# Sources in Unbounded Regions

## Statement of the Topic

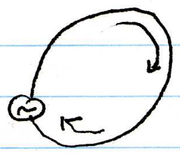
Long wire

All About Wire Antennas
Radiation Problems

CH2 2.12 & 2.14

2.16 & 15

2.24 E terms of A alone



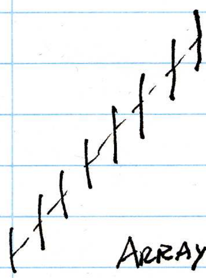
Loop

Ch 6 Balanis particularly SKIM

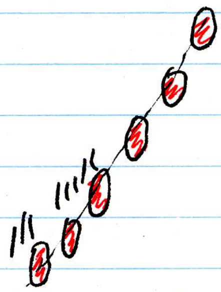
256 –

278 : 6-91 to 95

- 6.8.2 Far Field – 6.112



Array



Nicholas [illegible]

OPTICS 462 **Problem Set 10** NOT TO BE HANDED IN ~~Due April 28, 2000~~

**Nicholas George**

1. **Magnetic Dipole**: Consider an infinitesimal current loop $I_0 e^{i\omega t}$ of radius $a \ll \lambda$. Find the radiation field $E_\phi$ for this "small loop antenna."

2. Consider a larger radius loop antenna with a current $I_0 e^{i\omega t}$. Find the vector potential $\mathbf{A}(\mathbf{r})$ in the far-zone (approximate solution).

3. **Electric Dipole**: Consider this fundamental radiating structure [Hertz, 1893] with a dipole moment $\mathbf{p}$ given by

$$\mathbf{p} = q\mathbf{L}e^{i\omega t}$$

or the current density

$$\mathbf{J}(\mathbf{r}) = \hat{z}I\mathbf{L}\delta(\mathbf{r})\,.$$

Complete the details of the lecture derivation as follows:
(a) Find the radiation form for the electric field vector.
(b) Find the exact solution for the electric field vector $\mathbf{E}(\mathbf{r})$.

4. For an open-circuited transmission line of length $L$ and characteristic impedance $\eta_0$,
(a) Write the transmission line equations, and
(b) Write an expression for the current as a function of $L$,

5. Derive the equation for the standing wave of current in a long thin wire, i.e.,

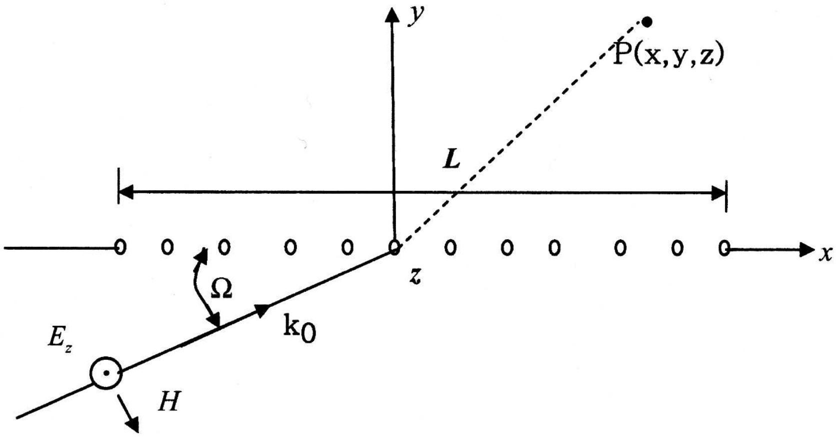
$$I(z) = I_0 \sin k(L - |z|)\,.$$

Contributions to this result are attributed to Pocklington(1897) and J. Labus(1933). The "picture" by Schelkunoff is that arising from problem 4.

6. Make computer plots of the radiation patterns for center driven thin-wire antennas of lengths $kl = \pi/2;\ \pi;\ 3\pi/2.$

7. A monochromatic plane wave is incident at an angle $\Omega$ on a closely-spaced ($\lambda/4$ or less) array of half-wave dipoles, as shown. The incident electromagnetic wave is polarized along the z-axis, and it is scattered by the thin, perfectly conducting wires also oriented along the z-axis.
(a) Calculate the scattered radiation in the farfield at $P$, neglecting any interaction between the dipoles that are a number $n_0$ per unit length for a length $L$.

(b)Plot the scattered farfield radiation in the x-y plane at $z = 0$ when $\Omega = 20^0$ and $L = 40\lambda$.




8. Considering radiation into the right-half-space, you are to summarize the solutions of the Rayleigh-Sommerfeld-Smythe derivation, providing the derivation for $E_Z(x,y,z)$.

9. Also for the R-S-S derivation, (a) write the Green's function $G_S$ that was used in the derivation and (b) show the details for the computation of $\frac{\partial G_S}{\partial z}$.

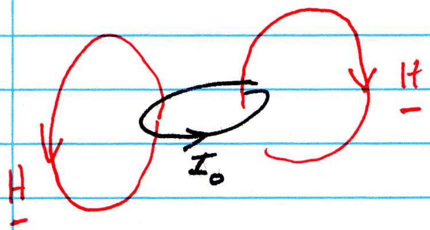
10. The original derivation of a vector form for the radiation from "apertures in plane conducting screens" by W. R. Smythe, Phys. Rev. 72, 1066 (1947) is given in the third edition of his book. While not covering this derivation in class, you may read it in SM 12.18. The result in an $\exp(j\omega t)$ formulism is given by

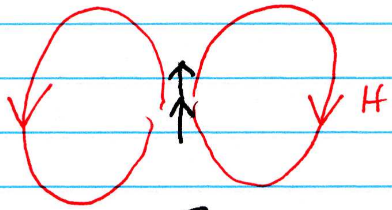
$$\mathbf{E}(\mathbf{r}) = \frac{1}{2\pi} e^{j\omega t} \nabla \times \int_S (\mathbf{n} \times \mathbf{E}') \frac{e^{-j\beta r}}{r} dx'dy',$$

in which $\mathbf{E}'$ is the aperture field, $r$ is the same as our $R_1$ in class, and $\beta$ is $k$. Find the component value $E_x, E_y, E_z$ for comparison to R-S-S formulas.

11. Find an expression for the on-axis field strength at arbitrary z when a perfectly-conducting, thin circular disc is placed in the plane at z=0. Consider a normally incident plane wave. Hint: See Applied Optics 26, 2360 (1987) R. English and N. George. Compare your result for small z to that obtained using the approximate Fresnel-zone forms.

# MAGNETIC DIPOLE



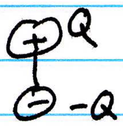

SYMBOL

⊕ m

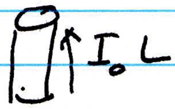
⊖ m

magnetic monopole would give same field

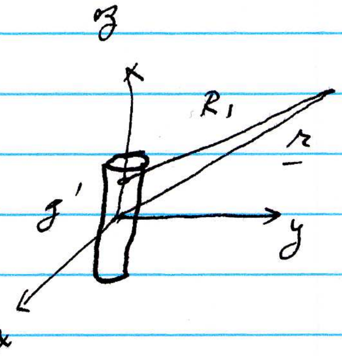
# ELECTRIC DIPOLE




# CURRENT



# ELECTRIC DIPOLE    CALLED HERTZIAN ELECTRIC DIPOLE (1893)    3




Gn: Thin linear current of very short length

Current dipole $I_0 L_z e^{i\omega t}$

SOURCE DESCRIPTION:

$$\underset{e}{I}(z) = I_0 \quad \text{for tiny } L_x, L_y, L_z$$

$$\hat{z}\, I_0 \,\text{rect}\,\frac{z}{L_z} \quad \text{IS A START}$$

Balanis 279-80

Example 6.3

NEED TO LIMIT $x, y$

$$\underline{J}(\underline{r}') = \hat{z}\,\frac{I_0}{L_x L_y}\,\text{rect}\,\frac{x}{L_x}\,\text{rect}\,\frac{y}{L_y}\,\text{rect}\,\frac{z}{L_z}$$

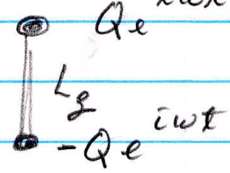
$$\underline{J}(\underline{r}') = \hat{z}\, I_0 L_z \lim_{L_{xyz}\to 0}\left(\frac{1}{L_x}\text{rect}\,\frac{x}{L_x}\right)\left(\frac{1}{L_y}\text{rect}\,\frac{y}{L_y}\right)\left(\frac{1}{L_z}\text{rect}\,\frac{z}{L_z}\right)$$

$$\boxed{\underline{J}(\underline{r}') = \hat{z}(I_0 L_z)\quad \delta(x')\,\delta(y')\,\delta(z')}$$

$$\underline{A}(\underline{r}) = \hat{z}\,\frac{\mu}{4\pi}\int \frac{J(\underline{r}')\,e^{-ikR_1}}{R_1}\,dx'dy'dz'$$

$$= \hat{z}\,\frac{\mu}{4\pi}(I_0 L_z)\int \frac{\delta(x')\delta(y')\delta(z')\,e^{-ikR_1}}{R_1}\,dx'dy'dz'$$

$$\underline{A}(\underline{r}) = \hat{z}\,\frac{\mu}{4\pi r}(I_0 L_z)\,\frac{e^{-ikr}}{r} \qquad \text{EXACT}$$




WHY DIPOLE MOMENT:

definition $\quad \underline{p} = \hat{z}\, Q L_z\, e^{i\omega t}$

$$i\omega p = \frac{dp}{dt} = \hat{z}\,\frac{dQ}{dt}L_z = \hat{z}\, I_0 L_z \qquad p = p_0 e^{i\omega t}$$

$$\boxed{i\omega \underline{p}_0 = \hat{z}\, I_0 L_z}$$

ELECTRIC DIPOLE - CONT

$$\underline{E} = -i\omega \underline{A} - \frac{i}{\omega\mu\epsilon}\nabla(\nabla\cdot\underline{A})$$

$$E_r = \eta \frac{(I_0 L_z)\cos\theta}{2\pi r^2}\left(1 + \frac{1}{ikr}\right)e^{-ikr}$$

$$E_\theta = i\eta \frac{\beta(I_0 L_z)\sin\theta}{4\pi r}\left[1 + \frac{1}{ikr} - \frac{1}{(kr)^2}\right]e^{-ikr}$$

$$E_\phi = 0$$

See (7) SM 1201

KONG, 238

$$\mu\underline{H} = \nabla\times\underline{A}$$

$$\underline{H} = \hat{\phi}\frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = i\frac{k(I_0 L_z)\sin\theta}{4\pi r}\left(1 + \frac{1}{ikr}\right)e^{-ikr}$$

$\frac{1}{r}$ radiation terms

$$E_\theta \cong i\eta \frac{k(I_0 L_z)\sin\theta}{4\pi r}$$

$$H_\phi \cong i\frac{k(I_0 L_z)\sin\theta}{4\pi r}$$

$$\frac{E_\theta}{H_\phi} \sim \eta$$

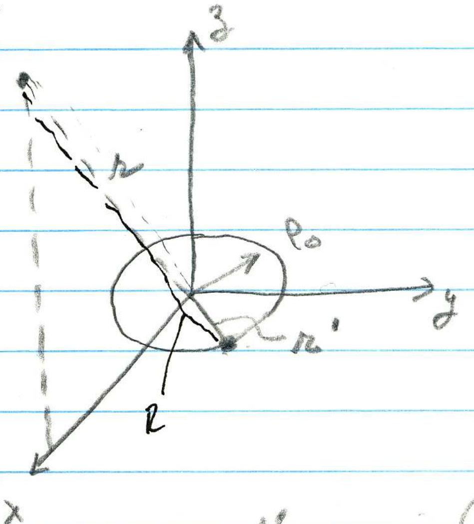
$e^{i\omega t}$

Gn: radius $\rho_0$

$I_0$

no $\phi$ variation

$\underline{J}\cdot \underline{\delta a}\; \rho d\phi$



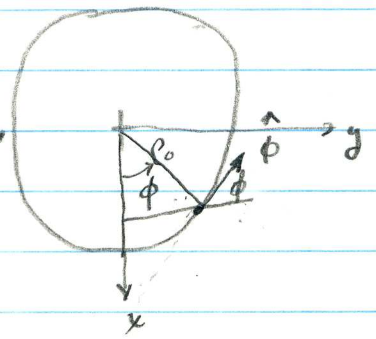
$R = r - r'\cos\psi$

$\cos\psi = \cos\phi\,\sin\theta$

$R = r - \rho_0\cos\phi'\sin\theta$

$$\underline{A} = \frac{\mu}{4\pi r}\int I_0\, e^{-ikR}\,\hat{\phi}\,\rho_0\, d\phi$$

$$\underline{A} = \frac{\mu I_0}{4\pi r}e^{-ikr}\int e^{ik\rho_0\sin\theta\cos\phi'}\,[-\hat{x}\sin\phi' + \hat{y}\cos\phi']\,\rho_0\, d\phi'$$



$$\int_0^{2\pi}\sin\phi'\, e^{i\alpha\cos\phi'}\, d\phi' = 0$$

$$\int_0^{2\pi}\cos\phi'\, e^{i\alpha\cos\phi'}\, d\phi'$$

$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$

$R = r - \rho_0\sin\theta\cos\phi$

$$\underline{A} = \frac{\mu I_0 e^{-ikr}}{4\pi r}\,\rho_0\,\hat{y}\; i\,2\pi\; J_1(k\rho_0\sin\theta)$$

We picked pt $P$ @ $\phi = 0$ in order to compute $\hat{\phi}$ component., Hence

$$\hat{\phi}A_\phi = i\,\frac{\mu I_0}{2}\frac{e^{-ikr}}{r}\,\rho_0\, J_1(k\rho_0\sin\theta)\,\hat{\phi}$$

$$\underline{E} = -i\omega\underline{A} - \frac{i}{\omega\mu\epsilon}\nabla(\nabla\cdot\underline{A}) \overset{\text{far zone}}{=} -i\omega A$$

$$\boxed{E_\phi = \hat{\phi}\,\frac{\omega\mu I_0}{2}\,e^{-ikr}\,\frac{\rho_0}{r}\,J_1(k\rho_0\sin\theta)}$$

Infinitesimal Loop result

$$\hat{\phi}\,\frac{\omega\mu I_0 k}{4\pi\, r}\,e^{ikr}(\pi a^2)\sin\theta$$

magnetic moment vs electric dipole

$j\omega M$ for $i_0 L$

$M = I \times \text{area} = I_0\pi\rho_0^2$

$$E_\phi = \hat{\phi}\,\frac{\overbrace{(I_0\pi\rho_0^2)}^{M}\,e^{-ikr}\sin\theta}{r}\,\frac{\pi\,\mu c}{\lambda^2} \rightarrow \eta$$

MAGNETIC DIPOLE $\quad E_\phi = \dfrac{M\sin\theta\, e^{-ikr}}{r}\,\dfrac{\eta\pi}{\lambda^2}$

(21) 12.01 SM

Details

$$\int_0^{2\pi} \cos\phi' \, e^{i k \rho_0 \sin\theta \cos\phi'} \, d\phi' = \int_0^{\pi} + \int_{\pi}^{2\pi}$$

$\boxed{4}$

$$\int_{\pi}^{2\pi} \cos\phi' \, e^{i k \rho_0 \sin\theta \cos\phi'} \, d\phi' = \int_0^{\pi} \cos(\phi+\pi) \, e^{i k \rho_0 \sin\theta \cos(\phi+\pi)} \, d\phi$$

let $\phi = \phi' - \pi$ $\quad \cos(\phi+\pi) = cc - ss$
$= -\cos\phi$

$$= -\int_0^{\pi} \cos\phi \, e^{-i k \rho_0 \sin\theta \cos\phi} \, d\phi$$

$$\int_0^{2\pi} = \int_0^{\pi} \cos\phi \left[\cos k\rho_0 \sin\theta \cos\phi + i \sin k\rho_0 \sin\theta \cos\phi\right]$$

$$- \cos\phi \left[\cos k\rho_0 \sin\theta \cos\phi - i \sin(k\rho_0 \sin\theta \cos\phi)\right] d\phi$$

$$= i 2 \int_0^{\pi} \cos\phi \, \sin(k\rho_0 \sin\theta \cos\phi) \, d\phi$$

$$\int_0^{\pi} \cos x \, \sin(z \cos x) \, dx = \pi \sin\frac{\pi}{2} J_1(z)$$ GR 3.71.13 ~~13.371~~

$$\int_0^{2\pi} = i 2\pi \, J_1(k\rho_0 \sin\theta)$$

Lecture Plan Monday

Far Zone & Arrays - General

Radiation from current in wires 4

5

6

7

You need to read sets & work on - ask questions

8

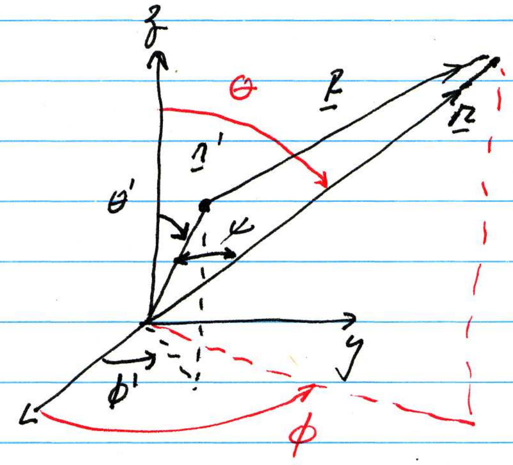
9

10

11

# FAR-ZONE FORMS

$$e^{ik|\underline{r}-\underline{r}'|} = e^{ik|\underline{R}|}$$




$\psi$ = angle between $\underline{r}, \underline{r}'$

$$R = \left(r^2 + r'^2 - 2rr'\cos\psi\right)^{1/2}$$

$$r\left(1+\frac{r'^2}{r^2} - \frac{2r'}{r}\cos\psi\right)$$

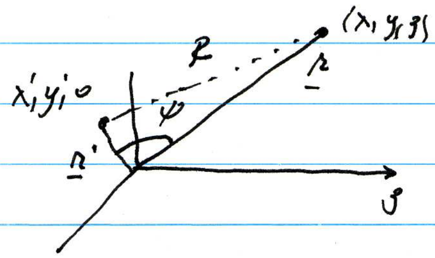
for $r' << r$ : $\quad \underbrace{r + \frac{r'^2}{2r} - \frac{\cancel{2}r'}{\cancel{2}}\cos\psi}\, + r\,O\left(\left(\frac{r'}{r}\right)^2, \left(\frac{r'^2}{r^2}\right)^3\right)$

$$R = r - r'\cos\psi$$

$$\frac{kr'^2}{r} << \frac{\pi}{2}$$

$$\frac{2\cdot 2\cancel{\pi}}{\lambda} r'^2 << \cancel{\pi}\, r$$

$$r >> \frac{4r'^2}{\lambda}$$




FOR OPT 461 = GOODMAN APERTURE CASE

$$\cos\psi = \frac{\underline{r}\cdot\underline{r}'}{rr'} = \frac{xx'+yy'+zz'}{(x^2+y^2+z^2)^{1/2}\,(x'^2+y'^2+z'^2)^{1/2}} \quad \text{general}$$

$$r'\cos\psi = \frac{1\;\; xx'+yy'\,|\cancel{r'}}{r\;\;\cancel{r'}} \qquad \text{exact} \quad z'=0$$

- $$R \cong r - \frac{xx'+yy'}{r} \qquad \text{FAR ZONE}$$

Directly: $R_1 = \left[(x-x')^2 + (y-y')^2 + z^2\right]^{1/2} = (x^2+y^2+z^2)^{1/2}\left[1 + \frac{\overset{-2x'x-2yy'}{\cancel{(x-x')^2}+(y-y')^2}}{r^2}\right]^{1/2}$

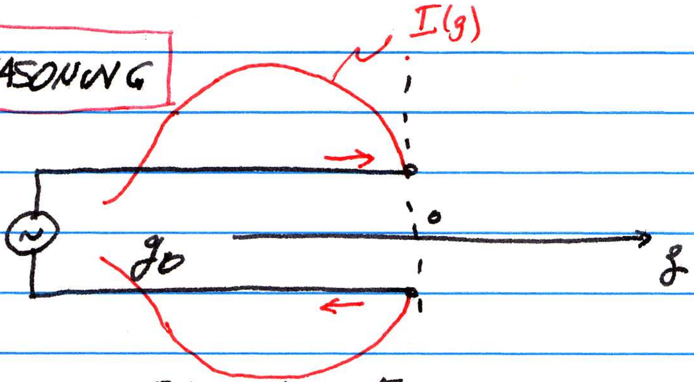
- $$R_1 \cong R_0 - \frac{xx'+yy'}{R_0}$$

same result if factor out $R_0 = (x^2+y^2+z^2)^{1/2}$

<u>not</u> $z - \frac{xx'+yy'}{z}$ !!!

# ANTENNA CURRENT

Sergei ~~SHELKUNOFF'S~~ Schelkunoff $\boxed{\text{REASONING}}$




$E_x$ Like $V(z)$

$H_y$ Like $I(z)$

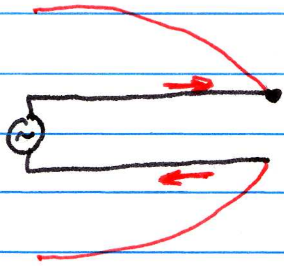
Telegrapher's Equation
2nd order

$$\left.\begin{aligned} V(z) &= A e^{-ikz} + B e^{ikz} \\ I(z) &= g_0 \left[ A e^{-ikz} - B e^{ikz} \right] \end{aligned}\right\} e^{i\omega t}$$

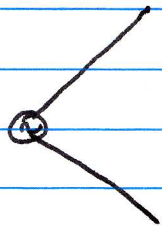
Standing waves: open circuit at $z = 0 \Rightarrow I(0) = 0$
$A = B$

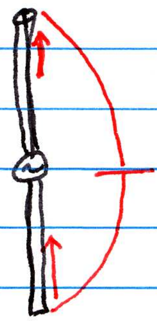
$$V(z) = A e^{-ikz} \left[ 1 + e^{i2kz} \right]$$

$$I(z) = g_0 A e^{-ikz} \left[ 1 - e^{ikz} \right]$$



Rabbit Ears





$$I(z) = I_0 \sin k(L - |z|)$$

See prob. 6.20 Balanis

→ Sergei A. Schelkunoff: Advanced Antenna Theory (1952) J. Wiley & Sons

Sec 3.5 Waves Guided by Parallel Wires    Insightful

Sergei A. Schelkunoff & Harald T. Friis    Antennas: Theory & Practice

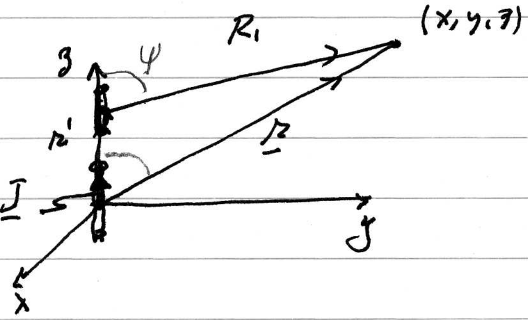
MTS    ( BTL Director )    J. Wiley 1952

# RADIATION: UNBOUNDED REGIONS - PROBLEM SOLVING

$e^{-i\omega t}$ HTD or Fourier time - use

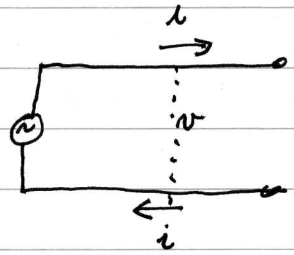
$$\underline{A}(\underline{r}) = \mu \int \underline{J}(\underline{r}') \frac{e^{ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|} dv'$$

$$\phi(\underline{r}) = \frac{1}{\epsilon} \int \rho(\underline{r}') \frac{e^{ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|} dv'$$




wire antenna, approximate current distribution

picture of travelling waves




$$\int J\, dx\, dy = I(z')$$

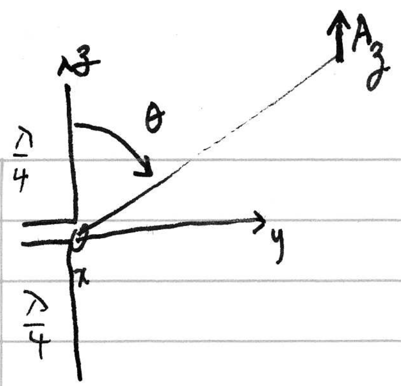
$$\underline{A}(\underline{r}) = \frac{\mu}{4\pi} \int \frac{I(z')\, e^{ikR_1}}{R_0} dz'$$

$R_1 = R_0 - r'\cos\psi$

neglect slight variation in $\psi$

$$= \frac{\mu e^{ikR_0}}{4\pi R_0} \int I(z')\, e^{-ikr'\cos\psi} dz'$$

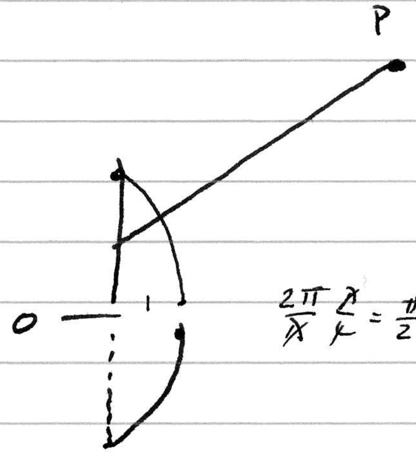
$$= \frac{\mu e^{ikR_0}}{4\pi R_0} \int I(z')\, e^{-ikz'\cos\theta} dz'$$



6-20
6-21 Balanis

6.

$$k\ell = \frac{\pi}{2}$$
$$\frac{2\pi}{\lambda}\ell = \frac{\pi}{2}$$
$$\ell = \frac{\lambda}{4}$$



$$\frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$I(z) = I_0 \sin k(L - |z|)$$
$$= I_0 \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{4} - z\right) \qquad z > 0$$

$$I(z) = I_0 \sin k(L + z) \qquad z < 0$$
$$= I_0 \sin k\left(\frac{\lambda}{4} - \frac{\lambda}{4}\right) \qquad z = -\frac{\lambda}{4}$$
$$= 0$$

$$A = \frac{\mu}{4\pi}\int \frac{J(r')e^{-ikR'}}{R'}\,dv'$$

$$\underline{A} = \frac{\mu}{4\pi}\int \hat{z}\, I_0 \sin k\left(\frac{\lambda}{4} - |z'|\right) e^{-ikR'}\,dz'$$

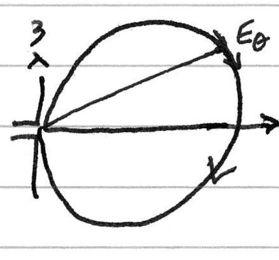
$$\int_{-\ell}^{0} I_0 \sin k\left(\frac{\lambda}{4} + z'\right)e^{-ikR} + \int_0^{\ell} I_0 \sin k\left(\frac{\lambda}{4} - z'\right)e^{-ik(r_0 - z'\cancel{\sin}\cos\theta')}\,dz'$$

$$A_\theta = -A_z \sin\theta = \frac{-\mu \sin\theta\, I_0\, \ell\, e^{-ikr_0}}{4\pi r}\left[\int_{-\ell}^{0} \sin k\left(\frac{\lambda}{4} + z'\right)e^{ikz'\cos\theta'}\,dz' + \int_0^{\ell} \sin k(\ell - z')e^{ikz'\cos\theta'}\,dz'\right]$$

Grind thru $\int_0^{\ell} \sin k(\ell - z')e^{ikz'\cos\theta'}\,dz'$

...

$E_\theta$ vs $z$




$$\boxed{E_\theta = -i\omega A_\theta = \frac{i\eta I_0 e^{-ikr_0}}{2\pi r_0}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}}$$

$E_\theta / H_\phi = \eta$

C. H. PAPAS

Papas (15) – (16) 3.2 p43 $e^{i\omega t}$

$$\int e^{a\xi} \sin(b\xi + c)\, d\xi = \frac{e^{a\xi}}{a^2+b^2}\left[a \sin(b\xi + c) - b\cos(b\xi+c)\right]$$

$$N = \hat{z}\, 2I_0 \frac{\cos(kl\cos\theta) - \cos kl}{k \sin^2\theta}$$

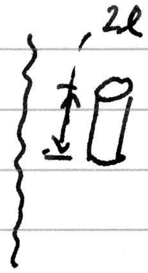
$$E_\theta = i\sqrt{\frac{\mu}{\epsilon}}\, \frac{e^{-ikr}}{2\pi r}\, I_0 \frac{\left[\cos(kl\cos\theta) - \cos kl\right]}{\sin\theta}$$

$l \to$ very short

$F(\theta) \to \frac{1}{2}(kl)^2 \sin\theta$

Hertzian dipole

$F(\theta) \to kl \sin\theta$




Not the same because ...

ARRAYS

Geometric progress

Gradshteyn & Ryzhik 0.112

$$\sum_{k=1}^{N} a q^{k-1} = a + aq + aq^2 + \dots aq^{N-1}$$

$$= \frac{a(q^n - 1)}{q - 1}$$

Look at 12.08 SM
Try a few problems
Read Antennas
12.08 (4)(5)

$$\sum_{m=0}^{m_x - 1} e^{im\psi} = \frac{1 - e^{im_x\psi}}{1 - e^{i\psi}}$$

$$= \frac{\sin(\frac{1}{2} m_x \psi)}{\sin \frac{\psi}{2}} e^{-i\frac{1}{2}(m_x - 1)\psi}$$

For antenna arrays (in 1, 2, 3d) & for mode locked lasers

BALANIS
SMYTHE $e^{i\omega t}$ which is the same in $\mathcal{F}_t$ as $\int_{-\infty}^{\infty} - e^{-i2\pi\nu t} dt$

in which case

$$\underline{A}(\underline{r}) = \mu \int \underline{J}(\underline{r}') \frac{e^{-ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|} d^3\underline{r}'$$

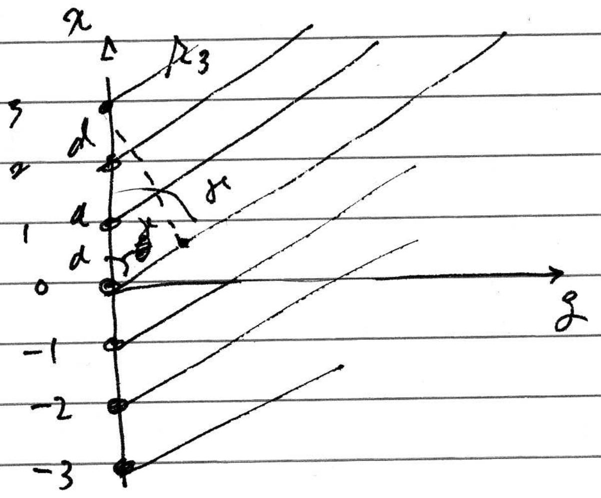
$$\phi(\underline{r}) = \frac{1}{\epsilon} \int \rho(\underline{r}') \frac{e^{-ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|} d^3\underline{r}'$$

Modern EO Imaging Systems - Take Opt 564

SYNTHETIC APERTURE How

PHOTOMIXING - WHY

PERIODIC STRUCTURE ARRAYS

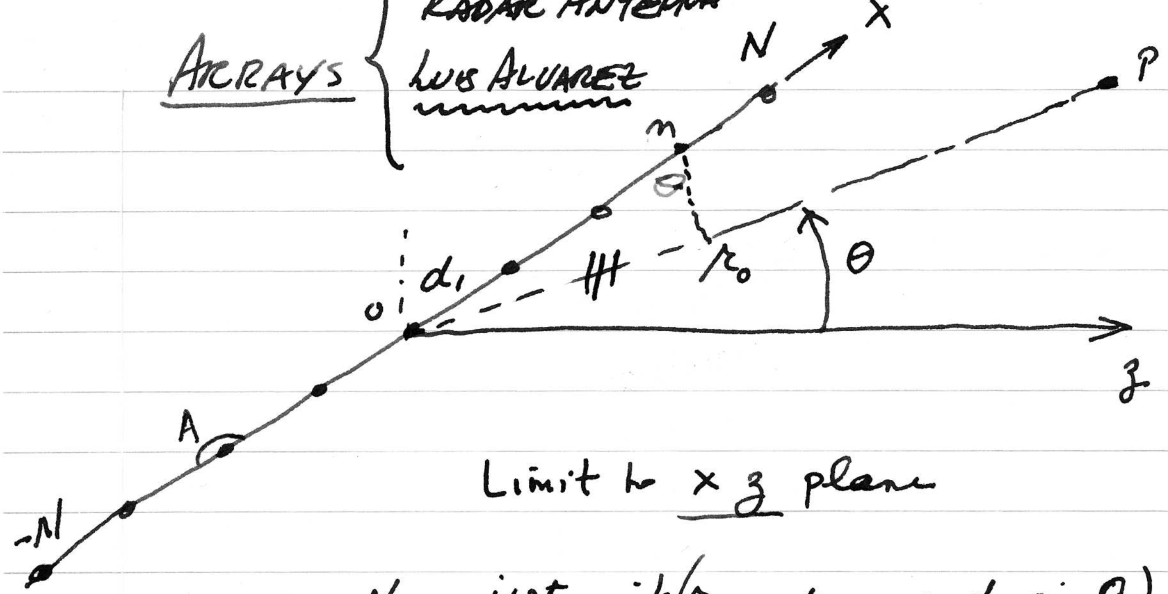



FAR ZONE: $r_3 = r_0 - 3d\cos\gamma$

$$\sum_{-N}^{N} e^{ikmd\sin\theta} = \frac{\sin\left(\frac{2N+1}{2}kd\sin\theta\right)}{\sin\left(\frac{kd\sin\theta}{2}\right)}$$

ARRAYS { INTERFEROMETER
RADAR ANTENNA
LUIS ALVAREZ

April 1991




Limit to $x\,z$ plane

$$V_p = \sum_{n=-N}^{N} A\, e^{i\omega t - ik(r_n = r_0 - n d_1 \sin\theta)}$$

} Suppress $\frac{1}{r}$ terms

$$= e^{i\omega t - ikr_0} \sum_{n=-N}^{N} e^{+(ikd_1 \sin\theta) n}$$

26. Dwight.
Geom Progression

$$= A e^{i(\omega t - kr_0)} \frac{\sin\left[\frac{2N+1}{2} k d_1 \sin\theta\right]}{\sin\left[\frac{k}{2} d_1 \sin\theta\right]}$$

$\theta \to 0$

peak value

$$V_p(\theta) = A \frac{\sin \frac{2N+1}{2} k d_1 \sin\theta}{\sin \frac{k}{2} d_1 \sin\theta} \cong A \frac{\frac{2N+1}{\cancel{2}} (\cancel{k} d_1 \theta)}{\frac{\cancel{k}}{\cancel{2}} d_1 \theta} = (2N+1) A$$

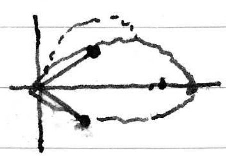
Angular bandwidth: $\frac{2N+1}{\cancel{2}} \cancel{k}^{\cancel{2\pi}/\lambda} d_1 \sin\theta' = \pi$

$\frac{2\pi}{\lambda}\frac{k}{2} d \sin\theta = m\pi$

$\sin\theta = \frac{m\lambda}{d}$

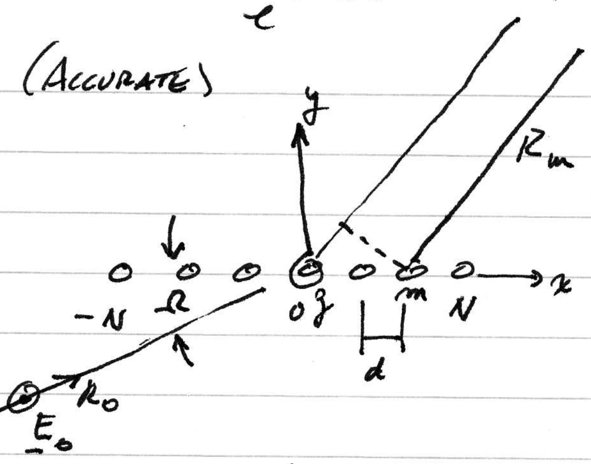
$$\sin\theta' = \frac{\lambda}{(2N+1) d_1}$$

LOTS OF LOBES IF $\frac{kd_1}{2} \gg \pi$



SUPPOSE FILM GRAIN WITH
$d_1 \to$ TINY $\quad \frac{2\pi}{\lambda 2} d_1 = 0.1 = \gamma'$ $\qquad \frac{V_p}{A} \to \frac{\sin\left(\frac{kL}{2}\sin\theta\right)}{\gamma' \sin\theta}$

$e^{i\omega t}$




# Wire Scattering - Discrete Sum (Accurate)

2004

$$\underline{A} = \mu \int \frac{\underline{J}(r')\, e^{-ikR}}{4\pi R}\, dv'$$

$n_o$ = # of halfwave dipoles / length

closely spaced

i) Simple way $\int dx'$ $\qquad x' = md$

ii) Precise way $\Sigma$ spacing $d$, as follows

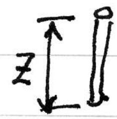
$m^{th}$ wire $\qquad I_o = \alpha E_o$

$$A_{gm} = \frac{\mu I_o}{4\pi} \iint \frac{e^{-i\underline{k_o}\cdot \underline{r}' - ikR_m}}{R_o}\, dg'$$

$$\underline{k_o}\cdot \underline{r}' = \underline{k_o}\cdot \underline{x}' = k_o x' \cos\Omega$$

$$A_{gm} = \frac{\mu(\alpha I_o)}{4\pi R_o}\, e^{-ikR_o} \int e^{i\frac{k}{R_o}xx' - ikx'\cos\Omega}\; e^{i\frac{k}{R_o}gg'}\, dg' \,\text{rect}\,\frac{g'}{Z}$$

$$R_m = R_o - \frac{xx' + gg'}{R_o}$$




$$e^{i2\pi\left(\frac{x}{\lambda R_o} - \frac{\cos\Omega}{\lambda}\right)x'} \qquad\qquad e$$

$$= e^{i2\pi\frac{d}{\lambda}\left(\frac{x}{R_o} - \cos\Omega\right)m} \qquad = \frac{\sin\frac{\pi}{\lambda R_o} g Z}{\frac{\pi g}{\lambda R_o}} \quad \left(\text{vs } \tfrac{\lambda}{4} \text{ pattern}\right)$$

$$\sum_{m=-N}^{N} e^{i\Delta m} = \frac{e^{i\frac{\Delta}{2}(2N+1)} - e^{-i\frac{\Delta}{2}(2N+1)}}{e^{i\frac{\Delta}{2}} - e^{-i\frac{\Delta}{2}}}$$

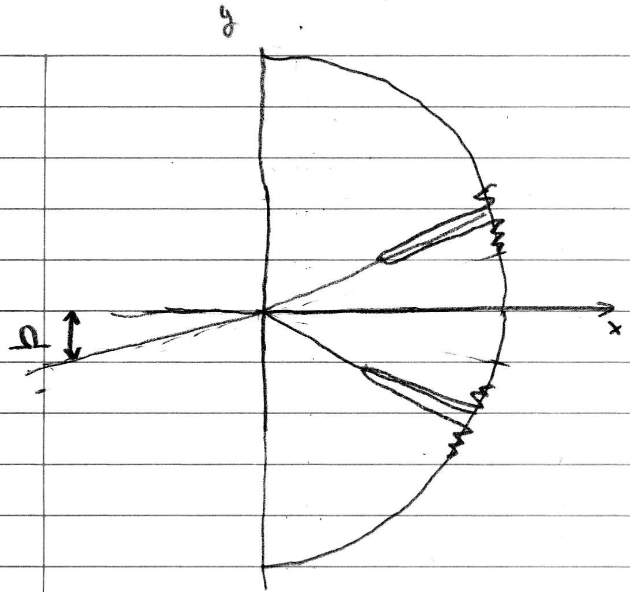
$$= \frac{\sin (2N+1)\frac{\Delta}{2}}{\sin \frac{\Delta}{2}}$$

Geom Progression
G+R 0.112

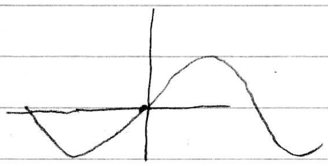
$$\sum_{k=1}^{n} aq^{k-1} = \frac{a(q^n - 1)}{q-1} \qquad q \neq 1$$

$$A_g = \frac{\mu(\alpha E_o)}{4\pi R_o}\, e^{-ikR_o}\, \frac{\sin(2N+1)\left(\frac{\pi d}{\lambda}\right)\left(\frac{x}{R_o} - \cos\Omega\right)}{\sin\frac{\pi d}{\lambda}\left(\frac{x}{R_o} - \cos\Omega\right)} \cdot \frac{Z\sin\left(\frac{\pi}{\lambda R_o} g Z\right)}{\frac{\pi g Z}{R_o}}$$

- Note $\frac{\sin(\ )}{\sin}$ is more accurate for any $d$. As we let $d$ get smaller the sin in denom. can be dropped!




When $\sin\phi = \sin\Omega$: $S_y = 3$



$kd = \frac{2\pi}{\lambda} d = \frac{2\pi}{\frac{1}{2}} 3 \sim 12\pi$

$\Omega = 20^\circ$ $\cos\phi = \cos 20^\circ = 0.940$

$\phi = \pm 20^\circ$

$\sin\phi = \sin\Omega = \sin 20^\circ$

$\phi = 20^\circ$

negative $\phi$

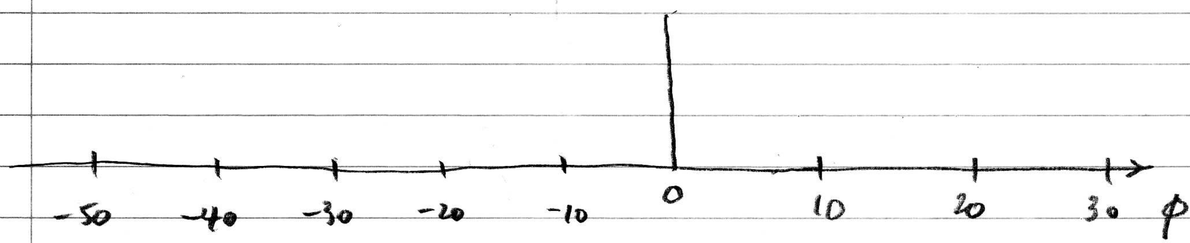
$2\pi 6 \mu = -2\pi$

$\mu = -\frac{1}{6}$

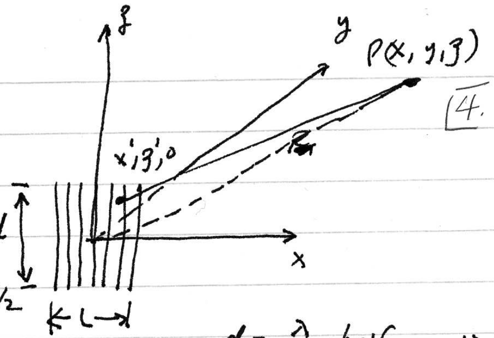
$d = 3\mu m$ $\lambda = \frac{1}{2}$

$1 + 2\cos\frac{2\pi}{\lambda} d (\sin\phi - \sin\Omega)$

$1 + 2\cos(2\pi \cdot 6)(\sin\phi - 0.342)$



$$\boxed{e^{-i\omega t}}$$




Fargon $\quad x', z' << R_1$

$$R_1 = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

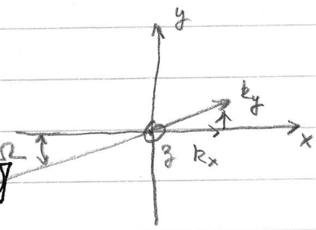
$$R_1 = \left[(x^2+y^2+z^2) - 2xx' - 2zz' + x'^2 + z'^2\right]^{1/2}$$

$d = \frac{\lambda}{2}$ halfwave dipole

$$R_1 = R_0\left[1 - \frac{2xx' + 2zz'}{R_0^2} + \frac{x'^2+z'^2}{R_0^2}\right]^{1/2}$$

$$R_1 = R_0\left[1 - \frac{xx'+zz'}{R_0^2} + O\left(\frac{x'^2+y'^2}{R_0^2}\cdots\right)\right]$$

$$R_1 = R_0 - \frac{xx'+zz'}{R_0}$$




$$i\underline{k}\cdot\underline{r} = ik\left[\hat{x}\,x\cos\Omega + \hat{y}\,y\sin\Omega\right]$$

Plane wave illum: $e$

$$\underline{A}(\underline{R}) = \frac{\mu}{4\pi}\int \frac{\underline{J}(\underline{r}')\,e^{+ikR_1}}{R_1}(dx'dy')\,dz' \qquad \underline{I}\cdot d\underline{a}' = I_0\,\hat{z}$$

$$\underline{A}(\underline{R}) = \frac{\mu_0 I_0}{4\pi R_0}\,\hat{z}\,e^{+ikR_0}\sum\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{d}{2}}^{\frac{d}{2}} e^{i\underline{k}\cdot\underline{r}_0'}\,e^{\mp ik_0\frac{xx'+zz'}{R_0}}\,dx'\,dz'$$

Closely spaced $\quad n_0 = \frac{N}{L}\ ; \int_{-\frac{L}{2}}^{\frac{L}{2}} dx' = L$ want $n_0 L$; $\frac{n_1}{4} \times n_0$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{d}{2}}^{\frac{d}{2}} e^{ik[x'\cos\Omega + \cancel{y_0'}\sin\Omega] - ik(x'\gamma_x + z'\gamma_z)}\,dx'\,dz'$$

$$\boxed{\begin{aligned}\frac{x}{R_0} &= \gamma_x\\ \frac{z}{R_0} &= \gamma_z\end{aligned}}$$

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} e^{-ikz'\gamma_z}\,dz' = \frac{e^{-ik\frac{d}{2}\gamma_z} - e^{+ik\frac{d}{2}\gamma_z}}{-ik\gamma_z} = 2\,\frac{\sin\left(k\frac{d}{2}\gamma_z\right)}{k\gamma_z}$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} e^{ik[\cos\Omega - \gamma_x]x'}\,dx' = \frac{e^{ik[\cos\Omega-\gamma_x]\frac{L}{2}} - e^{-ik[\cos\Omega-\gamma_x]\frac{L}{2}}}{ik[\cos\Omega-\gamma_x]}$$

$$'' = 2\,\frac{\sin k\frac{L}{2}[\cos\Omega - \gamma_x]}{k[\cos\Omega-\gamma_x]}$$

$$\boxed{\underline{A}(\underline{R}) = \frac{\mu_0 I_0\,e^{+ikR_0}}{4\pi R_0}\,\hat{z}\,n_0\,(L\,d)\,\frac{\sin k\frac{L}{2}[\cos\Omega-\gamma_x]}{k\frac{L}{2}[\cos\Omega-\gamma_x]}\times\frac{\sin\left(k\frac{d}{2}\gamma_z\right)}{k\frac{d}{2}\gamma_z}}$$