Analysis of Target Localization With Ideal Binary Detectors via Likelihood Function Smoothing

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***Abstract*—This letter deals with noncooperative localization of a single target using censored binary observations acquired by spatially distributed sensors. An ideal, noise-free setting is considered whereby each sensor can perfectly detect if the target is in its close proximity or not. Only those detecting sensors communicate their decisions and locations to a fusion center (FC), which subsequently forms the desired location estimator based on censored observations. Because a maximum-likelihood estimator (MLE) does not exist in this setting, current approaches have relied on heuristic, centrality-based geometric estimators such as the center of a minimum enclosing circle (CMEC). A smooth surrogate to the likelihood function is proposed here, whose maximizer is shown to approach the CMEC asymptotically as the likelihood approximation error vanishes. This provides rigorous analytical justification as to why the CMEC estimator outperforms other heuristics for this problem, as empirically observed in prior studies. Since the Cramér-Rao Bound does not exist either, an upshot of the results in this letter is that the CMEC performance can be adopted as a benchmark in this ideal setting and also for comparison with other more pragmatic binary localization methods in the presence of uncertainty.**

***Index Terms*—Centrality estimators, ideal binary detectors, noncooperative localization, performance analysis.**

I. Introduction

LOCALIZATION of a transmitter using distributed wireless sensors is a fundamental signal processing task that has received significant attention, see e.g., [1]–[3]. Typically, sensor observations either comprise measurements of angle of arrival (AoA) [4], [5], time difference of arrival (TDoA) [6], or received signal strength (RSS) [7]–[10]. The first two alternatives require sophisticated sensors, therefore not adhering to the stringent energy and complexity constraints imposed by wireless sensor networks (WSNs) [7], [11]. Binary observations based on thresholded RSS measurements are often preferred, because their communication to a fusion center (FC) is bandwidth efficient [3], [11]–[17]. A noncooperative scenario is considered here, where the target does not assist the FC with the localization process. Noteworthy application domains include primary user identification in cognitive radio networks [12], spectrum sensing [18], spectrum cartography [19], and localization of jammers in the battlefield [20].

In this WSN context, consider an ideal, noise-free setting described in Section II, whereby each sensor can perfectly detect if the target is in its close proximity or not, see, e.g., [21] for target tracking. These binary indicators are then communicated to a FC tasked with forming an estimator of the target's location [11]–[15]. Only those sensors detecting proximity to the target will communicate their decisions and locations to the FC [12], [13], [22]. This is motivated by savings in communication and processing cost, although part of the information might be lost as a result of such censoring [23]. This ideal scenario can be approximated in practice when sensors mitigate noise by, e.g., averaging the RSS measurements over a sufficiently long time period [15]. Moreover, study of this setting can shed valuable insights on the fundamental performance limits attainable by WSN localization algorithms based on censored observations in the presence of uncertainty such as noise and Rayleigh fading.

Interestingly, it is shown in Section III that the likelihood of the censored observations reduces to an indicator function over a convex region. Hence, a maximum-likelihood estimator (MLE) of the target location cannot be defined because there is no unique maximizer. For this reason, most existing approaches have cast the localization problem in this censored setting as a centrality problem, proposing heuristic estimators and comparing their performance [12], [13]. When the propagation model or transmission power are known, well-defined estimators have been proposed in [3], [21]. In lieu of such knowledge, empirical studies in [12] and [13] suggest that the center of a minimum enclosing circle (CMEC) outperforms other heuristic estimators for this problem. However, since theoretical analysis of CMEC performance is so far lacking, there is no formal explanation as to why it outperforms other competing alternatives.

Toward addressing this issue, a *smooth* surrogate to the original discontinuous likelihood function is proposed here, whose maximizer is shown to approach the CMEC asymptotically as the likelihood approximation error vanishes (Section IV). This in addition to maximum likelihood (ML) being an efficient estimator (asymptotically in the number of sensors) provides rigorous analytical justification as to why the CMEC estimator outperforms other heuristics for this problem, as empirically observed in [12] and [13]. A parametric family of near-optimal convex estimators is obtained as a byproduct, which approaches the CMEC as the parameter goes to infinity. Since the Cramér–Rao bound does not exist either, an upshot of the results in this letter is that the CMEC performance can be used as a benchmark in this ideal setting, and also for comparison with other more pragmatic binary localization methods in the presence of

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uncertainty. Smoothing of the likelihood function defined over a grid of potential target locations has been studied in [24]. For simplicity in exposition the discussion henceforth focuses on two-dimensional Euclidean space, although the proofs and deductions extend to $\mathbb{R}^3$ by considering minimum enclosing *ball*-based estimators instead of the CMEC. Numerical tests in Section V corroborate the analytical findings of this letter, while concluding remarks are given in Section VI.

II. System Model and Problem Formulation

Consider a noncooperative target located in $\mathbf{s}_t \in \mathbb{R}^2$ that transmits power isotropically in two-dimensional Euclidean space. Suppose that N wireless sensors are scattered uniformly at random over a region $\mathcal{A}$ of area $A := |\mathcal{A}|$, with density ρ sensors per unit area so that $N = \lfloor \rho A \rfloor$. Sensor i, say, is located at position $\mathbf{s}_i \in \mathbb{R}^2$, and compares the received signal power from the target with a prescribed detection threshold τ. This way sensor i can make a binary decision $d_i \in \{0, 1\}$ about the presence ($d_i = 1$) or absence ($d_i = 0$) of the target in its vicinity. An ideal scenario is considered here, where sensors are assumed capable of noise-free target detection in the absence of Rayleigh fading. Assuming an isotropic pathloss model, the local detection problems boil down to whether sensors are located within a detecting radius R from the target or not. The value of R naturally depends on the pathloss model and threshold value τ, both assumed fixed and given. This ideal scenario can be approximated in practice by averaging the received power over a sufficiently long period of time before comparing it with τ.

Only those sensors that detect the target is present (i.e., $d_i = 1$) will communicate their own location and decision to the FC, while the remaining ones transmit nothing to save energy. Given these censored data, the goal is to localize the target and to this end the FC forms a judicious estimate of $\mathbf{s}_t$. Under the adopted noncooperative setting, the transmitted power and propagation model are unknown to the FC.

III. Centrality-Based Estimators

In this section, we briefly describe the challenges facing a likelihood-based approach to localization in the setting of Section II, and outline the heuristic centrality-based geometric estimators that have been proposed in lieu of a MLE.

Recall that the per-sensor detection problems translate to whether sensors are located within distance R from the target or not. Thus, treating $\mathbf{s}_t$ and R as unknown parameters, the degenerate probability of detection of the ith sensor conditioned on its location $\mathbf{s}_i$ is

$$\mathrm{P}\left(d_i = 1 \mid \mathbf{s}_i; \mathbf{s}_t, R\right) = \mathbb{I}\left\{\|\mathbf{s}_i - \mathbf{s}_t\|_2 \leq R\right\} \tag{1}$$

where $\mathbb{I}\{X\}$ is the indicator function of the event X. Because sensors are deployed uniformly at random, the probability density function (pdf) that a *detecting sensor* is located at $\mathbf{s}_i$ becomes

$$f(\mathbf{s}_i | d_i = 1; \mathbf{s}_t, R) = \frac{1}{\pi R^2}\mathbb{I}\left\{\|\mathbf{s}_i - \mathbf{s}_t\|_2 \leq R\right\}. \tag{2}$$

Without loss of generality suppose that the indices of the detecting sensors are $\mathcal{M} = \{1, \ldots, M\}$, and let $\mathcal{S} = \{\mathbf{s}_i : i \in \mathcal{M}\}$ represent the collection of location vectors of those sensors reporting observations to the FC (whose decisions are all one). Then, the likelihood function of a given observation $\mathcal{S}$ is

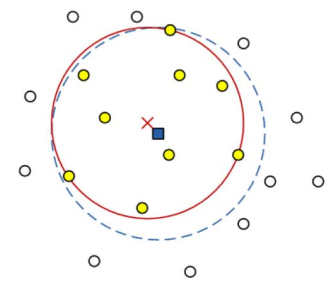

Fig. 1. Shown in red is the smallest enclosing circle of those detecting sensors, which are depicted in yellow. The CMEC (shown with a $\times$) is an estimate of the location of the target indicated by a square.

$$\mathcal{L}(\mathcal{S}; \mathbf{s}_t, R) = \left(\frac{1}{\pi R^2}\right)^M \prod_{i \in \mathcal{M}} \mathbb{I}\left\{\|\mathbf{s}_i - \mathbf{s}_t\|_2 \leq R\right\}. \tag{3}$$

Note that for a specific observation the number of detecting sensors M, is a known parameter. Moreover, upon defining the *possible target region* as [25], [26]

$$\mathcal{T}(\mathcal{S}) = \bigcap_{i \in \mathcal{M}} \mathcal{B}_R(\mathbf{s}_i)$$

where $\mathcal{B}_R(\mathbf{s}_i)$ denotes the disk of radius R centered at $\mathbf{s}_i$, it follows that the likelihood function can be rewritten as

$$\mathcal{L}(\mathcal{S}; \mathbf{s}_t, R) = \left(\frac{1}{\pi R^2}\right)^M \mathbb{I}\left\{\mathbf{s}_t \in \mathcal{T}(\mathbf{S})\right\}. \tag{4}$$

Thus, the binary-valued $\mathcal{L}(\mathcal{S}; \mathbf{s}_t, R)$ is maximized for all $\mathbf{s}_t \in \mathcal{T}(\mathcal{S})$, while it is zero when $\mathbf{s}_t \notin \mathcal{T}(\mathcal{S})$. All in all, the conclusion is that a MLE cannot be defined for this model.

In lieu of an MLE, heuristic *centrality-based estimators* were introduced in [12], [13], and [27], and are outlined here for completeness. For instance, the mean estimator simply adopts the centroid or barycenter of the detecting sensors [27], while the center of minimum enclosing rectangle (CMER) is the center of the smallest rectangle containing the sensors in $\mathcal{M}$ [12]. As CMER is dependent on the choice of axis, the Steiner center is a variant that averages the CMER over a $\pi/2$ axis rotation to remove that dependency [12], [28]. Finally, the center of the *minimum enclosing circle* (CMEC) is the solution to the following optimization problem [12], [13]:

$$\hat{\mathbf{s}}_{\text{CMEC}} = \arg\min_{\mathbf{s}} \left(\max_{i \in \mathcal{M}} \|\mathbf{s}_i - \mathbf{s}\|_2\right) \tag{5}$$

and several algorithms exist in the literature to find $\hat{\mathbf{s}}_{\text{CMEC}}$ [29], see also Fig. 1. Empirical studies in [12] and [13] suggest that the CMEC outperforms other aforementioned heuristic estimators for this problem. However, there is so far no formal explanation as to why this is the case, and we seek to provide an answer in the sequel.

IV. Likelihood Function Smoothing

In this section, we propose a smooth surrogate to the original discontinuous likelihood $\mathcal{L}(\mathcal{S}; \mathbf{s}_t, R)$, whose well-defined maximizer approaches the CMEC asymptotically as the likelihood approximation error vanishes (cf., Proposition 1). This result

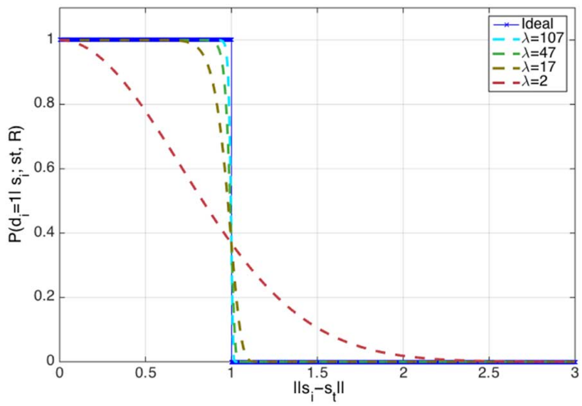

Fig. 2. Sensor detection probabilities versus distance from the target for $R = 1$ for the ideal [cf., (1)] and nonideal [cf., (6)] scenarios. As $\lambda \to \infty$, the smooth approximation approaches (1).

formally justifies why the CMEC outperforms other centrality-based estimators in this ideal target-localization setting.

A. *Smooth Detection Probability Approximation*

Consider now an auxiliary nonideal scenario where the probability of target detection $\mathrm{P}_\lambda\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right)$ 1) is modeled as a continuous function of the sensor distance from the target and 2) is parameterized by a constant λ that controls the pointwise approximation error to $\mathrm{P}\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right)$ in the ideal setting [cf., (1)]. Interestingly, it is shown that the approximation is tight as $\lambda \to \infty$.

Specifically, suppose the probability of detection for the ith sensor is given by the following continuous function [cf., (1)]:

$$\mathrm{P}_\lambda\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right) = e^{-\frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda}}. \tag{6}$$

Fig. 2 shows the plot of $\mathrm{P}_\lambda\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right)$ versus distance of the ith sensor to the target, for several increasing λ values along with $\mathrm{P}\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right)$ in (1). As the figure suggests, one can show that the approximation error vanishes as $\lambda \to \infty$, namely

$$\lim_{\lambda\to\infty} \mathrm{P}_\lambda\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right) = \mathrm{P}\left(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R\right). \tag{7}$$

Recalling that sensors are randomly deployed over a region $\mathcal{A}$, the pdf that a *detecting sensor* is located at $\mathbf{s}_i$ becomes

$$f_\lambda(\mathbf{s}_i|d_i = 1; \mathbf{s}_t, R) = \frac{f_\lambda(\mathbf{s}_i, d_i = 1; \mathbf{s}_t, R)}{f_\lambda(d_i = 1; \mathbf{s}_t, R)}$$
$$= \frac{f_\lambda(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R) f(\mathbf{s}_i)}{\int_{\mathcal{A}} f_\lambda(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R) f(\mathbf{s}_i) d\mathbf{s}_i} = \frac{\frac{1}{A} f_\lambda(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R)}{\frac{1}{A}\int_{\mathcal{A}} f_\lambda(d_i = 1|\mathbf{s}_i; \mathbf{s}_t, R) d\mathbf{s}_i} \tag{8}$$

where $f(\mathbf{s}_i) = 1/A$ is the uniform pdf over $\mathcal{A}$. To simplify (8), suppose the region is arbitrarily large ($A \to \infty$) to obtain

$$f_\lambda(\mathbf{s}_i \mid d_i = 1; \mathbf{s}_t, R) = \frac{e^{-\frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda}}}{\int_0^\infty \int_0^{2\pi} r e^{-\frac{r^\lambda}{R^\lambda}} d\theta dr}. \tag{9}$$

The normalizing constant in the denominator can be readily calculated with a change of variable $x = \frac{r^\lambda}{R^\lambda}$ to yield

$$\int_0^\infty \int_0^{2\pi} r e^{-\frac{r^\lambda}{R^\lambda}} d\theta dr = \frac{2\pi R^2}{\lambda} \Gamma\left(\frac{2}{\lambda}\right) \tag{10}$$

where the gamma function is $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$. Thus, the desired smooth pdf approximation to (2) takes the form

$$f_\lambda(\mathbf{s}_i|d_i = 1; \mathbf{s}_t, R) = \frac{\lambda}{2\pi R^2 \Gamma(\frac{2}{\lambda})} e^{-\frac{|\mathbf{s}_i - \mathbf{s}_t|^\lambda}{R^\lambda}}. \tag{11}$$

Using (7) while noting that $\frac{2\Gamma(\frac{2}{\lambda})}{\lambda} = \Gamma(\frac{2}{\lambda} + 1)$ and $\Gamma(1) = 1$, it follows that the approximation is asymptotically tight

$$\lim_{\lambda\to\infty} f_\lambda(\mathbf{s}_i \mid d_i = 1; \mathbf{s}_t, R) = f(\mathbf{s}_i \mid d_i = 1; \mathbf{s}_t, R). \tag{12}$$

Accordingly, one can for instance show that as $\lambda \to \infty$, the probability that a detecting sensor is located outside the detection region $\mathcal{B}_R(\mathbf{s}_t)$ (in the ideal scenario) vanishes.

The main usefulness of (11) is in that it allows to construct smooth counterparts of the likelihood function $\mathcal{L}(\mathcal{S}; \mathbf{s}_t, R)$ in (4), which offer well-defined MLEs as discussed next.

B. *Asymptotic MLE Meets CMEC*

Like in Section III, suppose that the indices of the detecting sensors for the nonideal scenario are $\mathcal{M}_\lambda = \{1, \ldots, M_\lambda\}$. Note that from the closing arguments in the previous section, $\mathcal{M}_\lambda \to \mathcal{M}$ as $\lambda \to \infty$. In the censoring context of interest here whereby only those sensors in $\mathcal{M}_\lambda$ communicate their location to the FC, the *smooth likelihood function* for a given vector of observations $\mathcal{S}_\lambda = \{\mathbf{s}_i : i \in \mathcal{M}_\lambda\}$ is given by

$$\mathcal{L}_\lambda(\mathcal{S}_\lambda; \mathbf{s}_t, R) = \prod_{i\in\mathcal{M}_\lambda} \frac{\lambda}{2\pi R^2 \Gamma(\frac{2}{\lambda})} e^{-\frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda}}.$$

The log-likelihood $\ell(\mathcal{S}_\lambda; \mathbf{s}_t, R) := \log \mathcal{L}_\lambda(\mathcal{S}_\lambda; \mathbf{s}_t, R)$ is

$$\ell(\mathcal{S}_\lambda; \mathbf{s}_t, R) = M_\lambda \ln \frac{\lambda}{2\pi R^2 \Gamma(\frac{2}{\lambda})} - \sum_{i\in\mathcal{M}_\lambda} \frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda}$$
$$= M_\lambda \ln \frac{\lambda}{2\pi \Gamma(\frac{2}{\lambda})} - 2M_\lambda \ln R - \sum_{i\in\mathcal{M}_\lambda} \frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda} \tag{13}$$

and accordingly the MLEs of the target location $\mathbf{s}_t$ and the detection range R are

$$\{\hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda}, \hat{R}_{\mathrm{ML},\lambda}\} = \arg\min_{\mathbf{s}_t, R} \left(2M_\lambda \ln R + \sum_{i\in\mathcal{M}_\lambda} \frac{\|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda}{R^\lambda}\right). \tag{14}$$

It is worth noting that in (14) the minimization with respect to $\mathbf{s}_t$ is independent of R. Thus, the joint optimization problem (14) decouples into separable minimization tasks

$$\hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda} = \arg\min_{\mathbf{s}_t} \sum_{i\in\mathcal{M}_\lambda} \|\mathbf{s}_i - \mathbf{s}_t\|_2^\lambda \tag{15}$$

$$\hat{R}_{\mathrm{ML},\lambda} = \arg\min_{R} \left(2M_\lambda \ln R + \sum_{i\in\mathcal{M}_\lambda} \frac{\|\mathbf{s}_i - \hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda}\|_2^\lambda}{R^\lambda}\right). \tag{16}$$

Moreover, (15) is a convex optimization problem offering a computationally appealing family of near-optimal target-location estimators parameterized by λ. Interestingly, as asserted in the following proposition $\hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda} \to \hat{\mathbf{s}}_{\mathrm{CMEC}}$ as $\lambda \to \infty$:

Proposition 1: Consider the MLEs in (14). Then, as $\lambda \to \infty$ the following hold:

1) The target location MLE approaches the CMEC in (5), i.e.,
$$\lim_{\lambda\to\infty} \hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda} = \hat{\mathbf{s}}_{\mathrm{CMEC}}.$$
2) The detection radius MLE approaches the radius of the minimum enclosing circle of the detecting sensors in $\mathcal{M}$, i.e.,
$$\lim_{\lambda\to\infty} \hat{R}_{\mathrm{ML},\lambda} = \max_{i\in\mathcal{M}} \|\mathbf{s}_i - \hat{\mathbf{s}}_{\mathrm{CMEC}}\|_2.$$

Proof: To prove 1), introduce the vector $\boldsymbol{\rho} \in \mathbb{R}_+^{|\mathcal{M}_\lambda|}$ with ith entry $\rho_i = \|\mathbf{s}_i - \mathbf{s}_t\|_2$, and notice that (15) can be written as
$$\hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda} = \arg\min_{\mathbf{s}_t} \|\boldsymbol{\rho}\|_\lambda^\lambda = \arg\min_{\mathbf{s}_t} \|\boldsymbol{\rho}\|_\lambda$$
where $\|\boldsymbol{\rho}\|_\lambda$ is the norm λ of $\boldsymbol{\rho}$ and the second equality follows because $(\cdot)^\lambda$ is monotonically increasing in $\mathbb{R}_+$. Now, as $\lambda \to \infty$ then $\|\boldsymbol{\rho}\|_\lambda \to \|\boldsymbol{\rho}\|_\infty := \max_{i\in\mathcal{M}} \rho_i$ implying that
$$\lim_{\lambda\to\infty} \hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda} = \arg\min_{\mathbf{s}} \left(\max_{i\in\mathcal{M}} \|\mathbf{s}_i - \mathbf{s}\|_2\right) = \hat{\mathbf{s}}_{\mathrm{CMEC}}.$$

To obtain $\hat{R}_{\mathrm{ML},\lambda}$, differentiate the cost in (16) with respect to R and equate the result to zero, to obtain the equation
$$\frac{2M_\lambda}{\hat{R}_{\mathrm{ML},\lambda}} - \frac{\lambda}{\hat{R}_{\mathrm{ML},\lambda}^{\lambda+1}} \sum_{i\in\mathcal{M}_\lambda} \|\mathbf{s}_i - \hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda}\|_2^\lambda = 0$$
with root
$$\hat{R}_{\mathrm{ML},\lambda} = \sqrt[\lambda]{\frac{\lambda\left(\sum_{i\in\mathcal{M}_\lambda} \|\mathbf{s}_i - \hat{\mathbf{s}}_{t_{\mathrm{ML}},\lambda}\|_2^\lambda\right)}{2M_\lambda}}. \tag{17}$$
Taking limits in (17) as $\lambda \to \infty$ and using 1) one obtains
$$\begin{aligned}\lim_{\lambda\to\infty} \hat{R}_{\mathrm{ML},\lambda} &= \lim_{\lambda\to\infty} \sqrt[\lambda]{\frac{\lambda}{2M}} \times \left(\max_{i\in\mathcal{M}} \|\mathbf{s}_i - \hat{\mathbf{s}}_{\mathrm{CMEC}}\|_2\right) \\ &= 1 \times \left(\max_{i\in\mathcal{M}} \|\mathbf{s}_i - \hat{\mathbf{s}}_{\mathrm{CMEC}}\|_2\right)\end{aligned} \tag{18}$$
since $\lim_{\lambda\to\infty} \lambda^{1/\lambda} = 1$, which establishes 2). ∎

Proposition 1 asserts that as $\lambda \to \infty$ (arbitrarily close to the ideal setting in Section III), the MLE maximizing the smooth likelihood function $\mathcal{L}_\lambda(\mathcal{S}_\lambda; \mathbf{s}_t, R)$ approaches the CMEC of the detecting sensors.

V. Numerical Tests

To support the analytical result of Section IV-B, a set of corroborating simulations are carried out here. The target is assumed to be located at the origin and a number of sensors are distributed uniformly at random with density ρ over the square $\mathcal{A} = [-50, 50] \times [-50, 50]$. For each λ and R, sensors make decisions on the presence of the target based on the probabilistic model in Section IV-A, and the corresponding set $\mathcal{M}_\lambda$ is determined. The MLEs are then calculated by solving the pair of problems (15) and (16). Moreover, sensor decisions in the ideal setting of Section III are also determined for each sensor-placement realization, and the CMEC of these detecting sensors is compared with the MLEs for four different values of λ. To compute the CMEC, a built in MATLAB function based on the Megiddo algorithm is adopted [30]. Results are averaged over 600 independent trials.

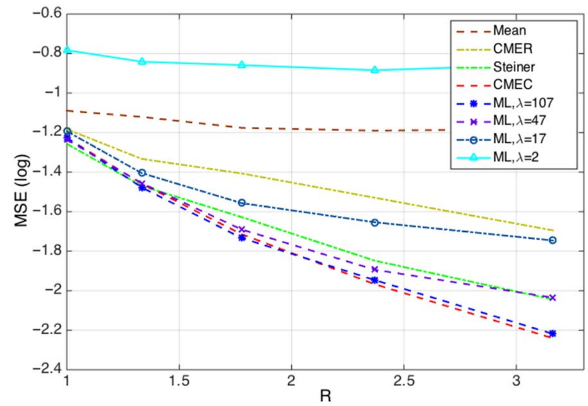

Fig. 3. MSE versus detection radius for fixed $\rho = 2.3$.

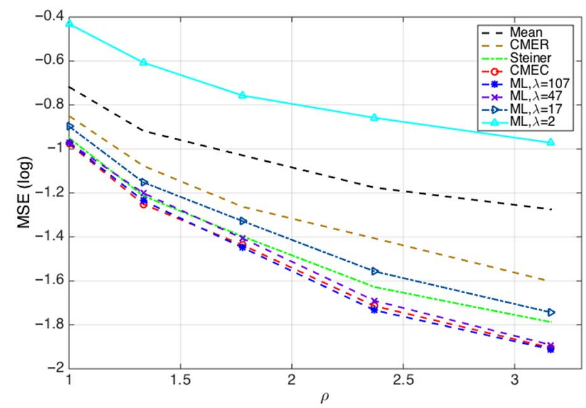

Fig. 4. MSE versus sensor density for fixed $R = 1.77$.

The simulations are run for different values of the parameters ρ and R. Fig. 3 depicts the mean-square estimation error (MSE) of the obtained MLEs versus R, along with the MSE of the CMEC and other centrality-based estimators for fixed density $\rho = 2.3$. As R increases, all MSE values approach zero except for the mean estimator and the MLE with small $\lambda = 2$. Most importantly, notice how the MLEs for large λ attain the MSE of the CMEC, as asserted by Proposition 1. Fig. 4 shows a similar comparison but now as a function of the density ρ, when the detection radius is fixed to $R = 1.77$. Once more, it is apparent that the MSE performance of the MLEs based on likelihood function smoothing follows closely that of the CMEC, for sufficiently large λ.

VI. Conclusion

We showed that the CMEC estimator obtained using censored WSN observations in an ideal noise-free target localization setting, is equivalent to a limiting MLE that maximizes a smooth, approximate likelihood function. This result addresses the lingering question of why the CMEC outperforms most heuristic centrality-based estimators proposed in lieu of a well-defined MLE. As a useful byproduct, the CMEC MSE can be used to benchmark the performance of all location estimators in the presence of uncertainty, such as when additive receiver noise or fading are present. It would also be interesting to investigate alternative approximating functions [cf., (6)], and study their respective rates of convergence to the CMEC.

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