Problem statement

• **Given**: Stream of graph observations

• **Model**: Random Dot Product Graph (RDPG) [1]

• **Goal**: Efficiently track nodes latent positions

Contributions and impact:

⇒ Track representation for dynamic graphs

⇒ Scalable and fast computation of nodal representations

⇒ Embedding dynamic networks with varying number of nodes

### RDPGs

- Node $i = 1, \ldots, n$ has associated latent vector $x_i \in \mathbb{R}^d$
- Edge $(i, j)$ exists with probability $P_{ij} = x_i^T x_j$
- Notation:
  - $A \in \{0, 1\}^{n \times n}$: adjacency matrix
  - $X = [x_1, \ldots, x_n]^T \in \mathbb{R}^{n \times d}$
  - $P = XX^T \in \mathbb{R}^{n \times n}$

- Model is invariant to rotations in $X$:
  - $P = XX^T = XW(XW)^T$ for any orthogonal $W$

- Expressive model, SBM a special case of RDPG

### Adjacency Spectral Embedding

- Estimate $X$ via LS regression: $X_{LS} = \arg\min_X \|XX^T - A\|_F^2$ ⇒ $X_{LS} = QA^{1/2}$

- **Gradient descent (GD) approach**: Estimate $X_{t+1} = X_t - \alpha \nabla f(X_t)$ with $f(X) = \|M \circ (A - XX^T)\|_F^2$
- Mask matrix $M := I - I$ accounts for all-zero diagonal of $A$
- Convergence: if $X_0$ is close to the solution, the iteration converges with linear rate to $X_{LS}$ [2]

### Node addition and deletion

- Project to subspace spanned by columns of $X$, then run GD

- **Alternative approach**: Compute out-of-sample extension [4]

- **Limitation**: Larger estimation error

- Setup: ER with $p = 0.1$. Initially, 100 nodes. 1000 new nodes are sequentially added

### References


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Tracking the Adjacency Spectral Embedding for Streaming Graphs
Paola Bermolen, Marcelo Fiori, Federico Larroca, Bernardo Marenco, Gonzalo Mateos
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### Changes in memberships

- **Alternative approach**: Update SVD decomposition using [3]

- **Limitation**: Does not preserve alignment. Larger estimation error

- Setup: Initially, two-community SBM with 200 nodes. At each timestep, one random node’s community assignment is changed

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### Real data

- Dataset of yearly football matches between national teams [5]

- Australia played in the Oceania Football Confederation until 2005, when it joined the Asian Football Confederation

- **Reproducibility** ⇒ Try it @ github.com/git-artes

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### References