Change Point Detection in Weighted and Directed Random Dot Product Graphs

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Introduction

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  - Neuronal activity
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  2. Probabilistic generative models: theoretically sound results but lack of generality of classic models (e.g. ER or SBM)

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- Two possible approaches:
  1. Embedding-based: limited interpretability and lack of theoretical guarantees
  2. Probabilistic generative models: theoretically sound results but lack of generality of classic models (e.g. ER or SBM)
- We resort to the very versatile Random Dot-Product Graph (RDPG) model:
  - Each node i has an associated vector $\mathbf{x}_i \in \mathbb{R}^d$
  - A link exists between nodes i and j with probability $\mathbf{x}_i^T \mathbf{x}_j$
  - **Attention: rotation ambiguity**

RDPG: intuition

- Vectors may be estimated by spectral decomposition of the adjacency matrix
- Example: the Zachary's Karate Club graph

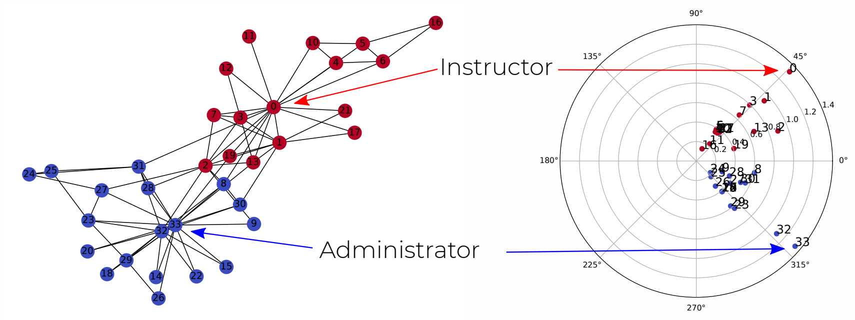

Figure: Zachary's Karate Club graph and its resulting $\mathbf{x}_i$ for $d = 2$.

- Intuition: larger vectors tend to be more connected and angle between vectors indicate affinity

Weighted graphs?

- We propose to extend the RDPG model to the weighted case
- Assume weights $\omega_{i,j}$ are independently drawn from distributions with moment-generating function $M_{\omega_{i,j}}(t) = \mathbb{E}[e^{t\omega_{i,j}}] = \sum_{m=0}^{\infty} \frac{t^m \mathbb{E}[\omega_{i,j}^m]}{m!}$
- **Weighted RDPG**: Each node has a sequence of vectors $\mathbf{x}_i[m] \in \mathbb{R}^{d_m}$ where $\mathbb{E}[\omega_{i,j}^m] = \mathbf{x}_i[m]^T \mathbf{x}_j[m]$
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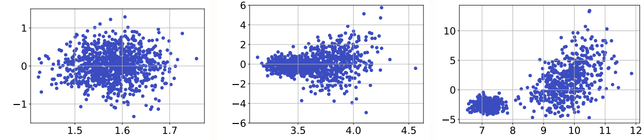
  - Vanilla RDPG is recovered by setting $\mathbf{x}_i[m] = \mathbf{x}_i \, \forall m$
  - Vectors $\mathbf{x}_i[m]$ may be estimated as in the RDPG case by considering $\mathbf{A}^{(m)} = [\omega_{i,j}^m]$
  - Example: weighted SBM with $p = 0.5$ and weights $N(\mu = 5, \sigma = 0.1)$ except between a group of nodes where the distribution is Poisson$(\lambda = 5)$



(a) $m = 1$ **(b)** $m = 2$ **(c)** $m = 3$

Application: Change-Point Detection

- Due to the rotation ambiguity, vectors $\mathbf{x}_i[m]$ cannot be used directly to detect changes:
  - Use the entries of $\hat{\mathbf{Y}}[m] = \hat{\mathbf{X}}[m]\hat{\mathbf{X}}[m]^T$ instead (in particular, the entries that do not share nodes such as $(i,j) \in \mathcal{O} = \{(i, i+n/2)\, \forall i = 1, \ldots, n/2\}$)

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- Example: Wi-Fi network with $n = 6$ APs, with measurements collected hourly during almost four weeks, where an AP was moved at $t \approx 310$

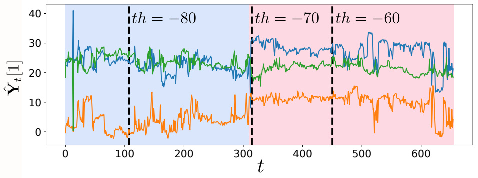

Figure: The evolution of $\hat{\mathbf{Y}}_t[1]$ for entries $(i, j) \in \mathcal{O}$ in the RSSI graph. The background color indicates the change-point estimated through our method and the vertical lines by applying different thresholds *th* to the graph.