RADIATION TOPICS

- FULL 4π SOLID ANGLE

$$\underline{A} = \mu \int \underline{J}(\underline{r}') \frac{e^{-ikR_1}}{4\pi R_1} dx'dy'dz'$$

$$\underline{E} = -i\omega\underline{A} - \frac{i}{\omega\mu\epsilon} \nabla(\nabla\cdot\underline{A})$$

- 2π SOLID ANGLE HALF-SPACE
  RAYLEIGH-SOMMERFELD-SMYTHE $\iint$ FORMULA
  REVIEW OPT. 461

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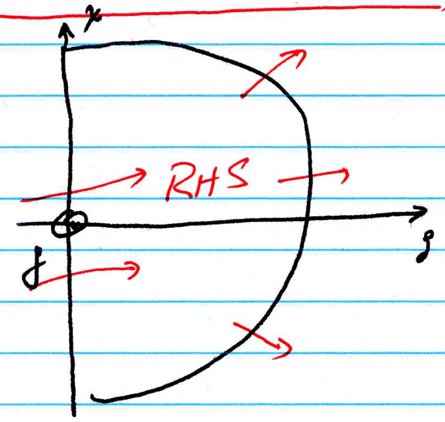
RAYLEIGH - SOMMERFELD - SMYTHE - OVERVIEW

$\boxed{2\pi\text{ - STERADIAN} \quad \text{RIGHT HALF SPACE}}$

STATEMENT OF THE PROBLEM



GIVEN: SOURCES $z < 0$

SOURCE FREE SIMPLE MEDIUM $z \geq 0$
OUTWARD RADIATION $R \to \infty$

EQUATIONS TO SOLVE: $ME \to \begin{cases} \nabla^2 \underline{E} + k^2 \underline{E} = 0 \\ \underline{H} = \frac{-1}{i\omega\mu} \nabla \times \underline{E} \end{cases}$

METHOD: GREEN'S FUNCTION
BOUNDARY CONDITION
SOMMERFELD $\begin{cases} E_{tangential} \ @ \ z = 0 \\ HTD \\ \text{Necessary \& Sufficient} \end{cases}$

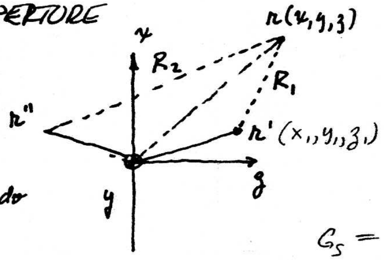
DO NOT NEED TO KNOW E_{normal}
(INCORRECT TO ASSUME ZERO)

Sommerfeld's Formula - Radiation From An Aperture

$G \times \{ \; (\nabla^2 + k^2)\, \phi(x,y,z) = 0$

$\phi \times \{ \; (\nabla^2 + k^2)\, G(\underline{r}, \underline{r}') = -\delta(\underline{r}-\underline{r}')$

$\int_{RHP} (G\nabla^2\phi - \phi\nabla^2 G)\, dv(z) = \int \phi(r)\,\delta(\underline{r}-\underline{r}')\, dv$



$\phi(x_1,y_1,z_1) = -\int_{z=0} (G\nabla\phi - \phi\nabla G)\cdot \underline{e}_z\, dx\, dy + \int (G\nabla\phi - \phi\nabla G)\cdot \underline{n}\, dA \;\; (ii) \to 0$

- $\phi(x_1,y_1,z_1) = \iint_{z=0} \phi(x,y,0)\nabla G\cdot \underline{e}_z\, dx\, dy$

SELECT GREEN'S FUNCTION :

$\nabla^2 G(x,y,z;x_1,y_1,z_1) + k^2 G = -\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)$

i) $G(x,y,0;x_1,y_1,z_1) = 0$

ii) Radiation condition: $\lim_{z\to\infty} f(\theta,\phi)\dfrac{e^{-ikr}}{r}$

LET

$G_s = g + G'$ & put singularity in g

$\int \nabla\cdot\nabla g\, dv + k^2 g = -\int_{V_0} \delta(x-x_1)\delta(y-y_1)\delta(z-z_1)\, dv$

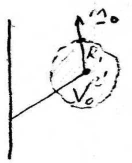
$\int \nabla g\cdot \underline{n}_0\, da + \int k^2 g\, \delta v = -1$

$g = \dfrac{A e^{-ikR_1}}{R_1}$

$\lim_{R_1\to 0}: \quad \nabla g = \dfrac{-A}{R_1^2}\, \underline{n}_0$

$-\dfrac{A}{R_1^2}\, 4\pi R_1^2 + O\left(\dfrac{k^2 A}{R_1}\, \dfrac{4}{3}\pi R^3\right) = -1$

$G_s = \dfrac{e^{-ikR_1}}{4\pi R_1} + G'$ No e^{ikR_1} OUTWARD TRAVELING.



$G_s = \dfrac{e^{-ikR_1}}{4\pi R_1} + \dfrac{B\, e^{-ikR_2}}{4\pi R_2}$

$R_1^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2$ $z_1 > 0$ in RHP

$R_2^2 = (x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2$ $z_2 < 0$ in LHP

$G_s(x,y,0;x_1,y_1,z_1) \equiv 0 \;\Leftarrow\; x_1 = x_2,\; y_1 = y_2,\; z_1 = -z_2$

$G_s = \dfrac{e^{-ik[(x-x_1)^2+(y-y_1)^2+(z-z_1)^2]^{\frac{1}{2}}}}{4\pi R_1} - \dfrac{e^{-ik[(x-x_1)^2+(y-y_1)^2+(z+z_1)^2]^{\frac{1}{2}}}}{4\pi R_2}$

$\dfrac{\partial G_s}{\partial z} = \dfrac{e^{-ikR_1}}{4\pi}\left[-\dfrac{1}{R_1^2}\dfrac{\partial R_1}{\partial z} - \dfrac{ik}{R_1}\dfrac{\partial R_1}{\partial z}\right] - \dfrac{e^{-ikR_2}}{4\pi}\left[-\dfrac{1}{R_2^2}\dfrac{\partial R_2}{\partial z} - \dfrac{ik}{R_2}\dfrac{\partial R_2}{\partial z}\right]$

$\dfrac{\partial R_1}{\partial z} = \dfrac{\frac{1}{2}}{R_1}\, 2(z-z_1)$ $\dfrac{\partial R_2}{\partial z} = \dfrac{\frac{1}{2}}{R_2}\, 2(z+z_1)$

$z = 0: \quad \dfrac{\partial R_1}{\partial z} = \dfrac{-z_1}{R_1}$ $\dfrac{\partial R_2}{\partial z} = +\dfrac{z_1}{R_2}$

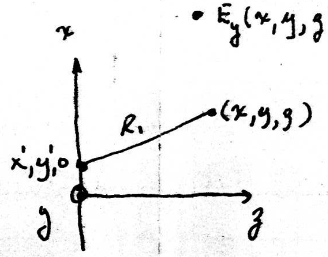
- $\dfrac{\partial G_s}{\partial z} = \dfrac{e^{-ikR_1}}{4\pi R_1}\left(ik + \dfrac{1}{R_1}\right)\dfrac{2z_1}{R_1}$ @ $z = 0$

$\phi(x_1,y_1,z_1) = \iint_{-\infty}^{\infty} \phi(x,y,0)\, \dfrac{e^{-ikR_1}}{2\pi R_1}\left(ik + \dfrac{1}{R_1}\right)\dfrac{z_1}{R_1}\, dx\, dy$

CHANGE NOTATION:

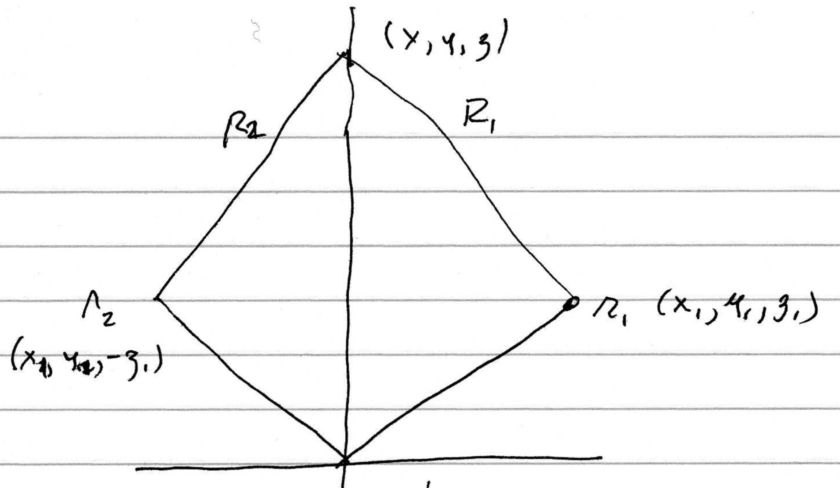
- $E_y(x,y,z) = \iint_{-\infty}^{\infty} dx'dy'\, E_y(x',y',0)\, \dfrac{e^{-ikR_1}}{2\pi R_1}\left(ik + \dfrac{1}{R_1}\right)\dfrac{z}{R_1}\, dx'\, dy'$

$R_1^2 = (x-x')^2 + (y-y')^2 + (z-0)^2$



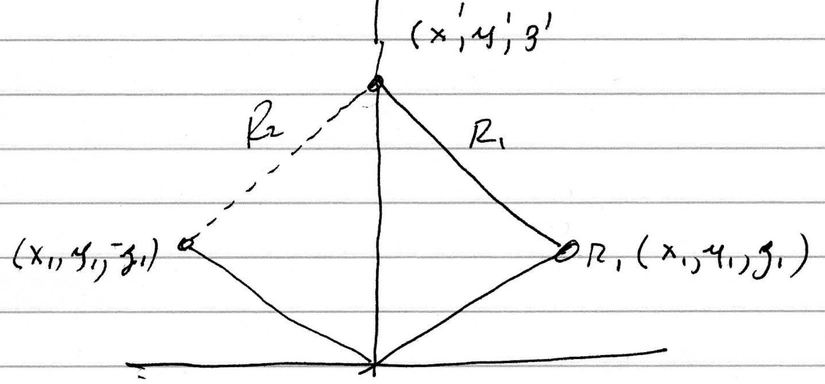
$R_1 \cong R_0 - \dfrac{\cancel{2}xx' + \cancel{2}yy'}{\cancel{2}R_0}$

Nicholas George

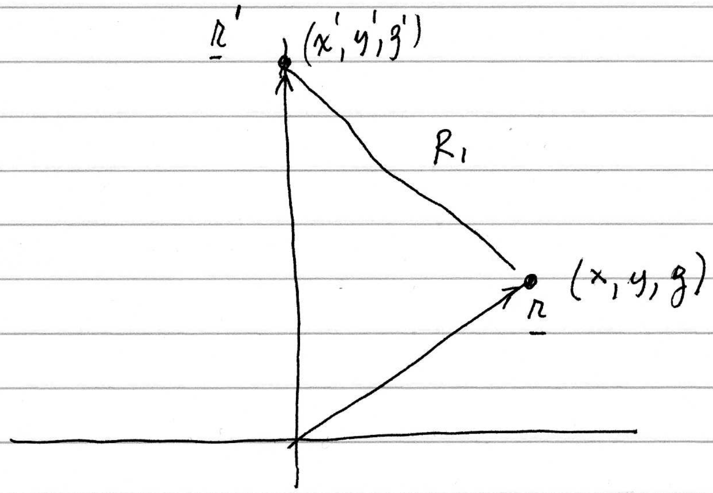


Coordinates

Green's Function Portion



Integration over z = 0 plane



Final Form

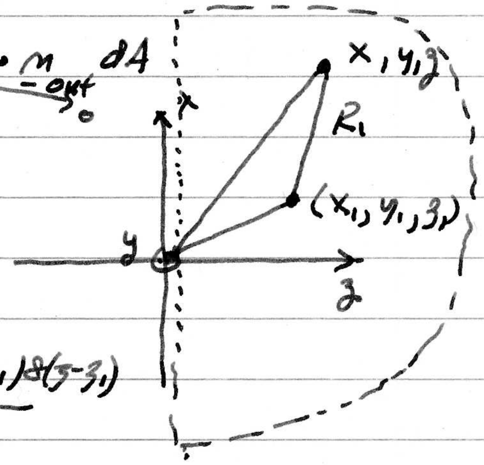
Summary of diagrams used in the derivation

DETAILS

Green's Function - Sommerfeld's Choice

$$\phi(x_1, y_1, z_1) = \iint_{z=0} \phi(x,y,z)\, \nabla G_s \cdot e_z \, dx\,dy \; +$$

$$\cancel{\iint_{II} (G\nabla\phi - \phi\nabla G)\cdot n_{out}}\, dA \to 0$$



FIND

$$\nabla^2 G_s + k^2 G_s = -\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)$$

Let

$$G_s = g^{+e} + G' \quad \Big\{ \text{singularity in } g$$

$$\underbrace{\nabla^2 G' + k^2 G'}_{\text{homog} \;=\; 0} + \underbrace{\nabla^2 g + k^2 g = -\delta(x-x_1')\delta(y-y_1)\delta(z-z_1)}$$

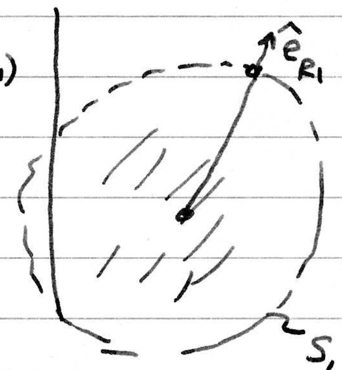
Assume $\quad g = \dfrac{A e^{-ikR_1}}{R_1}$ outward only

$$\nabla\cdot(\nabla g) + k^2 g = -\delta(x-x_1')\delta(y-y_1)\delta(z-z_1)$$

$$\oint_S \nabla g \cdot \hat{e}_{R_1}\, da + k^2 \frac{A e^{-ikR_1}}{R_1} \tfrac{4}{3}\pi R_1^3 = -1$$

$$\nabla \frac{Ae^{-ikR_1}}{R_1} = \hat{e}_{R_1} \frac{\partial \psi}{\partial R_1} = \hat{e}_{R_1} \frac{Ae}{R_1}^{-ikR_1} \left[-ik - \frac{1}{R_1}\right] \overset{1}{\underset{\parallel}{\frac{\partial R_1}{\partial R_1}}}$$

$$= \hat{e}_{R_1} \frac{-Ae^{-ikR_1}}{R_1}\left(ik + \frac{1}{R_1}\right) \to \frac{-Ae^{-ikR_1}}{R_1^2}\hat{e}_{R_1}$$



JACKSON

∇^2

$$-\frac{Ae^{-i0}}{\cancel{R_1^2}} \hat{e}_{R_1}\cdot\hat{e}_{R_1}\, 4\pi \cancel{R_1^2} = -4\pi A = -1$$

$$g = \frac{e^{-ikR_1}}{4\pi R_1}$$

Also an interesting identity (including $r = 0$):

$$\nabla^2 \frac{e^{-ikr}}{4\pi r} + k^2 \frac{e^{-ikr}}{4\pi r} = -\delta(x)\delta(y)\delta(z) = -\delta(\underline{r})$$

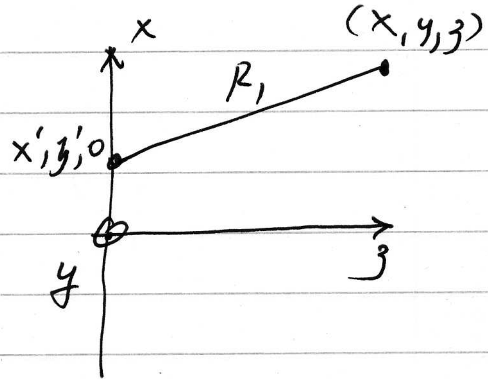
DETAILS

$$E_x(x,y,z) = \iint_{-\infty}^{\infty} E_x(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\,\frac{z}{R_1}\left(ik+\frac{1}{R_1}\right)dx'dy'$$

$$E_y(x,y,z) = \iint_{-\infty}^{\infty} E_y(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\,\frac{z}{R_1}\left(ik+\frac{1}{R_1}\right)dx'dy'$$

To find E_z we use $\nabla \cdot \underline{E} = 0$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$



Note that

$$\frac{\partial}{\partial x}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right) = \frac{e^{-ikR_1}}{2\pi R_1}\left[-ik-\frac{1}{R_1}\right]\frac{\partial R_1}{\partial x}$$

$$R_1^2 = (x-x')^2+(y-y')^2+z^2$$

$$2R_1\frac{\partial R_1}{\partial x} = 2(x-x')\cdot 1$$

$$\frac{\partial R_1}{\partial x} = \frac{x-x'}{R_1}, \quad \frac{\partial R_1}{\partial y} = \frac{y-y'}{R_1}, \quad \frac{\partial R_1}{\partial z} = \frac{z}{R_1}$$

$$\frac{\partial}{\partial z}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right) = \frac{-e^{-ikR_1}}{2\pi R_1}\left(ik+\frac{1}{R_1}\right)\frac{z}{R_1}$$

$$\frac{\partial E_z}{\partial z} = -\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} =$$

$$\frac{\partial E_z}{\partial z} = -\frac{\partial}{\partial x}\iint E_x(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\left(ik+\frac{1}{R_1}\right)\frac{z}{R_1}\,dx'dy' - \frac{\partial}{\partial y}\iint E_y(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\left(ik+\frac{1}{R_1}\right)\frac{z}{R_1}\,dx'dy'$$

$$\frac{\partial E_z}{\partial z} = \iint_{-\infty}^{\infty}\left[E_x(x',y',0)\,\frac{\partial^2}{\partial x\,\partial z}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right) + E_y(x',y',0)\,\frac{\partial^2}{\partial y\,\partial z}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right)\right]dx'dy'$$

$$E_z(x,y,z) = \iint_{-\infty}^{\infty}\left[E_x(x',y',0)\,\frac{\partial}{\partial x}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right) + E_y(x',y',0)\,\frac{\partial}{\partial y}\left(\frac{e^{-ikR_1}}{2\pi R_1}\right)\right]dx'dy'$$

$$E_z(x,y,z) = \iint_{-\infty}^{\infty}\left[E_x(x',y',0)\,\frac{x'-x}{R_1} + E_y(x',y',0)\,\frac{y'-y}{R_1}\right]\frac{e^{-ikR_1}}{2\pi R_1}\left(ik+\frac{1}{R_1}\right)dx'dy'$$

agrees with p18 - Notes 6

WHAT SOMMERFELD DID WAS PATH I

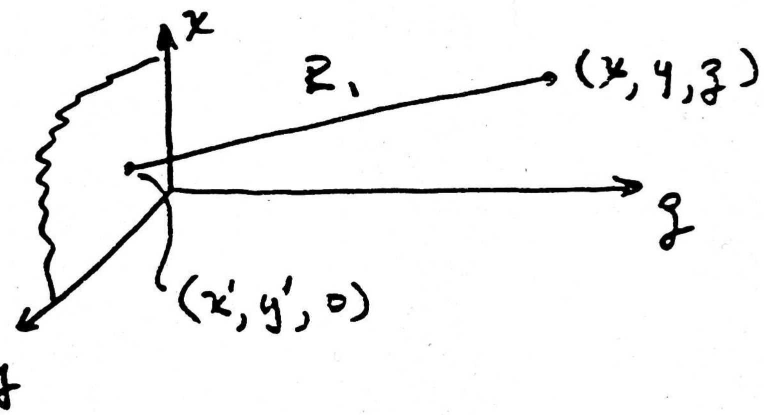
RIGOROUS SOLUTION, AS FOLLOWS:

$$E_x(x,y,z) = \iint_{-\infty}^{\infty} dx'\,dy'\; E_x(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\left(ik + \frac{1}{R_1}\right)\frac{z}{R_1}$$

$$E_y(x,y,z) = \iint_{-\infty}^{\infty} dx'\,dy'\; E_y(x',y',0)\,\frac{e^{-ikR_1}}{2\pi R_1}\left(ik + \frac{1}{R_1}\right)\frac{z}{R_1}$$

$$E_z(x,y,z) = \iint_{-\infty}^{\infty} dx'\,dy'\left\{E_x(x',y',0)\,\frac{x'-x}{R_1} + E_y(x',y',0)\,\frac{y'-y}{R_1}\right\}\frac{e^{-ikR_1}}{2\pi R_1}\left(ik + \frac{1}{R_1}\right)$$

SOURCES
<0



ISOTROPIC
μ, ϵ
$z \geq 0$

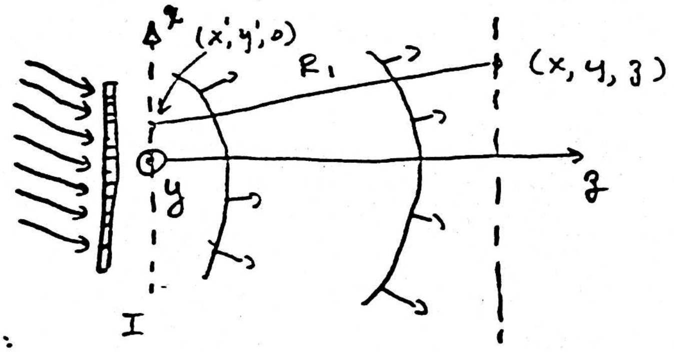
$$R_1 = \left[(x - x')^2 + (y - y')^2 + (z - 0)^2\right]^{1/2}$$

W.R SMYTHE FOUND A CONCISE VECTOR FORM FOR THIS RESULT
PHYS REV $\underline{72}$, 1066 (1947)

$$\underline{E}(\underline{r}) = 2\int (\underline{n} \times \underline{E}) \times \nabla' G\, da' \quad , e^{i\omega t}$$

$$G = \frac{e^{-ikR_1}}{4\pi R_1} \quad , (\underline{\text{Not}}\ G_s)$$

SO TO SUMMARIZE THIS DIFFRACTION THEORY, FOR SOURCES $e^{i\omega t}$ AT $z<0$, TRANSMISSION SCREEN AND ALL, IF THE INPUT FIELD $E_x(x', y', 0)$ IS KNOWN, THEN $E_x(x, y, z)$ IS DETERMINED EXACTLY FROM SIMPLY* INTEGRATING OVER I



GIVEN:

$E_x(x', y', 0)$ & $E_y = 0$

$E_x(x, y, z)$ TO FIND

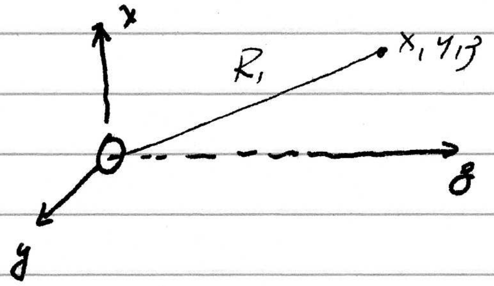
$$E_x(x,y,z) = \iint_I dx'dy'\, E_x(x',y',0)\, \frac{e^{-ikR_1}}{2\pi R_1}\left(ik + \frac{1}{R_1}\right)\frac{z}{R_1}$$

* BOOKS HAVE BEEN WRITTEN ON HOW TO APPROXIMATE & THEN INTEGRATE - BUT NO NEW IDEAS OF PHYSICS-SORT OCCUR

Sommerfeld's basic result above applies for any scalar, transverse component of the electric field. The basis of Fourier optics is the use of the equation above together with linear system theory. You should think in terms of an input field and an output field connected by a "linear" operation, i.e., the integration with some deterministic kernel that is determined from the configuration of the system.

ILLUSTRATIVE PROBLEMS

TINY PINHOLE IN CONDUCTING SHEET



Gm: $E_x(x,y,0) = \frac{E_0}{\Delta a}\,\delta(x,y)$ & $E_y = 0$

$$E_x(x,y,z \geq 0) = \iint dx'dy'\, E_x(x',y',0)\,\frac{e^{-ik\left[(x-x')^2+(y-y')^2+z^2\right]^{\frac{1}{2}}}}{2\pi R_1}\left(ik + \frac{1}{R_1}\right)\frac{z}{R_1}$$

$$E_x(x,y,z) = \frac{E_0}{\Delta a}\;\frac{e^{-ikr}}{2\pi r}\left(ik + \frac{1}{R_0}\right)\frac{z}{r}$$ Basic Result

Imaging and non-Imaging

$$\frac{|ik|}{\frac{1}{R_0}} \gg 10$$

$$kr = \frac{2\pi}{\lambda} r \gg 10$$

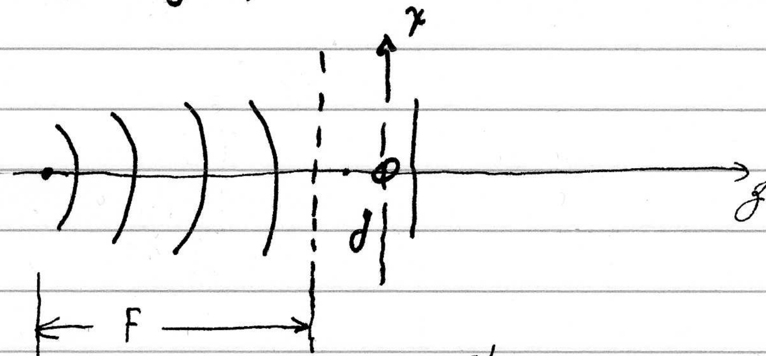
$$r \gg \lambda$$

The near-zone term is $\frac{1}{r_0}$ portion -

PERFECT/IDEALIZED LENS

Given: Maxwell's Equations
The RHS Solution to describe a point source

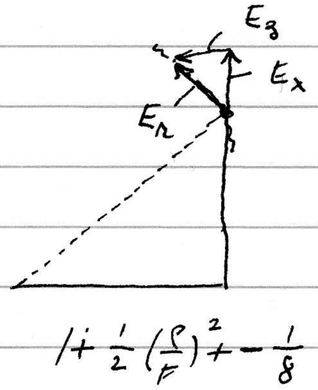
1) Expanding spherical wave is converted to a plane wave



$$E_r(x, y, F) = \frac{E_0}{\Delta a} \frac{e^{-ikr}}{2\pi r} ik \frac{F}{r}$$ $\rightarrow$ plane wave

define transmission function $t(x, y) = \dfrac{\text{exiting electric field component}}{\text{entering electric field}}$

$$t(x, y) = \frac{C}{\frac{E_0}{\Delta a} \frac{ikF}{2\pi r\, r} e^{-ikr}}$$



take major phase term
scalar fld $\underline{E_r \quad \text{a choice}}$

absorbs $\frac{F}{r}$ piece

$$\underline{1 + \frac{1}{2}\left(\frac{\rho}{F}\right)^2 + -\frac{1}{8}\left(\frac{\rho}{F}\right)^4 + \frac{1}{16}\left(\frac{\rho}{F}\right)^6}$$

$$t(x, y) = (\text{apodization})\, e^{ikr}$$

$$\cong e^{ik(F^2+\rho^2)^{1/2}} \qquad ik(F^2+\rho^2)^{1/2} = ikF\left(1+\left(\frac{\rho}{F}\right)^2\right)^{1/2}$$

$$\frac{\pi}{\lambda F}\rho^2 = \frac{\cancel{2}\pi}{\lambda}\cancel{F}\frac{1}{\cancel{2}}\frac{\rho^2}{\cancel{F}F} \qquad -\frac{2\pi}{\lambda}F\frac{1}{8}\frac{\rho^4}{F^4} = -\frac{\pi}{\lambda\, 4F^3}\rho \ \ldots$$

IDEALIZED PERFECT LENS

U. Gerg

DEFINITION: CONVERT EXPANDING PT SOURCE INTO A PLANE WAVE - FORM VALID IN NON-PARAXIAL REGIME
DROP APODIZATION - PHASE ONLY - THIN LENS FORM

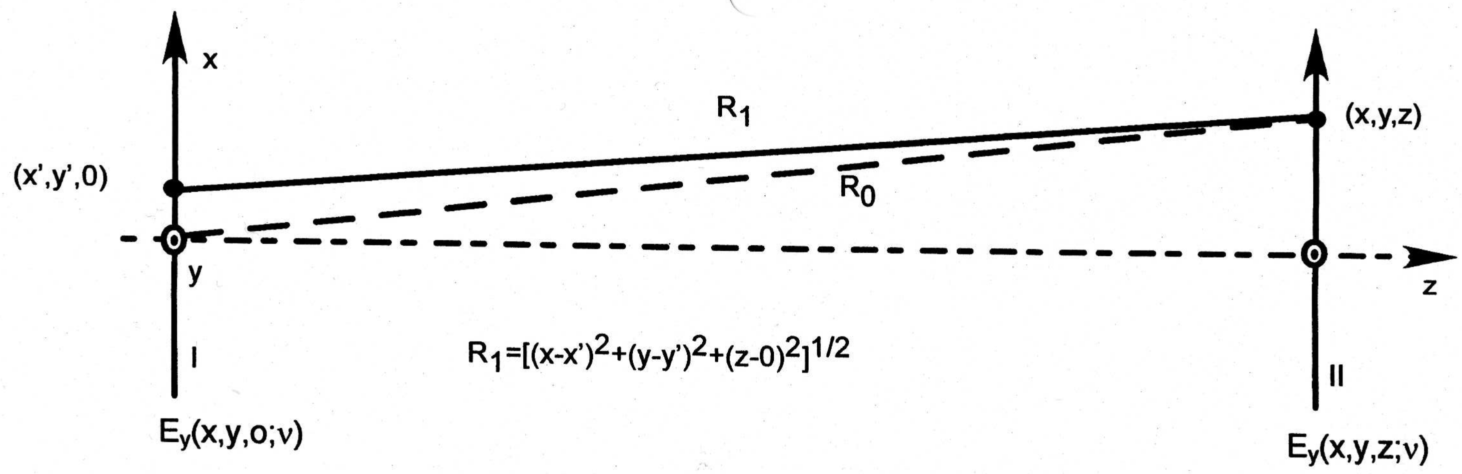
$$t(x,y) = t(\rho) = e^{ikF(1+(\frac{\rho}{F})^2)^{1/2} - ikF}$$

$$t(\rho) = e^{\frac{i\pi}{\lambda F}\rho^2 - \frac{i\pi \rho^4}{4\lambda F^3} + \frac{i\pi}{8\lambda F^5}\rho^6 - \cdots}$$

Interestingly, we see the non-zero "spherical aberration" term that is required for perfect imaging
This lens is technically called an asphere.

In Fourier optics in order to make the integrals for the Fresnel zone reasonable, one commonly uses a paraxial approximation to the lens alone - i.e., namely

$$t_{,p}(\rho) = e^{\frac{i\pi}{\lambda F}\rho^2}$$ dropping higher orders



$$R_1=[(x-x')^2+(y-y')^2+(z-0)^2]^{1/2}$$

$$\underset{x}{E_y}(x,y,z;\nu)=\int_{-\infty}^{\infty}\int dx'dy'\underset{x}{E_y}(x',y',0)\frac{e^{-ikR_1}}{2\pi R_1}(ik+\frac{1}{R_1})\frac{z}{R_1}$$

(E_x, E_y FORMS)

$$E_z(x,y,z;\nu)=\int_{-\infty}^{\infty}\int dx'dy'\{E_x(x',y',0)\frac{x'-x}{R_1}+E_y(x',y',0)\frac{y'-y}{R_1}\}\frac{e^{-ikR_1}}{2\pi R_1}(ik+\frac{1}{R_1})$$

FRESNEL ZONE (PARAXIAL)

$$E_y(x,y,z;\nu)=\frac{ie^{-ikz}}{\lambda z}\int_{-\infty}^{\infty}\int dx'dy'E_y(x',y',0)e^{\frac{-i\pi}{\lambda z}[(x-x')^2+(y-y')^2]}$$

FAR-ZONE (FRAUNHOFER) WIDE-ANGLE

$$E_y(x,y,z;\nu)=\frac{ie^{-ikz}}{\lambda R_0}(\frac{z}{R_0})\int_{-\infty}^{\infty}\int dx'dy'E_y(x',y',0)e^{i2\pi[(\frac{x}{\lambda R_0})x'+(\frac{y}{\lambda R_0})y']}$$

FOURIER OPTICS
SOMMEFELD-RAYLEIGH-SMYTHE FORMULA
RADIATION FROM A PLANAR APERTURE

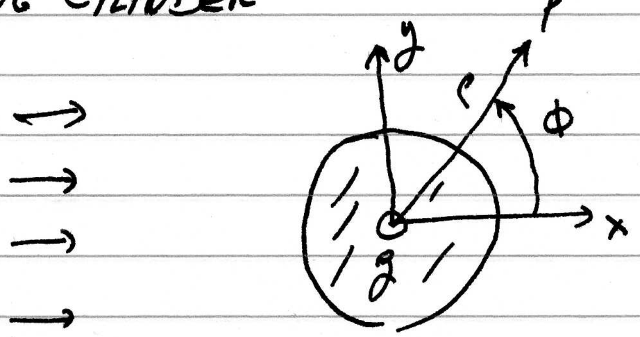
ILLUSTRATIVE PROBLEMS

OPT 461
Nicholas George

ILLUSTRATIVE SCATTERING PROBLEM

CYLINDER PROBLEM $\qquad e^{i\omega t}$

1. CONDUCTING CYLINDER



INCIDENT PLANE WAVE $\quad z$-polarized Electric Field

WE HAVE SHOWN FOURIER SERIES EARLIER:

$$e^{-ik(x=\rho\cos\phi)+i\omega t} = \sum_{m=-\infty}^{\infty} J_m(k\rho)\, e^{im(\phi-\frac{\pi}{2})}$$

AT POINT $P(\rho,\phi,z)$ we write the total electric field consisting of the incident wave and the scattered wave. Of course the scattered wave is in a form consistent with that learned from the separation of variables —

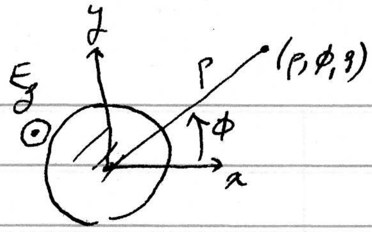
$$\rho \geq a \qquad E_z = P(\rho)\,\Phi(\phi)\,\cancel{Z(z)} \qquad \text{no } z\text{-variation}$$

$$\left\{ \cancel{H_m^{(1)}(k\rho)},\ H_m^{(2)}(k\rho) \right\} \quad \left\{ e^{\pm im\phi} \right\}$$

$$\text{in } \cancel{\xrightarrow{\infty} e^{ik\rho}} \qquad \xrightarrow{\infty} e^{-ik\rho} \text{ out}$$

$$E_z(\rho,\phi,z) = E_0 e^{-ik\rho\cos\phi} + \sum_{m=-\infty}^{\infty} a_m H_m^{(2)}(k\rho)\, e^{im(\phi-\frac{\pi}{2})}$$

CONDUCTING CYLINDER



$$0 = E_z(a,\phi) = E_0 e^{-ika\cos\phi} + \sum a_m H_m^{(2)}(ka)\, e^{im(\phi-\frac{\pi}{2})}$$

$$0 = E_0 \sum_{m=-\infty}^{\infty} J_m(ka)\, e^{im(\phi-\frac{\pi}{2})} + \sum a_m H_m^{(2)}(ka)\, e^{im(\phi-\frac{\pi}{2})}$$

$$a_m = -\frac{E_0 J_m(ka)}{H_m^{(2)}(ka)}$$

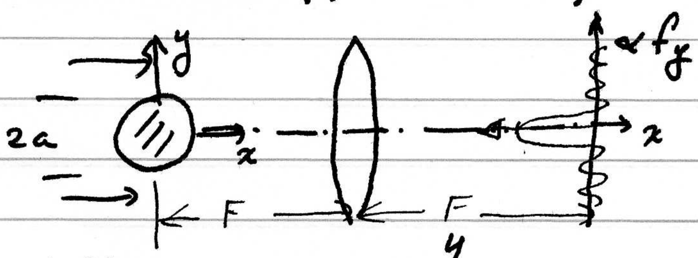
$$\underline{E}(\rho,\phi,z) = \hat{z}\, E_0 \sum_{m=-\infty}^{\infty} \left[J_m(k\rho) - \frac{J_m(ka)\, H_m^{(2)}(k\rho)}{H_m^{(2)}(ka)} \right] e^{im(\phi-\frac{\pi}{2})}$$

From the computer problems, we learn the details of $E_z(\rho,\phi)$

i) Fairly bland pattern $\frac{a}{\lambda}$ from 0 to 1

ii) Lots more "lobe-detail" $\frac{a}{\lambda}$ 10 to 100

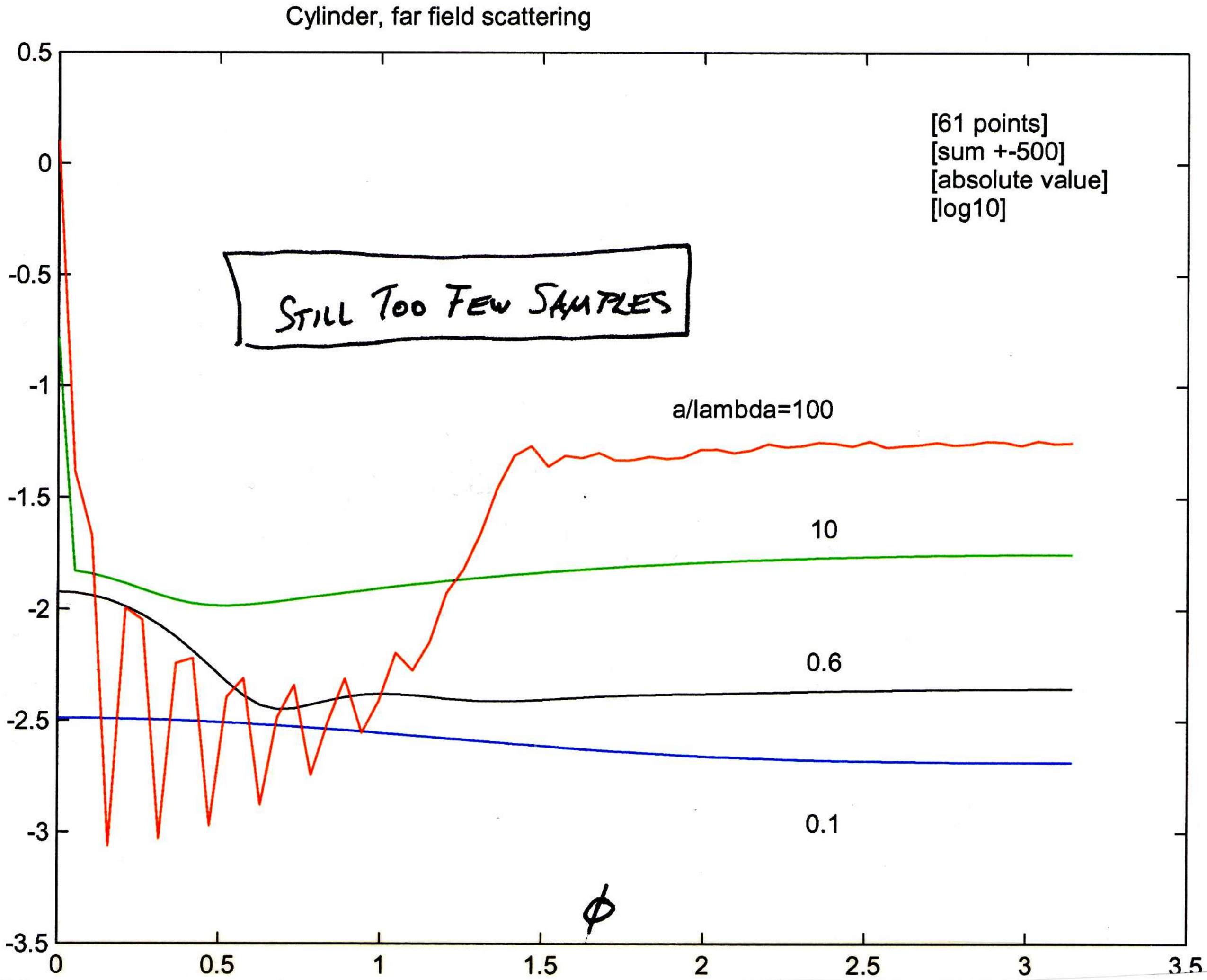
iii) In diffraction pattern sampling, the following configuration is typical:



input $\quad t \cong 1 - \text{rect}\,\frac{y}{2a}$

$$\mathcal{F}_y = \delta(f_y) - 2a\,\text{sinc}(2a f_y)$$

The lobes on sinc pattern (as with a slit) correspond only very roughly to the pattern of an actual wire.



The above figures are plots for the absolute value of scattered field in the far-zone for ϕ ranging from 0 to π. You need enough sampling points in the angle to see the detailed structure in the scattering pattern.

We also tried the plots for the real part of scattered field in the far-zone. They are not the same as the absolute value. In real world measurement, we measure the time average of the light intensity which is proportional to the square of the absolute value of scattered field. Therefore $|E_{scatt}|$ is more useful to plot.

FIG. 10-1

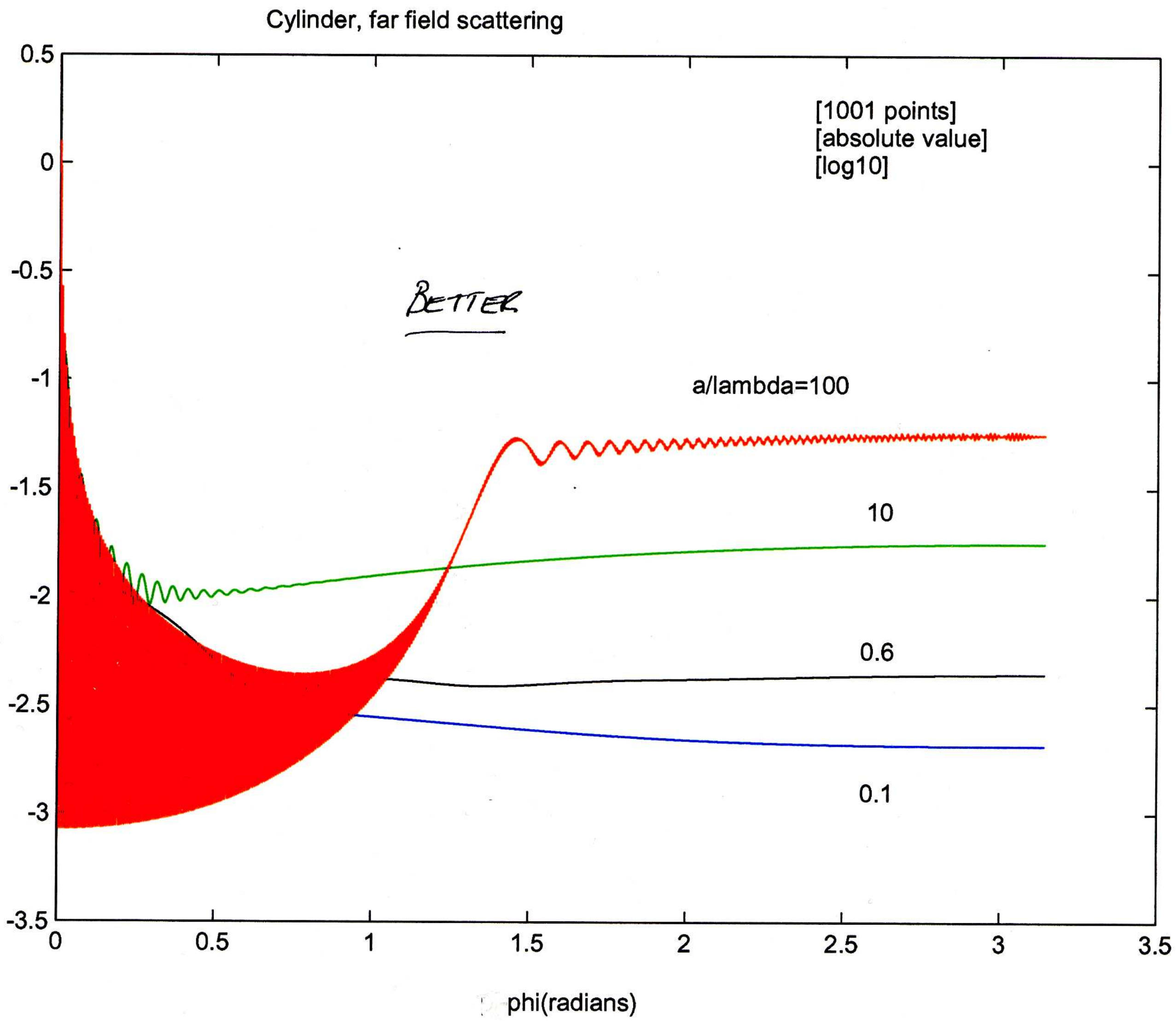
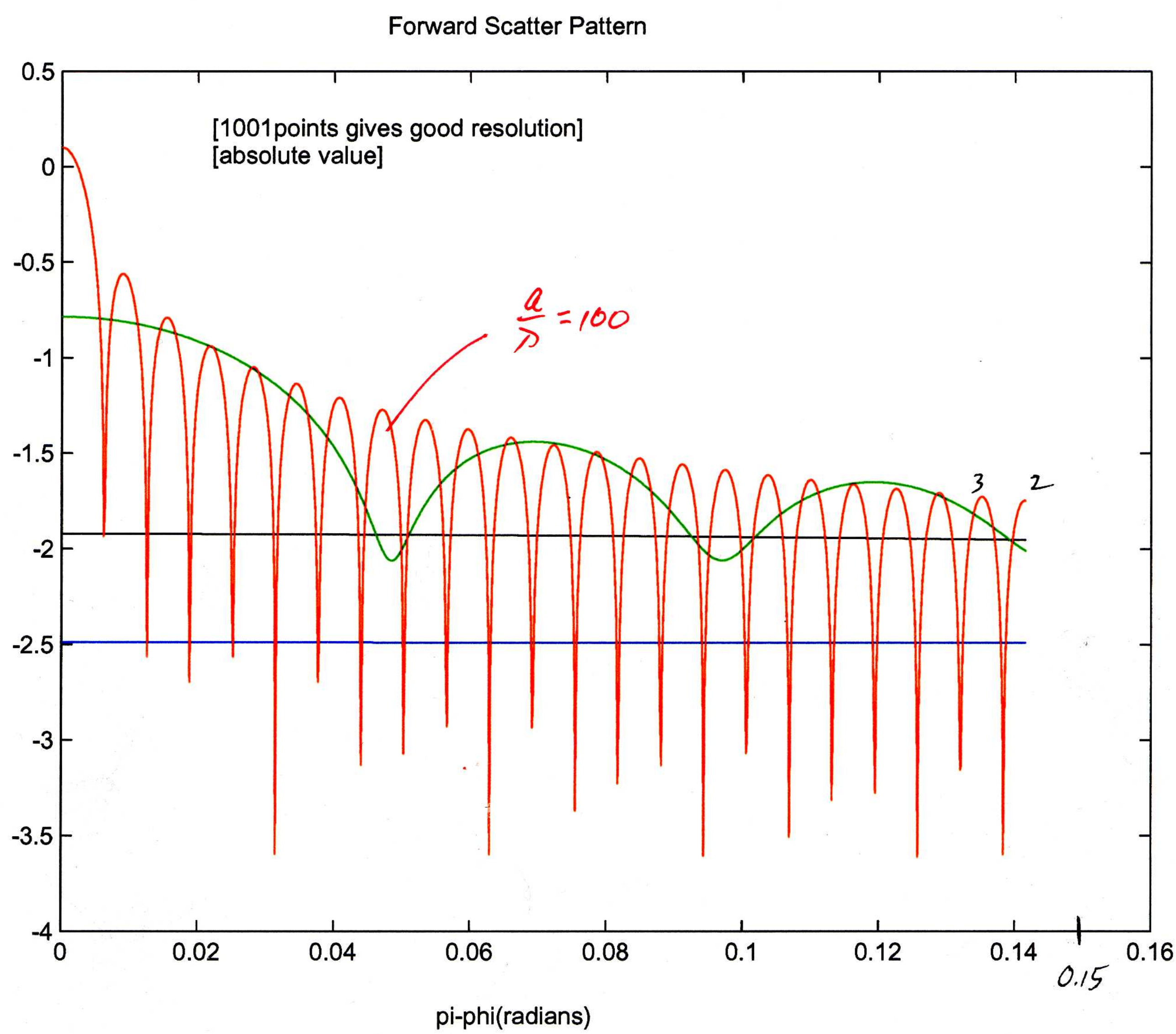

Cylinder, far field scattering
[1001 points]
[absolute value]
[log10]
BETTER
a/lambda=100
10
0.6
0.1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
0.5
1
1.5
2
2.5
3
3.5
phi(radians)

FIG 10-2



Exact calculation: 24 lobes from $\phi = 0$ to 0.15 rad.

DPS Fourier approx: 31 lobes from $\phi = 0$ to 0.15 rad.

FIG 10-3

WAVE EQUATION

CYLINDRICAL

$$\mathcal{P}(\rho)\ \Phi(\phi)\ Z(z) \qquad k_\rho^2 + k_z^2 = k^2 = \omega^2\mu\epsilon$$

$$\left\{ \begin{matrix} J_m(k_\rho \rho) \\ N_m(k_\rho \rho) \end{matrix} \right\} \text{ or } \begin{matrix} H_m^{(1)}(k_\rho \rho) \\ H_m^{(2)}(k_\rho \rho) \end{matrix} \rightarrow \left(\frac{2}{\pi \xi}\right)^{1/2} e^{-i(\xi - \frac{m\pi}{2} - \frac{\pi}{4})}$$

outward going

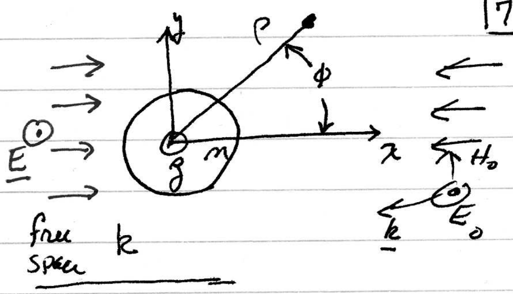
$\xi \to 0$ singularity $N_m(\xi)$, $H_m^{(1)}(\xi)$, $H_m^{(2)}(\xi)$ | $J_m(\xi)$ okay

$e^{\pm i m\phi}$

HELMHOLTZ WAVE EQUATION / DIELECTRIC CYLINDER $e^{+i\omega t}$

7

Travel $+x$; E_z polarized (TM)



$\nabla \times \underline{E} = -i\omega\mu \underline{H}$

$\nabla \times \underline{H} = \underline{J} + i\omega\epsilon \underline{E}$

BOUNDARY INDEX n when $\rho = a$

$\boxed{(1)}$ $E_{\text{tangential}}$ contin $\Rightarrow$ E_z $(\rho = a)$

$H_{\text{tangential}}$: $\nabla\times(\hat{z}E_z) = \hat{\rho}\,\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} + \hat{\phi}\left(-\frac{\partial E_z}{\partial\rho}\right) + \hat{z}\,0$

$\boxed{(2)}$ $H_\phi = \frac{1}{-i\omega\mu}\left(-\frac{\partial E_z}{\partial\rho}\right) = \frac{-i}{\omega\mu}\frac{\partial E_z}{\partial\rho}$ $\qquad \frac{1}{i^m} = \frac{1}{(e^{i\frac{\pi}{2}})^m} = e^{-im\frac{\pi}{2}}$

$$e^{-ikx = -ik\rho\cos\phi} = \sum_{m=-\infty}^{\infty} i^{-m} J_m(k\rho)\, e^{im\phi}$$ $e^{-ikx+i\omega t}$ ← INCIDENT

$$= \sum_{m=-\infty}^{\infty} J_m(k\rho)\, e^{i(m\phi - m\frac{\pi}{2})}$$ ✓

OUTSIDE : $\rho > a$

• $$E_z(\rho,\phi) = \sum_{m=-\infty}^{\infty} J_m(k\rho)\, e^{im(\phi-\frac{\pi}{2})} + \sum_{m=-\infty}^{\infty} a_m H_m^{(2)}(k\rho)\, e^{im(\phi-\frac{\pi}{2})}$$

INSIDE : $\rho < a$ diel n

• $$E_z(\rho,\phi) = \sum_{m=-\infty}^{\infty} b_m J_m(nk\rho)\, e^{im(\phi-\frac{\pi}{2})}$$

OUT $\qquad$ IN

(1) $$a_m H_m^{(2)}(ka) + J_m(ka) = b_m J_m(nka)$$ $\qquad \frac{\partial}{\partial\rho} = (1)$

(2) $$\cancel{k}\, a_m H_m^{(2)\prime}(k\rho) + \cancel{k}\, J_m'(ka) = n\cancel{k}\, b_m J_m'(nka)$$

Dielectric Cylinder

$$\begin{bmatrix} H_m(ka) & -J_m(nka) \\ H_m'(ka) & -n J_m'(nka) \end{bmatrix} \begin{bmatrix} a_m \\ b_m \end{bmatrix} = \begin{bmatrix} -J_m(ka) \\ -J_m'(ka) \end{bmatrix}$$

$$\begin{bmatrix} a_m \\ b_m \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -n \underline{J_m'}(nka) & J_m(nka) \\ -\underline{H_m'} & H_m(ka) \end{bmatrix} \begin{bmatrix} -J_m(ka) \\ -\underline{J_m'}(ka) \end{bmatrix}$$

$$\Delta = -n H_m^{(2)}(ka)\, \underline{J_m'}(nka) + \underline{H_m^{(2)\prime}}(ka)\, J_m(nka)$$

$$a_m = \left[n\, \underline{J_m'}(nka)\, J_m(ka) - J_m(nka)\, \underline{J_m'}(ka) \right] / \Delta$$

$$b_m = \left[\underline{H_m^{(2)\prime}}(ka)\, J_m(ka) - H_m^{(2)}(ka)\, \underline{J_m'}(ka) \right] / \Delta$$

Outside:

$$E_z(\rho, \phi) = \sum_{m=-\infty}^{\infty} \left\{ J_m(k\rho) + a_m H_m^{(2)}(k\rho) \right\} e^{im(\phi - \frac{\pi}{2})}$$

$e^{i\omega t}$

Inside

$$E_z(\rho, \phi) = \sum_{m=-\infty}^{\infty} b_m J_m(nk\rho)\, e^{im(\phi - \frac{\pi}{2})}$$