OPTICS 462

THIN FILM OPTICAL FILTERS

THIS TOPIC PROVIDES US WITH AN EXCELLENT EXAMPLE OF THE IMPORTANCE OF OUR STUDY OF PLANEWAVES

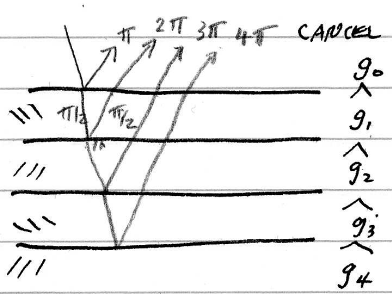
FOR ONE INTERESTED IN OPTICAL DEVICES⁺ THERE IS AN ENORMOUS PAYBACK OF UNDERSTANDING NOW THAT WE WILL SEE IN PROB SETS 6 & 7

⁺ MIRRORS
BEAM SPLITTERS
LASER CAVITIES
FILTERS
ETC.

REGULAR LECTURES & TWO WORKSHOPS
- MATLAB FOR BEGINNERS
- MATLAB FOR PROB SETS

CONSIDER ANTI-REFLECTION COATINGS

all $\lambda/4$



$\lambda_0 = 3.5$

g_0 1.0 air

g_1 1.35 magnes. fluoride

g_2 2.2 cerium oxide

g_3 3.3 silicon

g_4 4.0 germanium

$$\Gamma = \frac{g_1 - g_2}{g_1 + g_2}$$

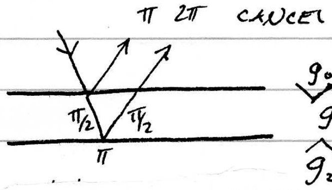
larger $g_2 \rightarrow \pi$

$$\frac{\lambda}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

This is an anti-reflection coating

Fig. 3.14 Macleod

$\lambda/4$



g_0 air

g_1 layer

g_2 substrate

MOST COMMON

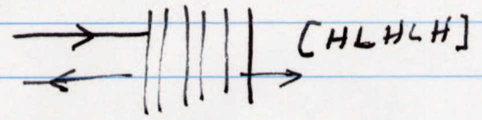
SINGLE ~~QUARTER~~ WAVE,

AS IN PROBLEM SET

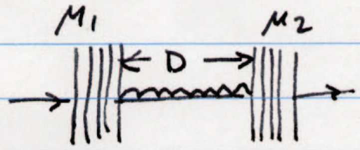
THIN FILM FILTERS

SIGNIFICANT APPLICATION OF {
- PLANEWAVE STUDY
- SEP OF VARIABLES WAVE EQ
- COMPUTER MATRICES – MATLAB

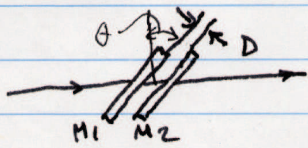
MANY DEVICE APPLICATIONS

1. 

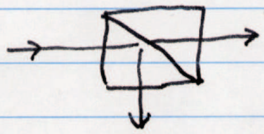
Very high reflectivity mirrors
With tiny controlled transmission

2. 

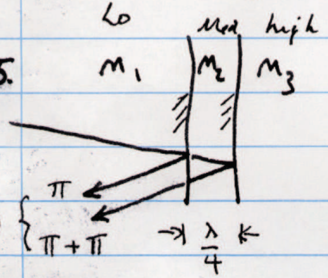
Laser Cavities
Two mirrors with large center section D

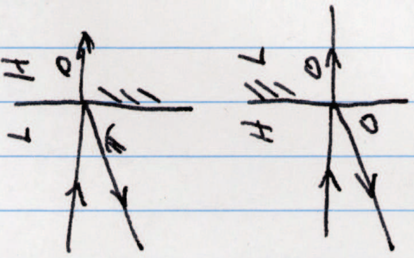
3. 

Tunable high transmission etalon
with 2 mirrors small separation D

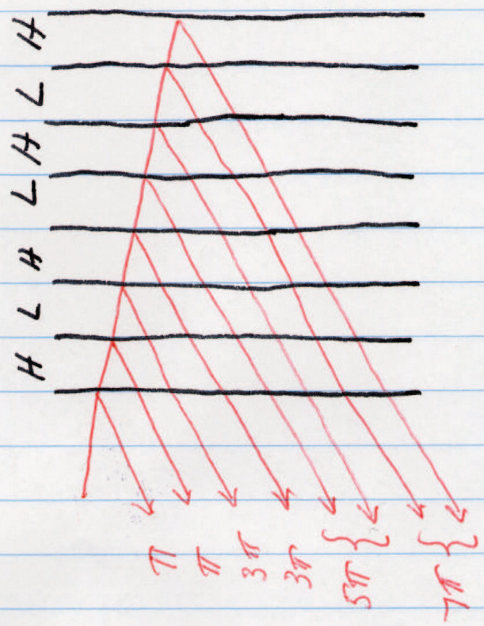
4. 

Various beam splitters: PBS,

5. 



WHY λ/4 ? ROUND TRIP π



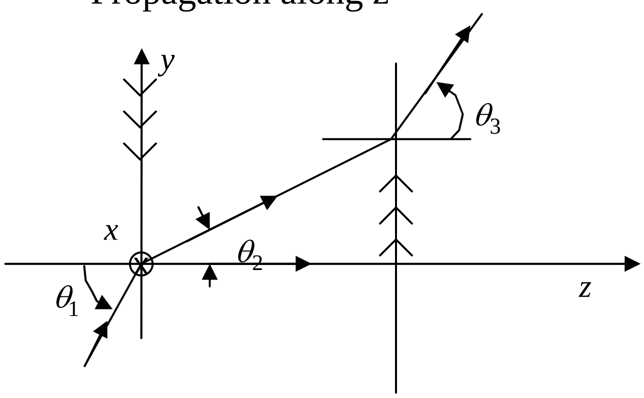
THIN FILM OPTICAL FILTERS $e^{i\omega t}$

N.George

Consider TE: Transverse Electric: E_x

Plane of Incidence yz plane

Propagation along z



$$\begin{cases} \boldsymbol{H} = \dfrac{-1}{i\omega\mu}\nabla\times\boldsymbol{E} \\ \nabla^2\boldsymbol{E}+\omega^2\mu\varepsilon\boldsymbol{E}=0 \end{cases}$$

Set $\boldsymbol{E}=\hat{x}E_x$ only

$$\frac{\partial}{\partial x}=0$$

$$E_y = E_z = 0$$

$$k^2=\omega^2\mu\varepsilon \qquad n\propto\sqrt{\varepsilon}$$

$$\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}+k^2E_x=0 \qquad\qquad k^2=n^2k_0^2$$

SEPARATION OF VARIABLES: LET $E_x=\mathbb{Y}\mathbb{Z}$

$$\frac{\mathbb{Z}\ddot{\mathbb{Y}}}{\mathbb{Z}\mathbb{Y}}+\frac{\mathbb{Y}\ddot{\mathbb{Z}}}{\mathbb{Y}\mathbb{Z}}+k^2\frac{\mathbb{Y}\mathbb{Z}}{\mathbb{Y}\mathbb{Z}}=0$$

$$\frac{\ddot{\mathbb{Y}}}{\mathbb{Y}}=-\frac{\ddot{\mathbb{Z}}}{\mathbb{Z}}-k^2=-\mathcal{K}^2$$

$$\ddot{\mathbb{Y}}+\mathcal{K}^2\mathbb{Y}=0 \qquad\qquad \mathbb{Y}(y)=e^{\pm i\mathcal{K}y};\begin{Bmatrix}\sin\mathcal{K}y\\ \cos\mathcal{K}y\end{Bmatrix}$$

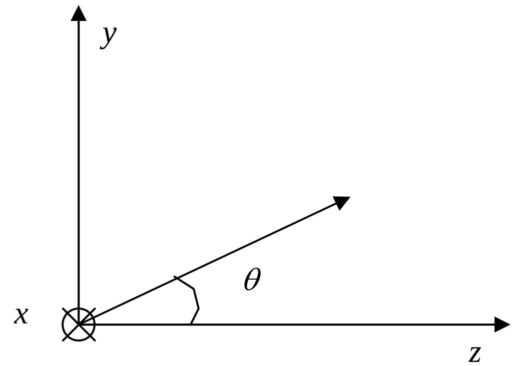
$$\ddot{\mathbb{Z}}+(k^2-\mathcal{K}^2)\mathbb{Z}=0 \qquad\qquad \mathbb{Z}(z)=e^{\pm i(k^2-\mathcal{K}^2)^{1/2}z};\begin{Bmatrix}\sin(k^2-\mathcal{K}^2)^{1/2}z\\ \cos(k^2-\mathcal{K}^2)^{1/2}z\end{Bmatrix}$$

Ref: 1.61 Born & Wolf

2.2 to 2.10 Angus Macleod Thin Films 3rd Edit 2001

F. Abeles Ann & Phys 5(1950) 596-640

MULTILAYERS: I. THE MATRIX



$$E_x(y,z) = \mathbb{Y}(y)\mathbb{Z}(z)$$

Pick $e^{-iky\sin\theta} = e^{-ik_0 ny\sin\theta}$

$$\mathcal{K} y = ky\sin\theta$$

$$k^2 - \mathcal{K}^2 = k^2(1-\sin^2\theta)$$

$$\sqrt{k^2 - \mathcal{K}^2} = k_0 n\cos\theta = k\cos\theta$$

- $E_x(y,z) = e^{-iky\sin\theta}(a\cos(kz\cos\theta) + b\sin(kz\cos\theta))$

$$H = \frac{-1}{i\omega\mu}\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{bmatrix} = \left\{\hat{x}0 + \hat{y}\frac{\partial E_x}{\partial z} - \hat{z}\frac{\partial E_x}{\partial y}\right\}\frac{-1}{i\omega\mu}$$

- $H_y = \frac{-1}{i\omega\mu}\frac{\partial E_x}{\partial z} = e^{-iky\sin\theta}[\frac{g\cos\theta}{-i}(-a\sin(kz\cos\theta) + b\cos(kz\cos\theta)]$

$$\frac{k\cos\theta}{-i\omega\mu} = \frac{\sqrt{\mu\varepsilon}\cos\theta}{-i\mu} = i\sqrt{\frac{\varepsilon}{\mu}}\cos\theta = ig\cos\theta$$

- $H_y(y,z) = e^{-iky\sin\theta}\frac{g\cos\theta}{-i}[-a\sin(kz\cos\theta) + b\cos(kz\cos\theta)]$

At $z = 0$ $E_x(y,0) = e^{-iky\sin\theta}[a+0]$

$$H_y(y,0) = e^{-iky\sin\theta}\frac{g\cos\theta}{-i}[0+b]$$

$$\begin{bmatrix} E_x(y,z) \\ H_y(y,z) \end{bmatrix}_{out} = e^{-iky\sin\theta}\begin{bmatrix} \cos(kz\cos\theta) & \frac{-i}{g\cos\theta}\sin(kz\cos\theta) \\ -ig\cos\theta\sin(kz\cos\theta) & \cos(kz\cos\theta) \end{bmatrix}\begin{bmatrix} a \\ b\frac{g\cos\theta}{-i} \end{bmatrix}_{in}$$

FOR EACH LAYER

MULTILAYERS: II. THE INVERSE MATRIX, M

With this basic result, we can arrange the matrices to give the output in terms of the input or vice versa, that is

$$\begin{bmatrix} E \\ H \end{bmatrix}_{out} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}_{in}$$

Inverting gives

$$\begin{bmatrix} E \\ H \end{bmatrix}_{in} = \begin{bmatrix} \frac{d}{\Delta} & -\frac{b}{\Delta} \\ -\frac{c}{\Delta} & \frac{a}{\Delta} \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}_{out} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix}_{out}$$

For a cascade of N layers:

We match the tangential electric and magnetic fields at each interface:

$\mathrm{In}_1 = M_1 \mathrm{Out}_1 = M_1 \mathrm{In}_2 = ... = M_1 M_2 ..... M_N \mathrm{Out}_N$

$$\begin{bmatrix} E \\ H \end{bmatrix}_{in} = M_1 M_2 ... M_N \begin{bmatrix} E \\ H \end{bmatrix}_{out} = M \begin{bmatrix} E \\ H \end{bmatrix}_{out}$$

Clearly the M_m matrix is the inverse of our derived result. Putting the film thickness $d_m = z_m$ (we suppress the y dependence):

- $$M_m = \begin{bmatrix} \cos(k_m d_m \cos\theta_m) & \frac{i}{g_m \cos\theta_m} \sin(k_m d_m \cos\theta_m) \\ i g_m \cos\theta_m \sin(k_m d_m \cos\theta_m) & \cos(k_m d_m \cos\theta_m) \end{bmatrix}$$

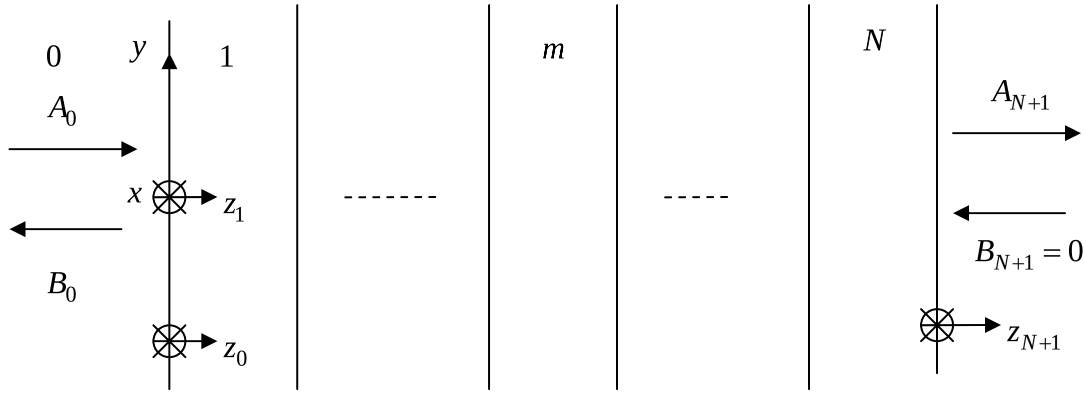
$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = ... = k_m \sin\theta_m = ... = k_N \sin\theta_N$$

Each $|M_m| = \cos^2\delta + \sin^2\delta = 1$

$$\begin{bmatrix} E_x' \\ H_y' \end{bmatrix}_{\mathrm{in}(1)} e^{-iky\sin\theta_0} = M_1 M_2 \cdots M_N \begin{bmatrix} E_x' \\ H_y' \end{bmatrix}_{\mathrm{outN}} e^{-iky\sin\theta_{N+1}}$$

$$M = M_1 M_2 \cdots M_N$$

III. MATCHING TO IN/OUT TRAVELING WAVES



First: find all θ's: $k_0 \sin\theta_0 = k_1 \sin\theta_1 = ... = k_N \sin\theta_N = k_{N+1} \sin\theta_{N+1}$

Skip above if normal incidence

$$E_{x0} = e^{-ik_0 y \sin\theta_0}[A_0 e^{-ikz_0 \cos\theta_0} + B_0 e^{ikz_0 \cos\theta_0}]$$

$$From \quad H_{y0} = \frac{1}{-i\omega\mu}\frac{\partial E_x}{\partial z}:$$

$$H_{y0} = e^{-ik_0 y \sin\theta_0}[g_0 \cos\theta_0][A_0 e^{-ikz_0 \cos\theta_0} - B_0 e^{ikz_0 \cos\theta_0}]$$

$$\frac{k_0 \cos\theta_0}{\omega\mu} = \frac{\sqrt{\mu\varepsilon_0}}{\mu}\cos\theta_0 = g_0 \cos\theta_0$$

$$e^{-ik_0 y \sin\theta_0}\begin{bmatrix} A_0 + B_0 \\ g_0 \cos\theta_0 (A_0 - B_0) \end{bmatrix} = Me^{-ik_{N+1} y \sin\theta_{N+1}}\begin{bmatrix} A_{N+1} \\ g_{N+1} \cos\theta_{N+1} A_{N+1} \end{bmatrix}$$

Reflection Coeff. :

$$r = \frac{B_0}{A_0} = \frac{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) g_0 \cos\theta_0 - (m_{21} + m_{22} g_{N+1} \cos\theta_{N+1})}{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) g_0 \cos\theta_0 + (m_{21} + m_{22} g_{N+1} \cos\theta_{N+1})}$$

Transmission Coeff :

$$t = \frac{A_{N+1}}{A_0} = \frac{2 g_0 \cos\theta_0}{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) g_0 \cos\theta_0 + (m_{21} + m_{22} g_{N+1} \cos\theta_{N+1})}$$

Algebraic details for *r* and *t* are on the next page.

These are in accord with Macleod and with 49, 50, Section 1.63Born &Wolf

ALGEBRA FOR REFLECTION-TRANSMISSION COEFFS.

$$\begin{bmatrix} A_0 + B_0 \\ g_0 \cos\theta_0 (A_0 - B_0) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} A_{N+1} \\ g_{N+1} \cos\theta_{N+1} A_{N+1} \end{bmatrix}$$

$$\gamma_0 = g_0 \cos\theta_0; \qquad \gamma_{N+1} = g_{N+1} \cos\theta_{N+1}$$

$$A_0 + B_0 = (m_{11} + m_{12}\gamma_{N+1}) A_{N+1}$$
$$\gamma_0 (A_0 - B_0) = (m_{21} + m_{22}\gamma_{N+1}) A_{N+1}$$
$$1 + B_0^{'} = (m_{11} + m_{12}\gamma_{N+1}) A_{N+1}^{'} = \alpha A'$$
$$1 - B_0^{'} = (\frac{m_{21}}{\gamma_0} + \frac{m_{22}}{\gamma_0}\gamma_{N+1}) A_{N+1}^{'} = \beta A'$$

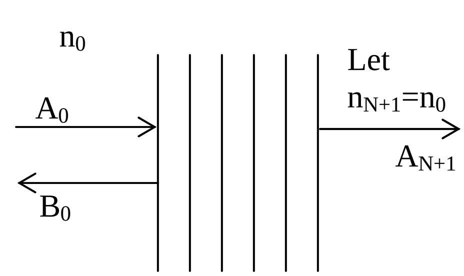
$$2 \qquad = [(m_{11} + m_{12}\gamma_{N+1}) + (\frac{m_{21}}{\gamma_0} + m_{22}\frac{\gamma_{N+1}}{\gamma_0})] A_{N+1}^{'}$$

$$t = A_{N+1}^{'} = \frac{A_{N+1}}{A_0} = \frac{2}{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) + (\frac{m_{21}}{g_0 \cos\theta_0} + m_{22}\frac{g_{N+1}\cos\theta_{N+1}}{g_0 \cos\theta_0})}$$

$$\beta(1 + B_0^{'}) = \beta\alpha A' \qquad \beta(1 + B_0^{'}) - \alpha(1 - B_0^{'}) = 0$$
$$\alpha(1 - B_0^{'}) = \alpha\beta A' \qquad (\beta - \alpha) - (\beta + \alpha) B_0^{'} = 0$$
$$\mathrm{B}_0^{'} = \frac{\alpha - \beta}{\alpha + \beta}$$

$$r = B_0^{'} = \frac{B_0}{A_0} = \frac{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) - (\frac{m_{21}}{g_0 \cos\theta_0} + m_{22}\frac{g_{N+1}\cos\theta_{N+1}}{g_0 \cos\theta_0})}{(m_{11} + m_{12} g_{N+1} \cos\theta_{N+1}) + (\frac{m_{21}}{g_0 \cos\theta_0} + m_{22}\frac{g_{N+1}\cos\theta_{N+1}}{g_0 \cos\theta_0})}$$

Consider normal incidence



$$\mathrm{Re}\, E \times H^* = \mathrm{Re}(A_0 + B_0)(A_0 - B_0)^* g_0$$
$$= A_0 A_0^* g_0 - B_0 B_0^* g_0 = A_{N+1} A_{N+1}^* g_0$$

Therefore, $|r|^2 + |t|^2 = 1$ for dielectric multilayer

Multi Layers Derivation Summary

Three main steps

1. **The matrix:** For the m^{th} layer $0 \le z_m \le d_m$

$$\begin{bmatrix} E_x(y,z) \\ H_y(y,z) \end{bmatrix}_{OUT} = \begin{bmatrix} \cos(kz\cos\theta) & \frac{-i}{g\cos\theta}\sin(kz\cos\theta) \\ -ig\cos\theta\sin(kz\cos\theta) & \cos(kz\cos\theta) \end{bmatrix} \begin{bmatrix} a \\ \frac{bg\cos\theta}{-i} \end{bmatrix}_{IN} e^{-iky\sin\theta}$$

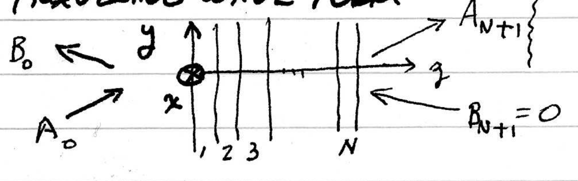
$$\begin{bmatrix} E_x(y,z) \\ H_y(y,z) \end{bmatrix}_{OUT} = \begin{bmatrix} \cos(kz\cos\theta) & \frac{-i}{g\cos\theta}\sin(kz\cos\theta) \\ -ig\cos\theta\sin(kz\cos\theta) & \cos(kz\cos\theta) \end{bmatrix} \begin{bmatrix} E_x(0) \\ H_y(0) \end{bmatrix}_{IN} e^{-iky\sin\theta}$$

2. **The Inverse Matrix** M_m & the overall $M = M_1 M_2 \cdots M_N$

$$\begin{bmatrix} E_x(z_m=0) \\ H_y(z_m=0) \end{bmatrix}_{IN\ m} = \begin{bmatrix} \cos(k_m d_m\cos\theta_m) & \frac{i}{g_m\cos\theta_m}\sin(k_m d_m\cos\theta_m) \\ i g_m\cos\theta_m\sin(k_m d_m\cos\theta_m) & \cos(k_m d_m\cos\theta_m) \end{bmatrix} \begin{bmatrix} E_x(d_m) \\ H_y(d_m) \end{bmatrix}_{OUT\ d_m}$$

$$\begin{bmatrix} E_x \\ H_y \end{bmatrix}_{IN_1} = M_1 M_2 \cdots M_N \begin{bmatrix} E_x \\ H_y \end{bmatrix}_{OUT_N} ; \quad e^{-ik_m y\sin\theta_m} \text{ IMPLICIT}$$

3. **Matching to Traveling Wave Form**



$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Reflection Coeff

$$r = \frac{B_0}{A_0} = \frac{(m_{11} + m_{12} g_{N+1}\cos\theta_{N+1}) g_0\cos\theta_0 - (m_{21} + m_{22} g_{N+1}\cos\theta_{N+1})}{(m_{11} + m_{12} g_{N+1}\cos\theta_{N+1}) g_0\cos\theta_0 + (m_{21} + m_{22} g_{N+1}\cos\theta_{N+1})}$$

Transmission Coeff

$$t = \frac{A_{N+1}}{A_0} = \frac{2 g_0\cos\theta_0}{(m_{11} + m_{12} g_{N+1}\cos\theta_{N+1}) g_0\cos\theta_0 + (m_{21} + m_{22} g_{N+1}\cos\theta_{N+1})}.$$

Thin Films Optical Filters ----Two Weeks

1. Rederive the multi-layer thin film cascade, use your class notes and summarize the solution form, as follows.

(a) **Consider normal incidence with N multilayers** with indices of refraction n_1, n_2, $\cdots$ n_N. What is the matrix M that describes

$$\begin{bmatrix} E_x \\ H_y \end{bmatrix}_{in} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_x \\ H_y \end{bmatrix}_{out}$$

(b) Match this solution to traveling waves in the input medium (n_0) and in the output medium (n_{N+1}). Compare your answers to the class references, either in Born and Wolf or in H. A. Macleod. If the algebra is a bit complicated, it is helpful to keep in mind a quote from Macleod (Third Edition p41). "This expression is of prime importance in optical thin-film work and forms the basis of almost all calculations" (in this field).

(c) Write expressions for the reflection and transmission coefficients in terms of the matrix elements.

2. Write a MATLAB program for a cascade of variable index and variable thickness media that is applicable for a large number of layers. Test the operation of your program, if possible.

3. Typically, one can gain a good understanding of laser mirrors and their high reflectivity by considering a deposition of high and low index quarter-wavelength films. Use zinc sulfide (ZnS) for the high index $n_H = 2.35$ at 550nm and magnesium fluoride (MgF_2) for the low index $n_L = 1.38$ at 550nm. Ignore the substrate to simplify matters taking the input and output indices to be unity.

(a) Using quarter wave stacks, plot curves for mirrors (HLH); (HLHLHLH); $\cdots$ and so on in order to gain an understanding of how many layers are required to get well into the 90 to 100% range.
(b) Provide a design for R>0.999.
(c) Provide a design for a transmission T=0.005 or one-half percent.
(d) Derive an expression for the reflection coefficients at midband and check them against your curves.

4. Read problems 4 and 5 before writing your programs, since the software requirement is quite similar. Also turn in your programs with some explanatory notes. Macleod derives a narrowband all-dielectric filter using quarter-wave layers:

$$[H, L, H, L, \cdots H, L, H, H, L, H, \cdots H, L, H].$$

You will notice that it is simply 2 mirrors of the type being analyzed in problem 3. Putting the two (H, H) pieces together gives a neat transmission peak of very narrow width in the center of the mirror's reflection band.

(a) Computer study using your 0.005 mirror design to see if you can obtain the attached curve.
(b) Make any reasonable variation and comment.

H. A. Macleod

274 *Band-pass filters*

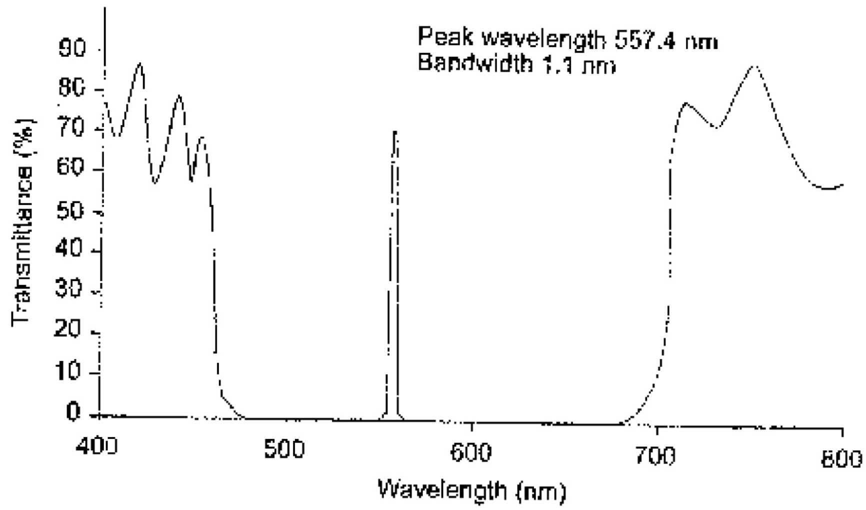
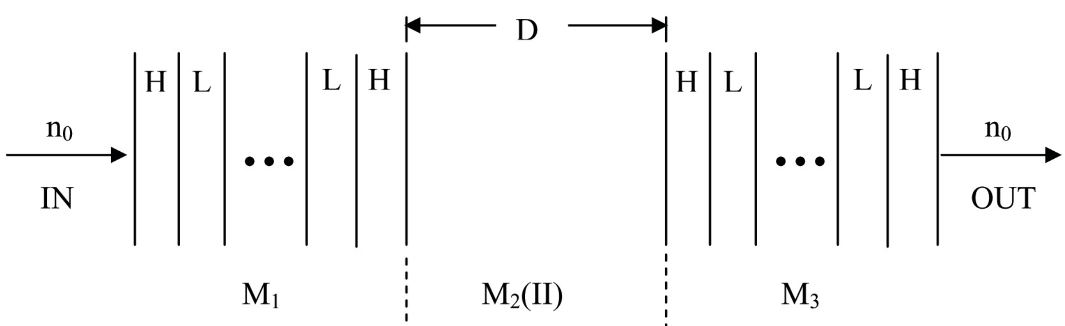


Figure 7.8. Measured transmittance of a narrowband all-dielectric filter with unsuppressed sidebands. Zinc sulphide and cryolite were the thin-film materials used. (Courtesy of Sir Howard Grubb, Parsons & Co. Ltd.)

5. Consider a laser cavity or a scanning interferometer that is formed by two multilayer stacks (high reflectivity mirrors) separated by a large distance D, which typically ranges from 10 to 100cm. Each of the mirrors is nine quarter-wave layers $(HL)^4H$ with the high index zinc sulfide $n_H = 2.35$ and the low index magnesium fluoride $n_L = 1.38$. Take the design center wavelength $\lambda_0 = 550.$ nm.



(a) Summarize pertinent equations for a computer study of this combination using normally incident monochromatic, plane-wave illumination with free space indices, n_0, for input, spacer D=10.00 cm to nm accuracy, and output. Include computation of both amplitude transmission and reflection coefficients for the entire configuration shown.

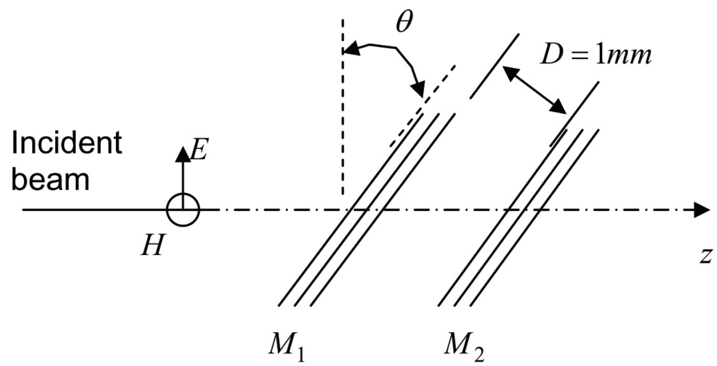
(b) Plot the transmission curve as a function of wavelength, as in problem 4. Start with a broad span of wavelengths suitable for study of the mirrors, but then increase the resolution greatly to observe fine scale features.

(c) Make labeled plot to show any features found.

(d) Print numerical values of the matrices M_1, M_3 at several wavelengths in the vicinity of $\lambda_0 = 550nm$ to gain an understanding of the element values.

(e) Explain the features in (c).

6. Fabry-Perot etalons are useful as tunable narrow-band filters. As shown, the filter consisting of 2 thin-film dielectric mirrors ($M_1 = M_2 =$ [HLHLHLHLH]) spaced by a distance $D = 1mm$ is tipped at an angle $\theta = 0^0, 5^0, 10^0$ and so on.



(a) Plot the transmission vs. wavelength at three or more values of θ in the range from 0^0 to 15^0.
(b) How does the bandpass deteriorate as the angle is varied?

Optics 462 **Problem Set** **Due 10 April 2006**

Multilayers **Nicholas George**

1. The recording of an optical interference pattern is at the heart of holography. Our study of plane waves would not be complete without considering the recording (and playback) of two important null-object structures, as follows:
   (a) For recording a holographic dielectric grating, it is common to use two plane waves incident on a photo-sensitive film plate. Draw a basic setup for making gratings and find an expression for the grating spacing as a function of wavelength and incident angles.
   (b) For recording a holographic multi-layer it is common as well to use two plane waves incident on a photo-sensitive film plate. Draw a basic setup for making multilayers, and find an expression for the multi-layer spacing as a function of wavelength and incident angles.

2. For a white light laser and in pollution sensing, it is important to have laser cavities operable with several narrow-band wavelengths that are spaced by tens or hundreds of nanometers. In this problem you are to consider the feasibility of a holographic dielectric mirrors operating at a single wavelength. As a starting point, consider a dielectric slab of thickness L along z-axis normal. Assume that it has a cosinusoidal stratification of the index of refraction given by

   $$n(z) = n_1 + \Delta n \cos(2\pi z / \Lambda), \qquad 0 \le z \le L$$

   in which $n_1 = 1.5$, $\Delta n = 0.05$ and operation at $\lambda_0 = 500nm$ is desired. Values of L in the range from $5\mu m$ to $30\mu m$ are ordinary/typical.
   (a) What is the value of Λ?
   (b) Using thin multilayer decomposition, you are to write a MATLAB program to plot the absolute value of the reflection coefficient vs. wavelength.
   (c) What value of L will give a reflection coefficient in excess of 0.95?
   (d) Discuss quantitatively the means of achieving a narrow-band of high reflectivity with this design.

3. Design a dielectric multilayer thin film mirror that provides competitive performance to the holographic multilayer.

Midterm Topics

Consider the mirror HL HL HL HL H

$$\begin{bmatrix} \cos(k_m d_m \cos\theta_m) & -\frac{i}{g_m \cos\theta_m} \sin(k_m d_m \cos\theta_m) \\ -i g_m \cos\theta_m \sin(k_m d_m \cos\theta_m) & \cos(k_m d_m \cos\theta_m) \end{bmatrix}$$

normal incidence, quarter wavelength

$$k_m d_m = \frac{2\pi}{\lambda_m} \frac{\lambda_{m0}}{4}$$

$$k_m d_m = \frac{2\pi}{\lambda_{\text{vacuum variable}}} \frac{n \lambda_{m d}}{4}$$

$$v = \nu \lambda$$

$$\frac{c}{n} = \nu \lambda_n$$

$$\lambda_n = \frac{\lambda_0}{n}$$

$$= \frac{1}{\lambda_{\text{vacuum variable}}} \left(\frac{\pi n}{2} \frac{\lambda_{\text{vacuum d}}}{n} \right)$$

$$= \frac{1}{\lambda_{\text{vacuum variable}}} \cdot \left(\frac{\pi}{2} \lambda_{\text{vacuum d}} \right)$$

center λ.

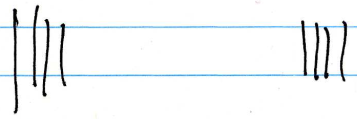
$$\frac{2\pi}{\lambda_{vac}} \frac{n \lambda_{00}}{n \, 4}$$

$$\frac{\pi}{2} \frac{\lambda_{00}}{\lambda_{vac}}$$

$$g_m = n \, g_{vac} = \frac{n}{377}$$

$$\begin{bmatrix} \cos\left(\frac{\pi}{2} \frac{\lambda_{00}}{\lambda}\right) & -i \frac{377}{n_m} \sin\left(\frac{\pi}{2} \frac{\lambda_{00}}{\lambda}\right) \\ -\frac{i n}{377} \sin\left(\frac{\pi}{2} \frac{\lambda_{00}}{\lambda}\right) & \cos\left(\frac{\pi}{2} \frac{\lambda_{00}}{\lambda}\right) \end{bmatrix}$$

Single Mirror



$(HL)^4 H$ $\qquad$ $(HL)^4 H$

$L = 1.38$

$H = 2.35$

$g_0 = \frac{1}{376.7}$

pair $\quad M = \begin{bmatrix} -\left(\frac{L}{H}\right) & 0 \\ 0 & \left(-\frac{H}{L}\right) \end{bmatrix} = (HL)$

$$(HL)^4 (HL)^4 = \begin{bmatrix} \left(-\frac{L}{H}\right)^4 & 0 \\ 0 & \left(-\frac{H}{L}\right)^4 \end{bmatrix}$$

$$(HL)^4 H = \begin{bmatrix} \left(\frac{L}{H}\right)^4 & 0 \\ 0 & \left(\frac{H}{L}\right)^4 \end{bmatrix} \begin{bmatrix} 0 & +\frac{i}{g_0\, 2.35} \\ +i\, g_0\, 2.35 & 0 \end{bmatrix}$$

$$(HL)^4 H = \begin{bmatrix} 0 & +\frac{i}{g_0}\frac{L^4}{H^5} \\ +i g_0 \frac{H^5}{L^4} & 0 \end{bmatrix} = \begin{bmatrix} 0 & +i\, 19.06218 \\ +i\, 0.05246 & 0 \end{bmatrix}$$

(98) 1.6 B&W $\qquad$ OK $\qquad$ 1.00000 ✓

Amplitude reflectivity

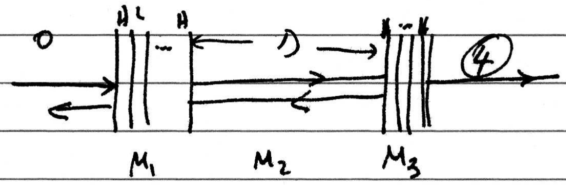
$$R = \frac{(m_{11} + m_{12} g_0) g_0 - (m_{21} + m_{22} g_0)}{(m_{11} + m_{12} g_0) g_0 + (m_{21} + m_{22} g_0)}$$

$$R = - \cdot \frac{-i \frac{L^4}{H^5} g_0 + i \frac{H^5}{L^4} g_0}{+i \frac{L^4}{H^5} g_0 + i \frac{H^5}{L^4} g_0} = \frac{1 - \left(\frac{H^5}{L^4}\right)^2}{1 + \left(\frac{H^5}{L^4}\right)^2}$$

$$= - \frac{1 - 390.5224}{1 + 390.5224} = -0.99489$$

(Computer 0.9949 right on)

Problem 5 -



$$\left.\begin{matrix}(A_0+B_0) & E_x \\ g_0(A_0-B_0) & H_y\end{matrix}\right|_{IN} = M_1 M_2 M_3 \left.\begin{matrix}E_x \\ H_y\end{matrix}\right|_{OUT} \begin{pmatrix}A_4 \\ g_4 B_4\end{pmatrix}$$

$$M_2 = \begin{bmatrix}\cos\left(\frac{2\pi}{\lambda}D\right) & \frac{i}{g_0}\sin\left(\frac{2\pi}{\lambda}D\right) \\ i g_0 \sin\left(\frac{2\pi}{\lambda}D\right) & \cos\left(\frac{2\pi}{\lambda}D\right)\end{bmatrix}$$

We take the point of view that one has made a few computations using the results of the multi-layer theory before making much of an algebraic effort to analyze a given thin film device. We wish to illustrate how by a study of the computer results one can guide himself/herself to develop a good understanding of the physics, i.e., from some computer guided, algebraic analysis.

COMPUTER MOTIVATED –

We find that the mirror itself does not change much on 10 nm intervals.

$$M = \begin{bmatrix} -.29 & +17.5i \\ +.052i & -.29 \end{bmatrix} \quad \sim 545 \text{ nm}$$

$$\begin{bmatrix} 0.0000 & +19.0\,i \\ +0.05i & 0 \end{bmatrix} \quad \sim 550 \text{ nm}$$

$$\begin{bmatrix} 0.285 & +17.5\,i \\ +.05\,i & 0.28 \end{bmatrix} \quad \sim 555$$

above $e^{+i\omega t}$

M= 1.0000

IN 1 nm: the change is

m_{11} .0000 to −19.0774i
(.0576) (−19.0162i)

m_{21} −.0524i 0000
(.0524i (.0576)

LIGHT - PHOTOGRAPHIC RECORD - INDEX MODULATION

$\dfrac{\text{Energy density electric field}}{\text{unit volume}}$ $\quad \underline{E}\cdot\underline{E}^*$ absorb photons

after developing AgBr 1 or 2 photons $\Rightarrow$ entire grain AgBr crystal goes to metallic silver

photopolymer very similar.

Explain concept of an artificial dielectric

$$\frac{\text{No. of scattering centers}}{\text{unit volume}} \quad N_0 = \alpha\, E\cdot E^*$$

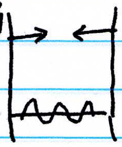
Origin of Refractive Index

$$n = 1 + \frac{N q_e^2}{2\epsilon_0 m(\omega_0^2 - \omega^2)} \qquad N = \frac{\#\text{ of atoms}}{\text{unit volume}}.$$

$$+ \quad N_0 \quad \frac{\text{scatter centers}}{\text{unit volume}} \text{ contribute to index}$$

Ch 31 Feynman
Ch 2 Balanis

$$E = A_0 e^{-ikz} + B e^{ikz} = A_0(e^{-ikz} + B e^{ikz})$$



$$E = 2A_0 \cos kz$$

$$EE^* = 4|A_0|^2 \cos^2 kz = 4|A_0|^2 \tfrac{1}{2}(1+\cos 2kz)$$

$$\cos 2kz \qquad 2kz = 2\,\frac{2\pi}{\lambda}\Delta z = 2\pi$$

$$\Delta z = \frac{\lambda}{2} \text{ for 1 cycle}$$