Time reversal of Berry's phase by optical phase conjugation

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We examine experimentally the time-reversal properties of Berry's geometrical phase in one of its optical manifestations, Pancharatnam's phase, through the use of optical phase conjugation. The time-reversal symmetry of the total optical system is broken by a nonreciprocal element, a Faraday rotator. Nevertheless, we find that the geometrical part of the phase acquired by an optical wave in passing through the system still respects time-reversal invariance.

The state of a quantum system can depend nontrivially on its past history.[1] The total phase acquired in a cycle of changes in any state is the sum of a dynamical component and a geometrical component. The dynamical phase describes the energy-dependent change of the phase of the wave function, whereas the geometrical phase is independent of the dynamics and depends only on the path taken in the space of all possible states of the system (state space). The geometrical phase expresses the system's geometrical memory of its past. Both of these phases are physically observable in interference experiments.[2] There has been much recent activity, both theoretical[2] and experimental,[3] concerning Berry's phase.

In optics, the geometrical phase has recently appeared in three different manifestations. In the first of these, this phase results from a cycle of changes in the direction of the spin of the photon.[4] The state space in this case is the sphere of all possible directions of the photon spin. In the second case this phase results from a cycle of changes in the polarization state of the light beam while the propagation direction is kept fixed; in this case the phase is known as Pancharatnam's phase.[5–9] The state space in this case is the Poincaré sphere. Pancharatnam's phase is equal to minus one half the directed solid angle subtended at the center of the sphere by the closed curve traced out by the polarization state as it undergoes a complete cycle. In the third case, this phase results from a cycle of squeezed states of light.[10] The state space in this case is a unit hyperboloid representing all possible squeezed states. The first and second manifestations differ from each other by a factor of 1/2 in their dependence on the solid angle subtended in their respective state spaces. The second and third manifestations differ from each other by a sign. The first and second have already been seen in interference experiments,[4–8] but the third has not yet been observed.

In this paper, we examine the behavior of Berry's phase (in the form of Pancharatnam's phase) under time reversal for the case of an optical system that does not possess time-reversal invariance. We perform the time-reversal operation through the use of optical phase conjugation. A phase-conjugate mirror produces a field whose amplitude is everywhere proportional to the complex conjugate of that of the incident field. This property implies that a phase-conjugate mirror generates a wave whose phase is the negative of that of the incident wave; this property leads to the well-known ability of phase conjugation to correct wave-front distortions in a double pass through an aberrating medium.[11] Optical phase conjugation can also be described in a formal sense as the generation of a time-reversed wave front.

We first consider the symmetry properties of Berry's phase under time reversal for a system whose Hamiltonian H is invariant under time reversal. The time evolution of the system is governed by the Schrödinger equation, $i\hbar\partial\psi/\partial t = H\psi$. Maxwell's equations can also be rewritten in the form of a Schrödinger equation, with ψ being a six-component spinor,[8] so that the following standard analysis of Berry's phase can be applied to them.[12] The system is assumed to evolve in such a manner that at time T the system has returned to its initial state with an acquired phase ϕ and hence can be expressed as

$$\psi(T) = e^{i\phi}\psi(0). \tag{1}$$

In order to decompose the phase ϕ into the sum of a geometrical and a dynamical part, we define a new state vector $\tilde{\psi}(t)$ by

$$\tilde{\psi}(t) = e^{if(t)}\psi(t), \tag{2}$$

with $f(t)$ chosen so that $\tilde{\psi}(T) = \tilde{\psi}(0)$ and hence that $f(0) - f(T) = \phi$.[12] The geometrical phase γ is then given by

$$\gamma = i \oint_c (\tilde{\psi}, \mathrm{d}\tilde{\psi}), \tag{3}$$

where $\mathrm{d}\tilde{\psi}$ is an infinitesimal change in $\tilde{\psi}$, and c is the closed path traced out in state space. The dynamical phase δ is given by

$$\delta = -\int_0^T (\tilde{\psi}, H\tilde{\psi})\mathrm{d}t, \tag{4}$$

where H is the Hamiltonian of the system. We assume that

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the time-reversal operation is applied to the system at time T. The time-reversed state $\psi'(T)$ is hence given by

$$\psi'(T) = \tau\psi(T) = e^{-i\phi}\tau\psi(0), \qquad (5)$$

where τ is the time-reversal operator and where we have used Eq. (1) and the fact that τ is antiunitary in obtaining the last form. The time-reversed state $\psi'(t)$ is allowed to evolve in the time for $t > T$ under the action of the time-reversed Hamiltonian H'. Since we have assumed that H is time-reversally invariant (i.e., $H' = H$) we deduce that

$$\psi'(t) = \tau\psi(2T - t) \qquad (6)$$

and in particular that

$$\psi'(2T) = \tau\psi(0). \qquad (7)$$

The evolution from time T to $2T$ is hence just the time-reversed version of the evolution from time 0 to T. In particular, the system evolves in such a way that at time $2T$ its state is the same as that at time T with an acquired phase ϕ', that is,

$$\psi'(2T) = e^{i\phi'}\psi'(T). \qquad (8)$$

We can represent ϕ' as the sum of the associated dynamical phase δ' and geometrical phase γ', given by

$$\gamma' = i\oint_{c'} {}_{c'}(\tilde{\psi}', \mathrm{d}\tilde{\psi}'), \qquad \delta' = -\int_T^{2T} (\tilde{\psi}', H'\tilde{\psi}')\mathrm{d}t, \qquad (9)$$

where by analogy with Eq. (2) $\tilde{\psi}'(t) = \exp[if'(t)]\psi'(t)$ with $\phi' = f'(0) - f'(T)$. By introducing expression (5) for $\psi'(T)$ into Eq. (8), we find that

$$\psi'(2T) = e^{i(\phi'-\phi)}\tau\psi(0). \qquad (10)$$

By comparing this equation with Eq. (7), we see that $\phi' = \phi$ (mod 2π). Since we have assumed that the Hamiltonian is time-reversally invariant, inspection of Eqs. (4) and (9) shows that $\delta' = \delta$, implying that γ' is identically equal to γ. Hence we have shown that Berry's phase is even under the time-reversal operation for a time-reversally invariant Hamiltonian.

For the case of a system whose Hamiltonian does not obey time-reversal invariance, there is no guarantee that the state of the time-reversed system will retrace the path through state space followed initially in the time interval 0 to T. However, if the system does retrace its path in state space (due to a judicious choice of experimental conditions such as those used in the experiment described in this paper), then the geometrical phase γ acquired by the time-reversed system will still be equal to that given by Eq. (3) for the initial system. It is evident from Eq. (3) that the geometrical phase is independent of the properties of the system's Hamiltonian, *including all its symmetries*. Hence Berry's phase is even under time reversal whether or not the Hamiltonian of the system obeys time-reversal invariance. This result is nontrivial, since not all forces in nature obey time-reversal invariance.

We have studied the time-reversal properties of Pancharatnam's phase by using the experimental arrangement shown in Fig. 1. A single-mode argon-ion laser beam with a power of 300 mW and a wavelength of 476.5 nm illuminates a Michelson interferometer. The reference arm of the interferometer consists of an ordinary mirror, and the test arm consists of a phase-conjugate mirror (which can be replaced by an ordinary mirror) and contains four quarter-wave plates and a Faraday rotator. The quarter-wave plates Q1 and Q4 are oriented with their fast axes at an angle of 45° with respect to the linear polarization direction of the incident light. Wave plates Q2 and Q3 are fixed together with their fast axes parallel so as to form a half-wave plate and can be rotated together so that their fast axes are oriented at an angle θ with respect to the fast axis of Q1. The Faraday rotator is inserted between wave plates Q3 and Q4 to break the time-reversal symmetry of the optical system. The phase-conjugate mirror is based upon degenerate four-wave mixing in fluorescein-doped boric acid glass. The four-wave mixing medium is ~150 μm thick, contains ~10^{17} molecules/cm^3, and has an $\chi^{(3)}$ nonlinear susceptibility of 0.05 esu at 476.5 nm.[13]

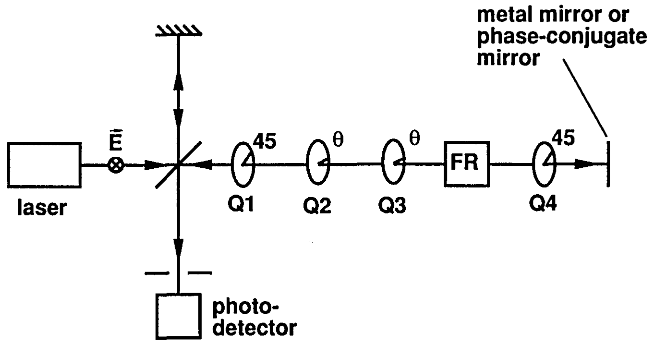

Fig. 1. Experimental setup used to study the time-reversal properties of a geometrical phase. The test arm of the Michelson interferometer contains four quarter-wave plates Q1–Q4 and a Faraday rotator (FR) of rotation angle β. The mirror that terminated the test arm can be either an ordinary mirror or a phase-conjugate mirror. A photomultiplier tube monitors the intensity at a fixed point in the interference pattern as the angle θ is changed.

In the experiment of Bhandari and Samuel,[7] which used a Mach–Zehnder interferometer, the state space of interest was the surface of the Poincaré sphere. However, our experiment uses a Michelson interferometer, and hence the light propagates along the system axis, first in the positive and then in the negative direction. In this case it is natural to introduce as the state space a generalized Poincaré sphere in which the polarization state is referred to a space-fixed axis and not to the propagation direction of the light. The generalized Poincaré sphere hence refers to the angular momentum of the light, whereas the usual Poincaré sphere refers to the helicity of the light. As in the experiment of Chiao *et al.*,[6] in the present case it is the angular momentum and not the helicity that is playing the central role, because it is angular momentum that is exchanged with the optical components of the system.

The closed path followed by the state of polarization in state space under our experimental conditions is shown in Fig. 1. As the beam of light propagates through the optical system, its polarization state is changed from linear (point A in Fig. 2), to right-hand circular (B), to linear at angle θ (C), to left-hand circular (D), and then returns to the incident polarization (A). After reflection off either the phase-conjugate or ordinary mirror, the polarization traces the same closed loop on the generalized Poincaré sphere, *ABCDA*, as

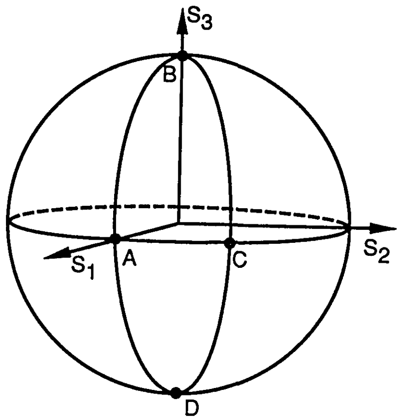

Fig. 2. Path traced by the state of polarization (*ABCDABCDA*) on the generalized Poincaré sphere as the beam propagates though the test arm of the interferometer. The coordinate axes S_1, S_2, and S_3 refer to the Stokes parameters for the beam propagating along the positive system axis.

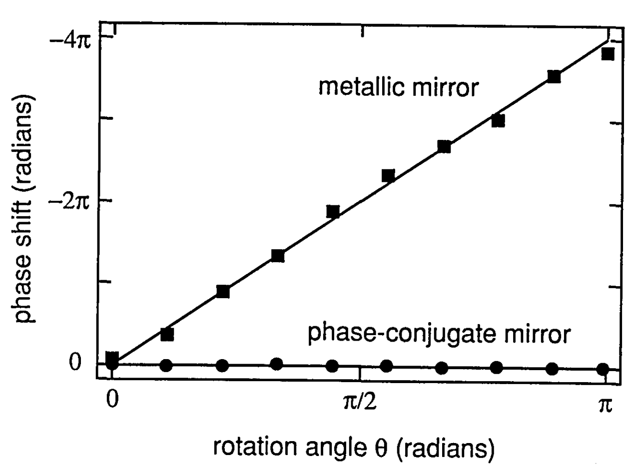

Fig. 3. Measured displacement of the interference pattern plotted as a function of the rotation angle θ of the quarter-wave plates, with the Faraday rotation angle β held fixed at ~45°.

the light propagates back through the test arm of the interferometer.

The fringe pattern produced by the interferometer is imaged onto a pinhole, and the transmitted light is detected by a photomultiplier tube. The displacement of the fringe pattern is measured as the rotation angle θ of the wave plates is varied from 0 to 180°. In Fig. 3 we plot the displacement of the interference pattern as a function of the rotation angle θ for the cases of an ordinary mirror and a phase-conjugate mirror. In both cases, the Faraday rotation angle β was set equal to 45°, although the results do not depend on the actual value of β. For the case of the ordinary mirror, the directed solid angle subtended on the generalized Poincaré sphere is readily shown to be equal to eight times the rotation angle θ. We see in Fig. 3 that the displacement of the fringe pattern is equal to minus one half the directed solid angle. This result confirms that the measured phase shift is equal to the geometrical phase acquired by the photons on cyclic changes of their polarization state. These results are also in agreement with the predictions of Jones calculus using a purely classical viewpoint.[14]

For the case in which the test arm is terminated by a phase-conjugate mirror, the closed path followed by the polarization state on the generalized Poincaré sphere is identical to the closed path followed for the case of the ordinary mirror (*ABCDABCDA*). However, as is shown in Fig. 3, in this case the measured phase shift, and hence the geometrical phase, is equal to zero for all values of the rotation angle θ. The closed path encountered after reflection from the phase-conjugate mirror is effectively traversed in a time-reversed sense, and the directed solid angle enclosed by this loop is equal in area to that of the loop traversed before the phase-conjugate mirror. The geometrical phase vanishes because the total phase is the difference of these two areas. This result confirms that the geometrical phase is even under time reversal.

Using a Michelson interferometer, Boyd *et al.*[15] showed that a uniform, time-reversible, dynamical phase shift can be negated by a phase-conjugate mirror. However, dynamical phase shifts imparted by the Faraday effect, which does not obey time-reversal invariance, cannot in general be negated by optical phase conjugation. We have verified this fact by measuring the phase shift due to the Faraday effect with the interferometer shown in Fig. 1. We see in Fig. 4 that as the magnetic field is varied to produce a rotation angle of β, the phase shift acquired to passage through the test arm of the phase-conjugate interferometer is equal to 2β. This result shows that phase conjugation is unable, in general, to negate the phase shift resulting from the Faraday effect, which breaks the time-reversal symmetry of the optical system. Since in the experiment whose results are shown in Fig. 3 the time-reversal symmetry of the system was broken by the Faraday effect, we see that the geometrical phase is canceled by phase conjugation even for a non-time-reversal-invariant system.

In conclusion, we have performed experimental studies of a geometrical phase for optical systems containing a phase-conjugate mirror. The total phase acquired in passing through this optical system was measured with a Michelson interferometer. We find that time reversal through phase conjugation corrects for the geometrical phase of the system, even when the entire optical system does not possess time-reversal invariance. We have thereby established one of the fundamental symmetry properties of Berry's phase.

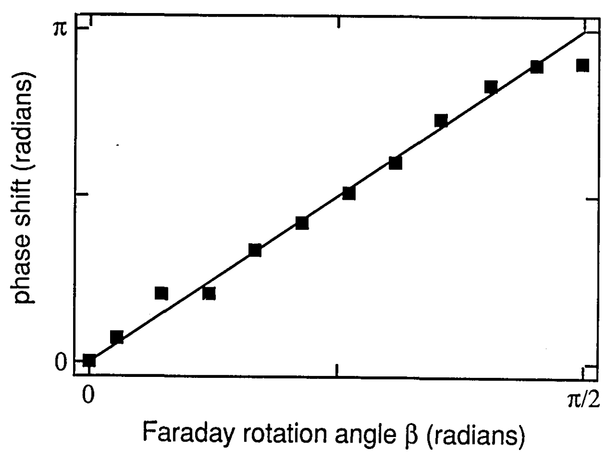

Fig. 4. The measured displacement of the interference pattern plotted as a function of the Faraday rotation angle β for θ held fixed at 45°.

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