributed among the modes, causing a modification in the transverse profile. Also, in the limit of high gain, those modes matching the pump will be amplified more efficiently, producing a probe beam with a mode structure similar to that of the pump, which could indiate why SBS leads to phase conjugation. The result is an amplified beam exhibiting spatial ringing in the near field with a considerably broader transverse profile than that of the input probe in the far field. In the mode-matched geometry, the amplification of a Gaussian input with a Gaussian gain profile produces a beam with a divergence angle greater than that of the input probe. Thus this beam has characteristics unlike those of either the pump or probe. The numerical model describing these effects have been confirmed experimentally for the narrow-pump and mode-matched geometries.

We gratefully acknowledge discussions of this work with P. Narum. This work has been supported by the sponsors of NYSCAOT.

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