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Energy network optimization using a Markov Chain-Monte Carlo algorithm

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Abstract

We describe a mathematical model and two-tier computational algorithm for finding network structures that best satisfy energy supply/demand constraints in the system. The method uses a Markov matrix for routing flows in the algorithm's 'inner' loop while elements of this matrix are periodically updated in an 'outer' loop using Monte Carlo sampling. We illustrate the efficacy of the method on two numerical examples.

Keywords: Markov matrix, Monte Carlo, simulated annealing, flow distribution networks

1. Introduction

The behavior of many complex systems can be modeled by processes resident on network (graph) structures where components (nodes) in the system are linked together in some fashion. Examples include computer networks [1], biological systems [2–8], power supply grids [9–17] and financial risk management networks [18, 19]. The behavior in these systems depends upon the pattern of interconnections between components in the network. In power supply applications, for example, the network architecture (i.e scale-free or homogeneous [20]) will have a significant effect on system reliability where intermittent equipment failure, demand overload, and flow congestion have to be dealt with to ensure proper system functioning [21], [20], [22]. Knowledge of the network structure is key to answering many practical questions e.g. how do we route energy flows to avoid flow overloads along network pathways? Where should additional energy storage capacity be incorporated into the network if energy demand is to be better satisfied at each point in the system?

In this paper we describe a novel two-tier computational method for addressing questions of this sort. The network model describes a system where energy is supplied to the network from fixed external sources. Within the network energy flow can be directed between any node in the system. In addition, a local independent battery backup-storage capability is assigned at each node. This facilitates local energy storage when excess energy is available at a given node from which energy could subsequently be drawn to re-supply to the system if necessary. The overall objective of the optimization algorithm is to establish routing information that attempts to satisfy overall supply-demand requirements throughout the system. A specific objective of the work was to study the effect of network rewiring [17][23].

Algorithms for modeling flow in networked systems usually fall into three main classes: either Markov matrix-based [24], linear/mixed integer linear programming methods

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[25][26] or solving conductance models, i.e. Kirchoff's laws [27]. In the hybrid approach we describe here we meld concepts from various fields eventually arriving at a novel optimization method for networked processes. We represent the switching properties of the network by a Markov transition matrix whose elements provide the network routing information. The values of these elements are unknowns and the objective of the algorithm is to find the matrix values that best satisfy supply-demand equilibrium while maintaining flows along the network's edges below a pre-set threshold value $I_{\max}$. Towards this end we employ Monte-Carlo sampling, reminiscent of the simulated annealing algorithm originally discussed in the papers of Cerny [28] and Kirkwood et al. [29]. Simulated annealing is a powerful metaheuristic optimization computational method [30] which has found application in a wide variety of fields including transport networks [31], failure analysis in power grids [24] and vehicle routing problems [32]. As discussed elsewhere [33, 34] requirements for its use are: (1) a model for the system being studied with variables that can be manipulated to effect changes in the process simulation outcomes (i.e. decision variables) (2) a scalar 'cost' objective function that measures progress towards some process goal (3) an annealing parameter which can be adjusted (if necessary) to promote numerical convergence during the course of the calculations and (4) a criterion for termination of the computations. These so-called metaheuristic algorithms make very few assumptions (e.g. linearity) about the particularities of the system being optimized [35] and, unlike many other optimization algorithms, they do not require 'nice' mathematical features (e.g. differentiability) that gradient based methods, for example, require [36].

The paper is organized as follows. In section 2 we describe the network model followed by a description of the Monte Carlo optimization algorithm mused for its solution in section 3. In section 4 we present computational results for two example problems followed by conclusions in section 5.

2. The network flow model

We consider an undirected graph $G = (V(G), E(G))$ with vertex set $V(G)$ and edge set $E(G)$ .The edge set $E(G)$ of the network can be represented by an adjacency matrix $\boldsymbol{A}$ [37]which can be used to construct a network flow model with the following canonical form [38]:

$$\boldsymbol{P} = \begin{array}{c} \\ TR. \\ ABS. \end{array} \begin{array}{c} \begin{array}{cc} TR. & ABS. \end{array} \\ \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} \end{array} \tag{1}$$

Here I is a K-by-K identity matrix, $\boldsymbol{0}$ is a K-by-N zero matrix, $\boldsymbol{R}$ is a nonzero N-by-K absorbing state matrix, and $\boldsymbol{Q}$ is the N-by-N transition probability (Markov) matrix which directs flows along the network from node i to node j according to the probability associated with the element q_{ij} . For an absorbing network where there is energy consumption at each node the sum of the matrix row elements q_{ij} are less than one along each row [38]. If the absorption probability at node i is q_{ai} then $1 - \sum_{j=1}^{N} q_{ij} = q_{ai}$.

If the required total power availability at each point in the network is denoted by the row vector $\boldsymbol{u}$ and the external power/ backup availability at each node is denoted by the row vector $\boldsymbol{s}$ then (assuming no losses) we can write the following energy flow conservation

equation relating these quantities:

$$\boldsymbol{u} = \boldsymbol{uQ} + \boldsymbol{s} \tag{2}$$

It is observed that $\boldsymbol{u}$ is a row vector that represents the available power at each node; some of this power is transmitted elsewhere in the network while part is consumed locally. The steady-state energy conservation law in the system is given by $\sum_{i=1}^{N} q_{ai} u_i = \sum_{i=1}^{N} s_i$.

We define a matrix $U_{n\times n}$ which has the elements of vector $\boldsymbol{u}$ arranged along its main diagonal as follows,

$$\boldsymbol{U}_{n\times n} = \begin{pmatrix} u_1 & & & 0 \\ & u_2 & & \\ & & \ddots & \cdots \\ 0 & & \cdots & u_n \end{pmatrix} \tag{3}$$

and a matrix D that represents the power flow between nodes. Each element d_{ij} of D is defined as follows,

$$d_{ij} = u_i \times q_{ij} \tag{4}$$

where q_{ij} is the ij element of $\boldsymbol{Q}$.

The matrix form of equation (4) is,

$$\begin{pmatrix} u_1 & & & 0 \\ & u_2 & & \\ & & \ddots & \vdots \\ 0 & & \cdots & u_n \end{pmatrix} \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{21} & \cdots & q_{2n} \\ & & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix} = \begin{pmatrix} u_1q_{11} & u_1q_{12} & \cdots & u_1q_{1n} \\ u_2q_{21} & u_2q_{21} & \cdots & u_2q_{2n} \\ & & \ddots & \vdots \\ u_nq_{n1} & u_nq_{n2} & \cdots & u_nq_{nn} \end{pmatrix} \tag{5}$$

The actual consumption at each node c_i is calculated from the $\boldsymbol{R}$ matrix and is given by the equation,

$$c_i = (1 - \sum_j^N q_{ij}) u_i = q_{ai} u_i \tag{6}$$

If we define the demand goal at each node as a_i then the 'residual' energy at each node is given by $\delta_i = c_i - a_i$ which can be either positive or negative.

2.1. The network roughness function

We introduce the concept of the 'network roughness' function $\omega(N)$ inspired by the use of an analogous property in systems involving stochastic processes [39] where it is used to measure the overall uniformity of a thin-film surface being randomly deposited on a substrate. Here, by analogy, it is used as a measure of the overall departure of the network's residual energy 'surface' from the equilibrium requirements as flow re-distribution occurs across the network.

The network roughness function $\omega_k(N)$ at the kth iteration is defined as:

$$\omega_k(N) = 1/N \sum_{i=1}^{N} \delta_i^2 \tag{7}$$

The magnitude of $\omega_k(N)$ is used as the Monte Carlo sampling function to decide whether or not to accept each attempt at a flow redistribution step in a manner analogous to the Metropolis method ubiquitous in statistical mechanical simulations-where the energy between states is usually the

property sampled [40, 41].

These ideas can be couched as the following general nonlinear optimization problem; a more formal formulation of the network optimization problem can be expressed as:

$$\text{Given } s,\ a, \text{ initial } Q_0,\ N, \text{ find the Markov matrix } Q \tag{8}$$

$$\text{Minimizing } \omega_k(N) = 1/N \sum_{i=1}^{N} \delta_i^2 \tag{9}$$

subject to the constraints

$$u_i \le l_{\max}, \forall i \tag{10}$$

$$0 \le q_{ij}, q_{ai} \le 1, \forall i \tag{11}$$

$$q_{ai} + \sum_j q_{ij} = 1, \forall i \tag{12}$$

Note that δ_i depends upon the elements of Q, and the constraints (11) and (12) ensure that Q is a valid Markov matrix.

3. Optimization algorithm

A given network structure gives rise to asymmetric adjacency matrix A,a 0-1 matrix, where 0 represents no edge between a pair of nodes and the value 1 denotes the existence of one. The initial transition matrix is defined as Q_0 where $q_{i0} = (q_{i10}, q_{i20}, \cdots, q_{ino})$ is its ith row. Although the network has an undirected structure, the sign of the magnitude of the power flow gives a net flow direction. In order to reflect this we define a new matrix $F = (D - D^T)$, which gives the algebraic sum of the flow along each edge. We set all the negative elements of F to be 0. In this way, all the information about the absolute flows is included in matrix F. We also define the maximum allowable power flow along any given edge which we call its limiting value $l_{\max}$. Each row of matrix F shows the flow from node i to the other nodes it is connected to. The purpose of Monte Carlo sampling is to iteratively re-distribute flows so that the maximum flow along any network edge is maintained below a maximum prescribed value while attempting to satisfy overall energy demand. The optimization algorithm proceeds as follows:

3.1. Initialization

Input Q_0, $l_{\max}$, β, a, $k = 0$

$u_0 = u_0 Q_0 + s_0$

Calculate ω_0

3.2. Order δ_i

Calculate residual supply-consumption vector δ and order from the highest value $\delta_{\max}$ to the lowest value $\delta_{\min}$

3.3. Find potential redistribution paths from $\delta_{\max}$

a. Start with the maximum δ indexed by x.

b. Find all the nodes that are connected with node x within the matrix Q.

3.4. Redistribute flows

$k = k + 1$

a. Redistribute the excess energy δ_x to its connected node with the lowest value of δ_y which is a 'greedy' heuristic [42]. That is replace q_{xy} with $q_{xy} + \delta_x / u_x$ and decrease q_{xa} by this amount to maintain the Markov property of the system

b. Calculate a uniform random number z in the range [0,1].

c. If $\exp(-\beta \omega_k - \omega_{k-1} / \omega_{k-1}) < z$, reject the move and redistribute the excess from δ_x to a connected node with the next higher value of δ_y. Re-calculate ω_k.

d. If for node x, all possible attempted re-distributions do not satisfy $\exp(-\beta \omega_k - \omega_{k-1} / \omega_{k-1}) > z$, replace node x with the next largest δ and repeat steps 3b and 4.

e. If $\exp(-\beta \omega_k - \omega_{k-1} / \omega_{k-1}) > z$, accept the redistribution move. Calculate new Q_k and $u_k = [I - Q_k]^{-1} s$. The matrix $[I - Q_k]^{-1}$ is positive definite therefore is guaranteed to have a non-singular inverse [43].

Calculate ω_k and check for stopping conditions: If $|\omega_k - \omega_{k-1}| < \varepsilon$ and foreach element of $u_k = [I - Q_k]^{-1} s < l_{\max}$, end calculations. Otherwise go to2. and repeat the calculations.

If all attempts at redistribution in steps 4a-4d fail or $|\omega_k - \omega_{k-1}| < \varepsilon$ and $u_k = [I - Q_k]^{-1} s > l_{\max}$, from 4e, decrease β and/or $l_{\max}$ and repeat the calculations until problem constraints are satisfied. At some point the second it ions will be satisfied since smaller β allows for wider sampling of the fluctuations and increasing $l_{\max}$ obviously relaxes the maximum flow restriction which will be satisfied at some point. In step 1 the annealing factor β is analogous to the Boltzmann temperature term in statistical thermodynamic simulations. We also note that in step 3 the restriction of ow distribution between nodes connected in the original network can be relaxed-if the 'hard-Ware' allowing for additional edges is in place. We investigate this in the next section.

4. Numerical results

For our examples we used random graph initial structures [37] with values for the edge ow transition probabilities in the Q, R matrices taken from a uniform distribution sampled over the range [0,1]. In the first problem we used a network with 15 nodes each with supply/demand values randomly allocated to the s and a vectors. These two vectors are not necessarily correlated.

A network capable of satisfying its energy needs requires $\sum_{j=1}^{N} s_i \geq \sum_{j=1}^{N} a_i$ .The initial values for c_0 and a_i are shown in table 1 and the value of the initial Q in table 2. The network structure of this system is shown in Figure1. From the entries in table 2 it is evident that the matrix has zero entries implying that this particular network is not a complete graph [37]. The other initialization parameters $l_{\max}$ were set at 2 times the initial $d_{\max}$ and annealing factor β =0.5. The algorithm was run with these inputs for the fixed network structure. The network roughness function is plotted in Figure 2and is insensitive to values of the annealing parameter β . This was found throughout calculations with this problem, and is not unusual with simulated annealing [33]. We note, however, that the denominator term in the exponential, ω_{k-1} , does change every time a new state is accepted lending a dynamic character to the annealing process. The initial and final histograms of the residual demand functions $\delta_i = c_i - a_i$ are shown in Figures 3 and 4, respectively. It is evident that the redistribution helps balance supply/demand throughout the network. However, nodes 7 and 9 are well below the required consumption goals. These nodes are not coupled to the most over supplied node in the original network (node 1) and a logical design step is to reconfigure the network to couple these respective nodes. We allowed this coupling in the algorithm and doing so leads to results given in Figure 5 with the network roughness function for this case shown in Figure 6. We observe that the new network structure allows for a modest improvement in the supply/demand equilibrium compared to the prior fixed structure case. We then modified the algorithm so as to enable connection between the nodes with the highest and lowest values of δ_i anywhere in the system. The re-wired network structure with additional connections (shown in red) is shown in Figure 7 with the histogram and roughness function shown in Figure 8 and 9. A very significant improvement in the supply/demand equilibrium in the system results. In fact, now all consumption goals are achieved and the excess available power could be sent to a local storage device. Furthermore, the algorithm provides the exact places in the network where new edge capacity should be added (or used if the link is already in place) to satisfy system demand. In all these cases we checked the numerical consistency of the calculations which should show that the overall supply equals the overall consumption in the system and found this to be always true.

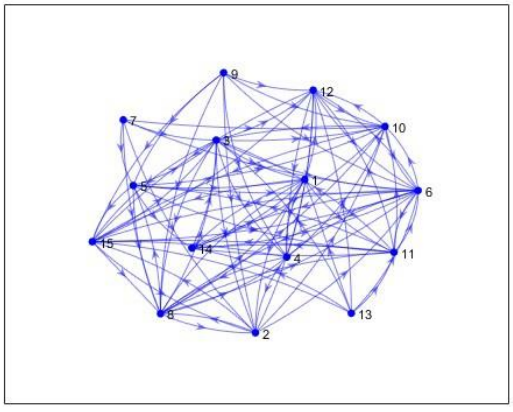

Figure 1: Initial Fixed Network Structure

Table 1: $c0$ and a vector

c_0	a
44.20	10.00
67.67	65.00
33.55	30.00
43.64	45.00
13.46	18.00
72.23	83.00
2.34	20.00
31.38	31.00
1.51	10.50
141.93	120.00
49.17	49.00
125.92	120.00
42.51	42.00
78.96	80.00
49.49	49.00

Table 2: Q0 matrix

0.000	0.073	0.000	0.072	0.000	0.000	0.000	0.068	0.000	0.000	0.091	0.124	0.000	0.182	0.123
0.000	0.000	0.000	0.000	0.000	0.033	0.000	0.125	0.000	0.056	0.000	0.116	0.000	0.000	0.125
0.241	0.097	0.000	0.096	0.000	0.044	0.000	0.092	0.000	0.074	0.000	0.153	0.000	0.000	0.076
0.000	0.069	0.000	0.000	0.000	0.000	0.000	0.181	0.000	0.225	0.000	0.117	0.000	0.000	0.054
0.121	0.000	0.126	0.000	0.000	0.110	0.000	0.082	0.000	0.001	0.061	0.000	0.000	0.122	0.044
0.000	0.000	0.000	0.057	0.000	0.000	0.000	0.097	0.000	0.188	0.072	0.091	0.000	0.145	0.098
0.000	0.056	0.026	0.000	0.059	0.095	0.000	0.053	0.000	0.125	0.000	0.000	0.000	0.000	0.000
0.145	0.058	0.151	0.058	0.062	0.132	0.000	0.000	0.000	0.235	0.000	0.000	0.000	0.000	0.000
0.000	0.062	0.000	0.061	0.065	0.105	0.000	0.058	0.000	0.138	0.000	0.097	0.000	0.000	0.105
0.000	0.000	0.250	0.000	0.000	0.213	0.000	0.000	0.000	0.000	0.000	0.016	0.000	0.000	0.108
0.000	0.000	0.085	0.000	0.042	0.091	0.000	0.068	0.000	0.162	0.000	0.069	0.000	0.101	0.032
0.000	0.000	0.000	0.071	0.000	0.033	0.000	0.000	0.000	0.236	0.000	0.000	0.000	0.000	0.065
0.104	0.000	0.109	0.000	0.000	0.148	0.000	0.000	0.000	0.000	0.053	0.006	0.000	0.106	0.038
0.109	0.000	0.114	0.000	0.000	0.175	0.000	0.000	0.000	0.145	0.055	0.000	0.000	0.000	0.000
0.000	0.106	0.225	0.000	0.000	0.191	0.000	0.100	0.000	0.000	0.000	0.181	0.000	0.000	0.000

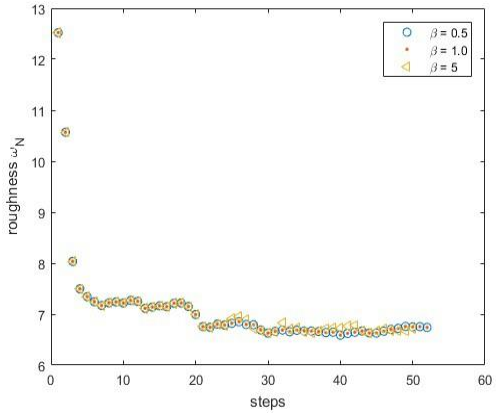

Figure 2: Roughness ωN *vs.* Steps for Fixed Network Structure

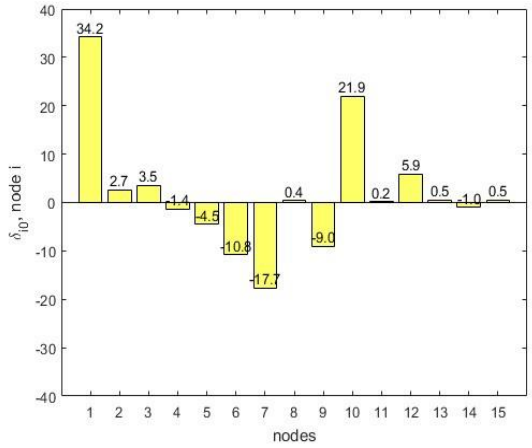

Figure 3: Initial Histogram for Fixed Network Problem, δi_0 *vs.* Nodes

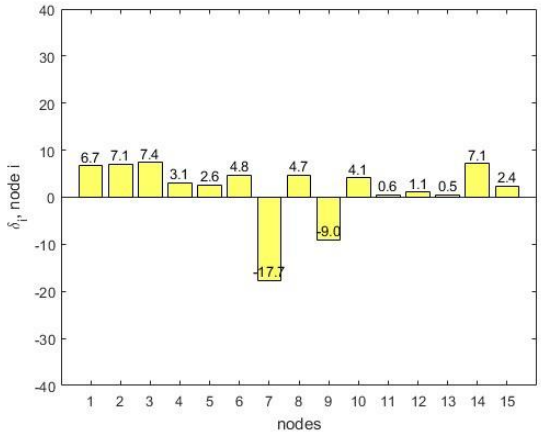

Figure 4: Final Histogram for Fixed Network Structure, δ_i vs. Nodes

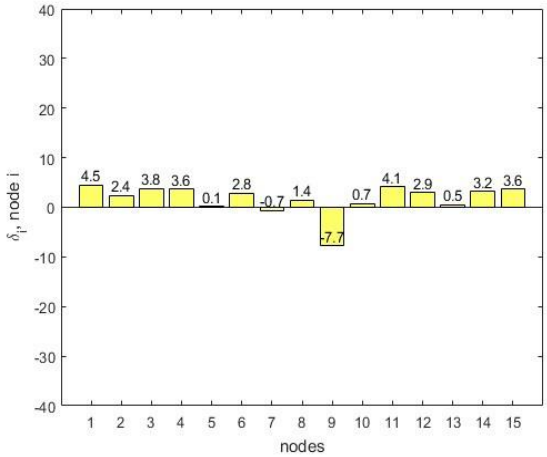

Figure 5: Histogram for Single Added Link Network, δ_i vs. Nodes

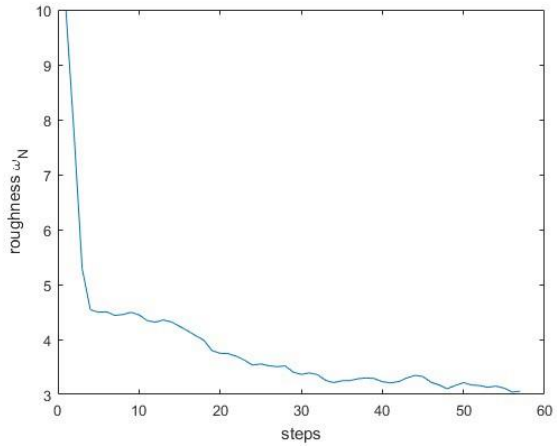

Figure 6: Roughness for Single Added Link Network, ω_N vs. Steps

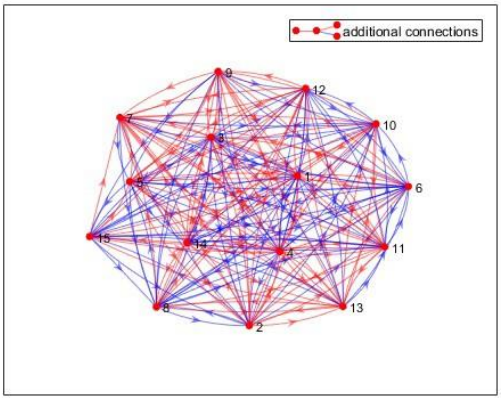

Figure 7: Re-Wired Network Structure with Additional Links Throughout

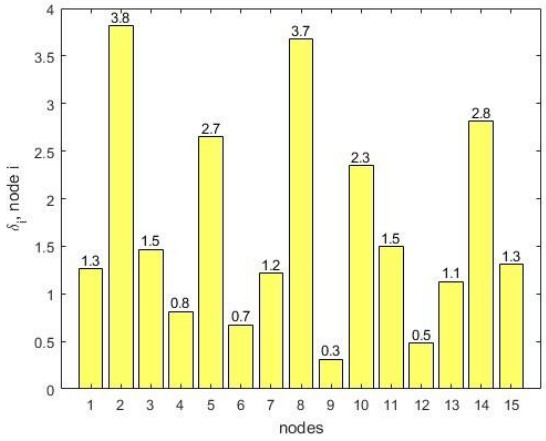

Figure 8: Histogram for Network in Figure 7, δ_i vs. Nodes

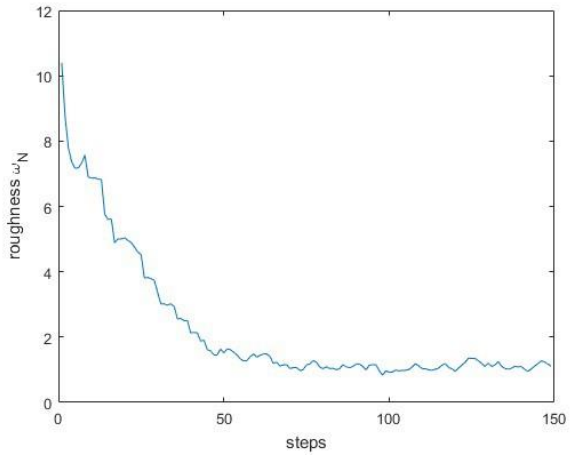

Figure 9: Roughness for Network in Figure 7, ω_N vs. Step

When optimizing systems, it is instructive to have benchmark problem(s) against which any algorithm(s) can be tested. These are not always easy to come up with especially for NP-complete combinatorial problems [42] or those with a large solution space like the problem we address here. We, however, conceived of such a benchmark for our algorithm using a two-step computation process carried out along the following lines. Firstly, we used the network structure shown in Figure 1 and for a given initial Q matrix and specified supply and consumption (residual) vectors s, a we solved for δ in the system. In the second part we attempted to solve the reverse problem by finding the Markov matrix Q for the values of the consumption found in the first set of calculations. We know that interalia the optimal solution to this problem should lead to a now known (by construction) Markov matrix that satisfies supply-demand in the system. Our method indeed does this very closely with the results shown in Figures 10, 11 and 12 for the roughness functions before and after consumption histograms, respectively.

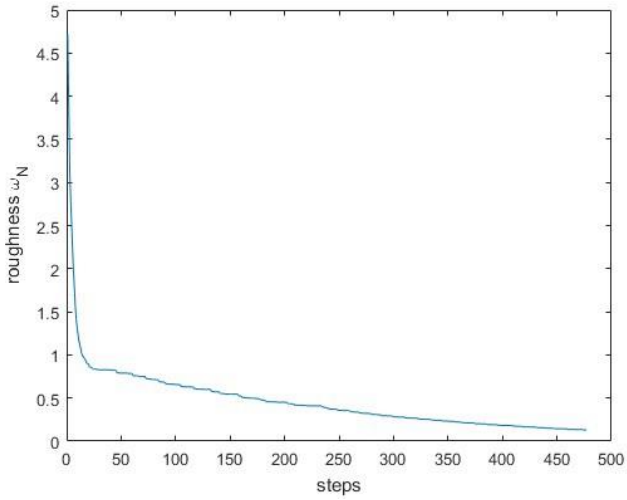

Figure 10: Roughness for Benchmark Problem, ω_N vs. Steps

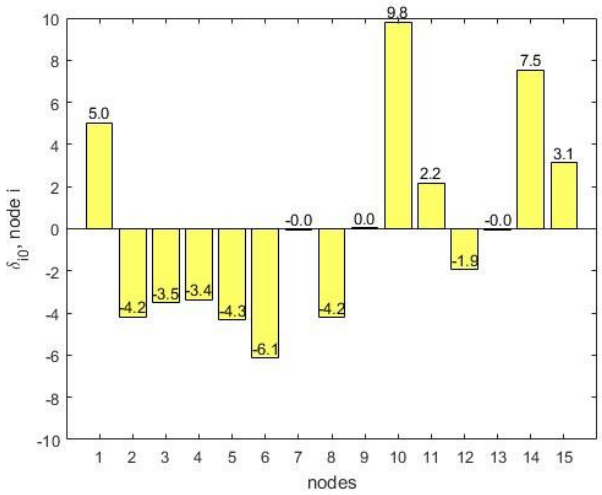

Figure 11: Before Histogram for Benchmark Problem, δ_{i0} vs. Nodes

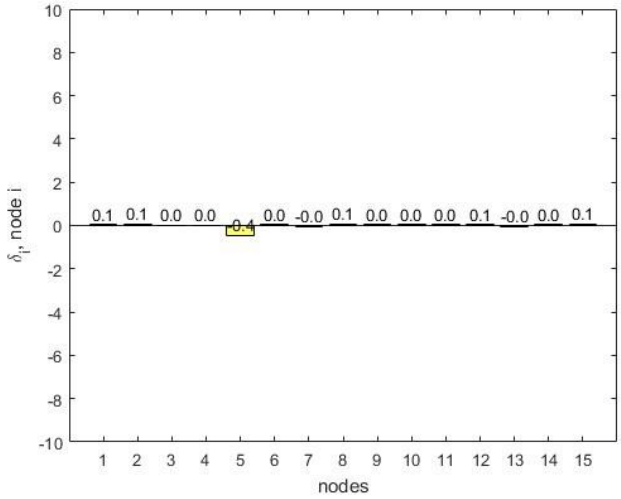

Figure 12: After Histogram for Benchmark Problem, δ_i vs. Nodes

5. Conclusions

We present novel results for a hybrid Markov Matrix-Monte Carlo annealing optimization algorithm whose objective is to find a set of Markov transition probabilities that route network flows to satisfy preset supply/demand goals. This, while keeping ow magnitudes along any path in the network below a prescribed maximum value. Computational results illustrate that the algorithm performs very well in achieving design objectives in two example problems involving network flow models. The first problem showed the efficacy of the algorithm in finding solutions that enhance the network's supply-demand equilibrium by dynamically establishing new network connections that switch energy from areas of excess to areas of deficit in the system. In the second problem we established a benchmark problem and then tasked the algorithm with finding the optimal solution. The algorithm succeeded in doing this and thus represents a potentially powerful general way of addressing important routing problems in complex networks where the flow 'entity' can be expansively interpreted (i.e.

energy, information, etc.). The results provide a good foundation for future work incorporating edge flow costs and dynamic demand conditions into the problem formulation.

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