# Partially coherent radiation from reverberant chambers

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Using a reverberant chamber, it is possible to radiate a partially coherent ultrasonic beam into a free field. This type of acoustic secondary source is shown to be analogous to the Lambertian source in optics. The features and potential uses of this type of field are in marked contrast to the case of fully coherent ultrasonic radiation produced by plane or focussed transducers. The partially coherent secondary sources are not difficult to construct in a laboratory setting, and provide a useful alternative in ultrasonic measurements of materials properties, and other applications such as imaging.

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## INTRODUCTION

The distinction between coherent and incoherent illumination can be easily visualized. Coherent sources of light from lasers are associated with speckle patterns and narrow beamwidths. In comparison, the relatively incoherent light produced by a heated filament is associated with a broad directivity pattern and diffuse illumination. In ultrasonics, the radiation patterns produced by common transducers are spatially and temporally coherent, and the vast majority of ultrasonic measurements are carried out using fields produced by these sources.

There are a number of instances where coherent beam effects are considered to be undesirable, such as the so-called speckle patterns or diffraction "ringing" seen in ultrasonic images. Attempts to overcome these effects have included the addition of a random phase shifting layer near the acoustic source,[1] or the use of randomly excited multiple transducers[2] to generate a more incoherent acoustic beam.

It will be shown that an incoherent, or more accurately, a partially coherent source of ultrasound can be constructed using reverberant chamber techniques. If a small aperture on the reverberant chamber wall is allowed to radiate acoustic energy into a free field, the partially coherent, secondary source formed will produce a farfield directivity pattern obeying Lambert's cosine law.

Reverberant chambers have been used extensively in room and underwater acoustics in various applications.[3–5] In ultrasonics they have been utilized for the measurement of attenuation coefficients of liquids,[6] and in a more recent development, to measure the total ultrasonic energy absorbed by a body during hyperthermia experiments.[7] To the authors' knowledge, the use of reverberant chambers to generate "incoherent" ultrasound has not been previously reported.

The first section of this paper will use acoustic theory with diffuse field assumptions to evaluate the autocorrelation function for velocity and pressure at the aperture between reverberant and free fields. The distance over which the autocorrelation function drops to a pre-determined value is called the coherence length in scalar optics theory.[8,9] The coherence length of the secondary source formed by the reverberant chamber is shown to be on the order of a wavelength. The farfield directivity function is then derived from the autocorrelation function of velocity at the aperture. The beam pattern is verified experimentally, and comparisons of the diffraction patterns caused by coherent and "incoherent" beams are shown. Finally, applications where use of the "incoherent" beam may provide an advantage over the traditional coherent sources are discussed.

## I. THEORY

The complex pressure $\hat{P}$ at a position $\mathbf{x}$ in a reverberant chamber can be thought of as the superposition of plane waves incident from all directions[3]

$$\hat{P}(\mathbf{x}) = \sum_q \hat{P}_q \exp\left[j(k\mathbf{n}_q \cdot \mathbf{x} - \omega_0 t)\right], \tag{1}$$

where the index $q$ represents direction, $\mathbf{n}_q$ are unit vectors uniformly distributed around $4\pi$ solid angle, $k$ and $\omega$ are the wavenumber and radial frequency of the plane waves, and the $\hat{P}_q$ are independent, identically distributed variables of random magnitude and phase. The corresponding velocity at a point is thereby given as

$$\mathbf{v}(\mathbf{x}) = \sum_q \mathbf{n}_q \hat{v}_q \exp\left[j(k\mathbf{n}_q \cdot \mathbf{x} - \omega_0 t)\right], \tag{2}$$

where, from the plane-wave impedance relations

$$\hat{v}_q = \hat{P}_q / \rho c, \tag{3}$$

where $\rho$ is the media density, and $c$ the speed of sound.

If an aperture is made in a wall of the reverberant tank, then energy is allowed to escape into a free field as shown in Fig. 1. We will assume that the reverberant chamber walls are perfectly rigid and that no transmission takes place into the free field except at the aperture.

We further assume the plane waves $\hat{P}_q$ are slowly time varying (quasi-monochromatic conditions) due to breaking up of standing wave patterns by source phase modulation, for example. It has been shown that the time averaged farfield intensity directivity pattern from a randomly vibrating area is given by the Fourier transform of the autocorrelation function of the normal velocity component across the aperture[10]

$$I(\psi)/I(0) = F\{Bv_x\, v_x(r)\}, \tag{4}$$

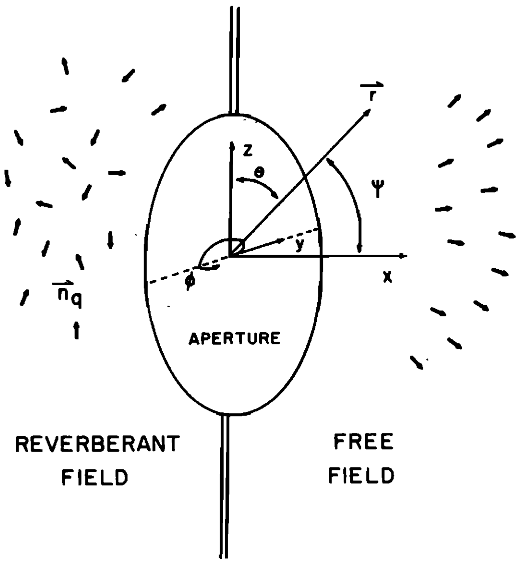


FIG. 1. Diagram of an aperture which allows energy from a reverberant field to escape into a free field. The coordinate system is positioned at the center of the aperture, and the wall separating the two fields is aligned with the $z$–$y$ plane of the coordinate system.

where $I(\psi)$ represents the time averaged intensity in the farfield at an angle $\psi$ off the normal axis (see Fig. 1); $F\{\,\}$ represents the Fourier transform operator, and $Bv_xv_x$ represents the spatial autocorrelation function for the normal component of velocity on the aperture. In using the above expression, we are tacitly assuming that the autocorrelation function $Bv_xv_x$ drops to zero over a small characteristic length compared to the aperture size, but the aperture is considered to be sufficiently small so as to not void the presumption of a diffuse field within the reverberant chamber.

In order to calculate the autocorrelation function, we write the $x$ component of velocity at some position $\epsilon$ on the aperture (the $y$–$z$ plane of Fig. 1)

$$\hat{v}_x(\epsilon) = \mathbf{e}_x \cdot \mathbf{v}(\epsilon) = \sum_q n_{xq} \hat{v}_q \exp\,[j(k\mathbf{n}_q \cdot \epsilon - \omega_0 t)], \tag{5}$$

where $\mathbf{e}_x$ is a unit vector in the $x$ direction and

$$n_{xq} = \mathbf{n}_q \cdot \mathbf{e}_x. \tag{6}$$

Since we are on the border between the reverberant field and a free field, the plane waves are incident from the left hemisphere only, and the summation on $q$ is now understood to be taken over this hemisphere, or $2\pi$ solid angle.

By writing the correlation function definition, then substituting Eq. (5), we obtain

$$\begin{aligned} Bv_xv_x(\Delta t,\Delta\epsilon) &= E\,\{\hat{v}_x(t,\epsilon)\hat{v}_x^*(t+\Delta t,\epsilon+\Delta\epsilon)\}, \qquad (7)\\ &= E\left\{\left(\sum_q n_{xq}\,\hat{v}_q \exp\,[j(k\mathbf{n}_q\cdot\epsilon - \omega_0 t)]\right)\right.\\ &\quad\times\left(\sum_{q'} n_{xq'}\,\hat{v}_{q'}^* \exp\{-j[k\mathbf{n}_{q'}\cdot(\epsilon+\Delta\epsilon)\right.\\ &\quad\left.\left. - \omega_0(t+\Delta t)]\}\right)\right\}, \qquad (8)\end{aligned}$$

where $E\,\{\,\}$ represents an ensemble average and the asterisk represents conjugation. The product of the two series will include cross terms of the form

$$E\,\{n_{xq}\,\hat{v}_q\,n_{xq'}\,\hat{v}_{q'}^*\,e^{(\ldots)}\}. \tag{9}$$

But since the $n_{xq}$ and $\hat{v}_q$ are independent, and the $\hat{v}_q$ are uncorrelated, this term vanishes. Thus

$$Bv_xv_x(\Delta t,\Delta\epsilon) = E\left[\sum_q n_{xq}^2\,v_q^2 \exp\,[j(\omega_0\Delta t - k\mathbf{n}_q\cdot\Delta\epsilon)]\right]. \tag{10}$$

Taking the real part of the above expression, we obtain

$$Bv_xv_x(\Delta t,\Delta\epsilon) = (V^2)_{\text{avg}} E\left\{\sum_q n_{xq}^2 \cos\,(\omega_0\Delta t - k\mathbf{n}_q\cdot\Delta\epsilon)\right\}, \tag{11}$$

where, since the $V_q$ are independent of the $n_{xq}$ and cosine terms we have taken the mean squared value of the velocity out from the curly braces. Since an ideal, diffuse field is assumed to be present in the reverberant chamber, then the ensemble or spatial averaging will assign equal weighting to all directions of incident sound. Thus the average of the summation over discrete directions becomes the average over all directions of incident waves,[3,11] in this case, the left hemisphere of Fig. 1

$$Bv_xv_x(\Delta t,\Delta\epsilon) = \frac{V_{\text{avg}}^2}{2\pi}\int_{\text{hemisphere}} n_{xq}^2 \cos\,(\omega_0\Delta t - k\mathbf{n}_q\cdot\Delta\epsilon)d\Omega. \tag{12}$$

Without loss of generality, we align the vector $\Delta\epsilon$ with the $z$ axis in Fig. 1. Using spherical coordinates:

$$(n_{xq})^2 = (\mathbf{n}_q\cdot\epsilon_x)^2 = (\sin\theta\cos\phi)^2, \tag{13}$$

$$\mathbf{n}_q\cdot\Delta\epsilon = \Delta\epsilon\cos\theta, \tag{14}$$

$$d\Omega = \sin\theta\, d\theta\, d\phi, \tag{15}$$

so

$$\begin{aligned} Bv_xv_x(\Delta t,\Delta\epsilon) &= \frac{V_{\text{avg}}^2}{2\pi}\int_{\phi=-\pi/2}^{\pi/2}\int_{\theta=0}^{\pi} (\sin\theta\cos\phi)^2\\ &\quad\times\cos(\omega_0\Delta t - k\Delta\epsilon\cos\theta)\sin\theta\, d\theta\, d\phi. \end{aligned} \tag{16}$$

Integrating over $\phi$ and expanding the cosine term yields

$$\begin{aligned} Bv_xv_x(\Delta t,\Delta\epsilon) &= \frac{V_{\text{avg}}^2}{4}\cos\omega_0\Delta t\int_0^{\pi}\sin^3\theta\cos(k\Delta\epsilon\cos\theta)d\theta\\ &\quad+\frac{V_{\text{avg}}^2}{4}\sin\omega_0\Delta t\int_0^{\pi}\sin^3\theta\sin(k\Delta\epsilon\cos\theta)d\theta. \end{aligned} \tag{17}$$

The second integral is zero because the integrand is an odd function over the range $0 \leqslant \theta \leqslant \pi$. The first integral can be realized[12] as a Bessel function of the first kind, of fractional order

$$\begin{aligned} Bv_xv_x(\Delta t,\Delta\epsilon) &= (V_{\text{avg}/4}^2)\cos\omega_0\Delta t\\ &\quad\times[\sqrt{\pi}(2/k\Delta\epsilon)^{3/2} J_{3/2}(k\Delta\epsilon)]. \end{aligned} \tag{18}$$

The spatial dependence of this function is similar to the $\sin(k\Delta\epsilon)/k\Delta\epsilon$ autocorrelation of pressure in the reverberant field. The first zero of the Bessel function occurs when the argument equals 4.49. Thus, if we define the coherence length as being the minimum separation distance $\Delta\epsilon$ required to zero the correlation function, then this length is equal to approximately $0.7\lambda$, where $\lambda$ is the acoustic wavelength.

The farfield intensity beam pattern is given by the two-dimensional Fourier transform of the spatial correlation function.[10] Since $Bv_x v_x$ is a function of separation distance $\Delta\epsilon$ (without directional dependence), we are seeking a special case; the Fourier transform of a circularly symmetric function. This is given as a Hankel transform

$$F\{Bv_x v_x\} = \int_0^\infty Bv_x v_x(r)\, J_0(rk \sin \psi) r\, dr, \tag{19}$$

where $r$ replaces $\Delta\epsilon$ as separation distance, and $\psi = 0$ aligns with the $x$ axis of Fig. 1.

Placing the Bessel function term into the Hankel transform gives

$$F\{Bv_x v_x(r)\} = \int_0^\infty \sqrt{\pi}\left(\frac{2}{kr}\right)^{3/2} J_{3/2}(kr) \times J_0(kr \sin \psi) r\, dr. \tag{20}$$

Fortunately, closed form expressions exist for definite integrals of mixed Bessel functions, and the result for this integrand is[13]

$$F\{Bv_x v_x(r)\} = (4/k^2)\,(1 - \sin^2 \psi)^{1/2}, \tag{21}$$

$$= (4/k^2)\,(\cos \psi). \tag{22}$$

Thus the farfield intensity distribution falls off as $\cos \psi$, as does a Lambertian source in optics

$$I(\psi)/I(0) = \cos \psi. \tag{23}$$

This result can be derived in two additional ways. A simple argument begins with the description of plane waves incident on the aperture of Fig. 1, whose propagation directions are uniformly distributed over a half hemisphere. The direction of acoustic energy flux or intensity may also be taken as uniformly distributed over a half hemisphere. The acoustic power escaping in any direction will be proportional to the scalar product of the intensity with the unit vector which is normal to the aperture surface. This scalar product leads naturally to a $\cos \psi$ term. A rigorous alternative derivation can be obtained by considering the pressure, instead of velocity over the aperture. The major points will be summarized. First, the farfield intensity beam pattern can be shown to be given by the Fourier transform of the pressure autocorrelation function, times a $\cos^2 \psi$ term.[8,14]

Thus

$$I(\psi) \cong F\{B_{pp}(r)\} \cos^2 \psi. \tag{24}$$

The pressure autocorrelation function can be shown (in a manner analogous to the derivation for velocity) to be given by a "sinc" function

$$B_{pp}(r) = \sin(kr)/kr. \tag{25}$$

The Fourier (or Hankel) transform of the two-dimensional sinc function yields a $[1/(1 - \sin^2 \psi)]$ term which equals $1/\cos \psi$. Thus

$$I(\psi)/I(0) = (1/\cos \psi) \cos^2 \psi = \cos \psi \tag{26}$$

which is the same as our previous result.

As a final note, as we extend the analysis from the single frequency case to a narrow or broadband signal, the time average intensity angular distribution retains a $\cos \psi$ term since this beam shape is independent of $\omega$. However, the correlation length at the aperture will decrease as higher frequency components are added. The ability to vary correlation length is an advantage which is not shared by other schemes for producing "incoherent" sound.[1,2]

## II. METHODS

A reverberant underwater chamber was constructed using an aluminum pot with a wall thickness of approximately 1 mm and a height and diameter of approximately 30 cm.

A 2.5-cm-diam piezoelectric, 1-MHz transducer was used to produce the acoustic signal. The formation of standing waves was avoided by indenting the aluminum wall in an irregular manner, scattering the direct beam off an irregularly shaped reflector, and frequency modulating the 1-MHz signal using a random signal generator as in input to a voltage-controlled frequency oscillator (see Fig. 2).

A second, anechoic tank was connected by a 2-cm-diam aperture cut in each tank. These apertures were lightly soldered together, while the tank walls kept isolated by a paper layer which insured that transmission of sound to the "free-field" tank would occur only through the aperture. Natural rubber absorbers were used in the "free-field" tank to ensure anechoic conditions. A 1.0-mm-diam broadband PVDF receiver was positioned in the free field to measure the acoustic pressure, and the received and amplified signal was measured by an oscilloscope, a spectrum analyzer, or an rms voltmeter. Overall, the transmitted ultrasonic bandwidth was measured to be 34 kHz centered at 1 MHz. The narrow bandwidth of the signal was due to the use of a high-$Q$ transducer, and permits the assumption of quasi-monochromatic sound used in the theory. The fluctuations in output frequency were very effective in breaking up standing wave patterns, as indicated by the low variance of rms pressure measured at

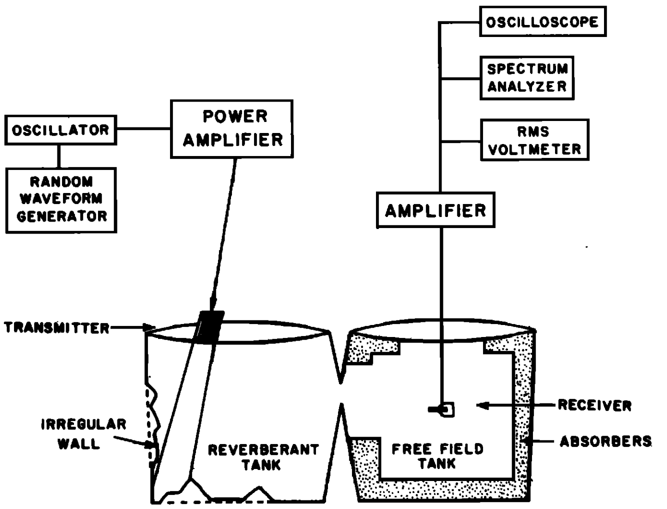


FIG. 2. Schematic drawing of experimental setup used to produce and measure acoustic fields.

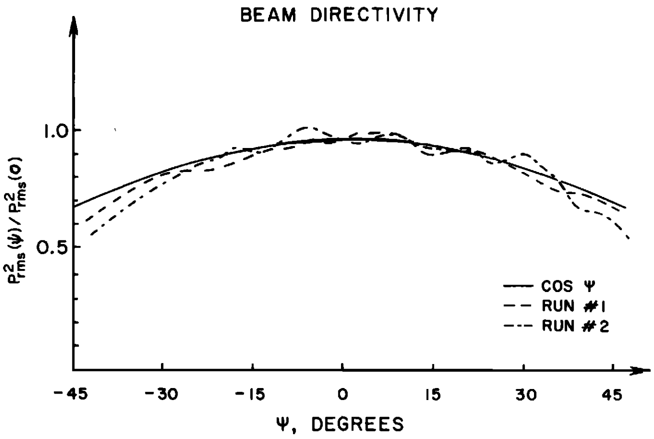


FIG. 3. Measured intensity (rms pressure, squared) as a function of angle, compared to the theoretical beam pattern given by cos $\psi$.

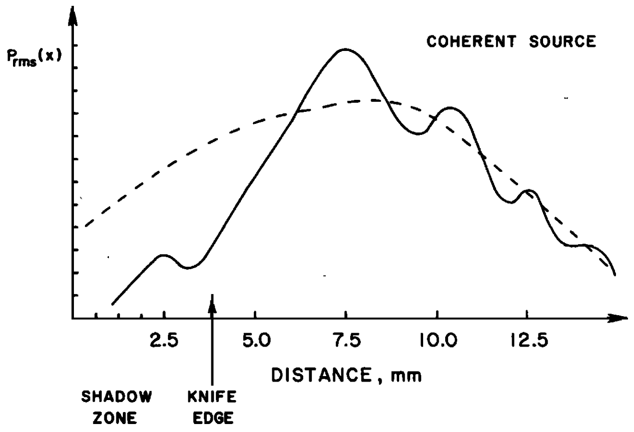


FIG. 5. The diffraction pattern from a knife edge in the farfield of a coherent ultrasonic field. Dotted line—measured rms pressure in the farfield with no obstructions present. Solid line—measured diffraction pattern with knife edge in place.

random locations in the reverberant field.[3] Another measure of the quality of the reverberant field is the decay constant after the source is turned off. This was measured to be on the order of 10 ms, comparing favorably with other reverberant chambers using degassed water and low megahertz ultrasound.[6,7]

## III. RESULTS AND DISCUSSION

To determine the farfield directivity, the receiver was positioned in the free-field tank and aligned with the center of the aperture. It was moved off axis, in the $\psi$ direction, at a constant range of 15 cm from the aperture, and the rms signal voltage was recorded over an excursion of $\psi = \pm 45°$. The square of this value, normalized with respect to the $\psi = 0$ measurement, should display the cos $\psi$ dependence. These data are shown in Fig. 3, where experiments were repeated with slightly altered orientation of the source transducer and the irregular reflector used to scatter the main beam (Fig. 2). The variations in measured beam patterns with source reflector positions reveal that the reverberant chamber acoustic field is not ideally "diffuse," however, the general agreement with theory is good.

Another experiment which displays the unique behavior of the "incoherent" beam is the one-dimensional image of the diffraction pattern created by a knife edge. Theoretically, the Fresnel diffraction pattern at a semi-infinite plane bounded by a straight edge has an oscillatory behavior shown in Fig. 4 for the case of coherent illumination.[8] With "incoherent" illumination, however, the diffraction pattern ringing is smoothed to produce a monotonically increasing intensity from the shadow region to the illuminated plane (see Fig. 4). Although the ideal conditions of a semi-infinite shadow region and uniform coherent or incoherent irradiation could not be met, the general features of coherent and incoherent diffraction patterns could be demonstrated experimentally.

A knife edge was constructed using thin sheets of aluminum separated by a paper film, then lightly soldered and smoothed around the edges. The edge was placed in the free-field tank, 8 cm from the aperture and perpendicular to the beam axis. The diffraction pattern was measured with the receiver probe positioned 3 mm distal to the knife edge and moved transaxially in 0.625-mm increments. Incoherent il-

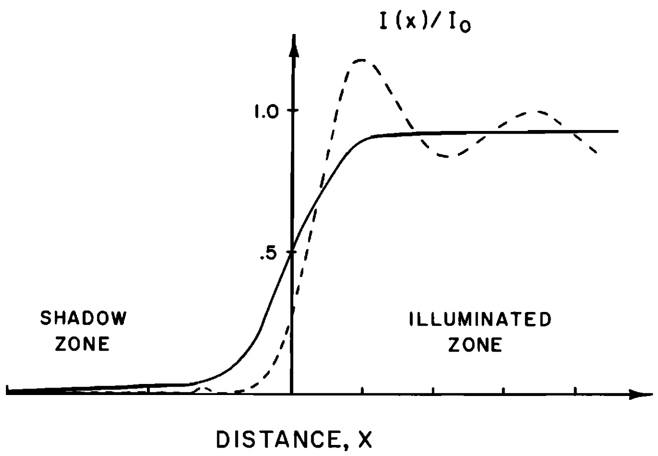


FIG. 4. Theoretical intensity diffraction patterns created by a knife edge under conditions of coherent (dotted line) and incoherent (solid line) irradiation. (Adapted from Refs. 8 and 9.)

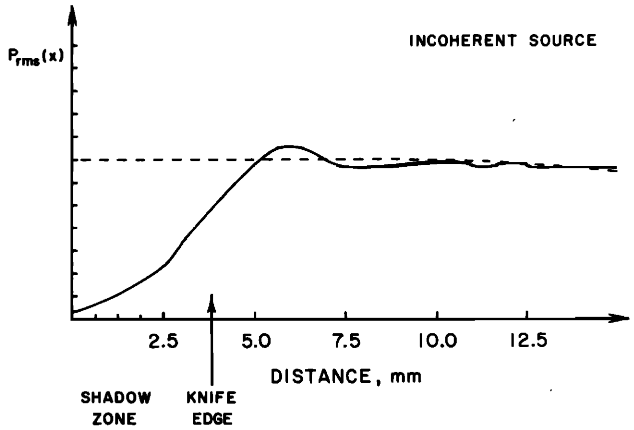


FIG. 6. The diffraction pattern from a knife edge under incoherent irradiation. Dotted line—measured rms pressure in the farfield with no obstructions present. Solid line—measured diffraction pattern with knife edge in place.

lumination was obtained using the reverberant chamber, and coherent illumination was obtained by aligning the transmitter so the direct beam was normally incident on the aperture, and propagating into the free-field tank. Frequency modulation of the source was identical in both cases, so the difference in diffraction patterns cannot be attributed to frequency smearing or averaging effects. Figure 5 shows the measured beam patterns for the coherent sound source, before and after introduction of the knife edge. The diffraction pattern oscillations are prominent. Figure 6 shows the beam patterns obtained, with and without the knife edge, using the incoherent source. The undisturbed beam pattern with its cos $\psi$ dependence is much broader than the coherent beam pattern. The diffraction pattern measured with the knife edge in place is markedly smoothed compared to the previous case. Some oscillations are present, indicating that the beam has a nonzero correlation length. It was shown in the theory section that the correlation length at the aperture was on the order of the wavelength $\lambda$, in this case approximately 1.5 mm.

## IV. CONCLUSION

A partially coherent ultrasonic source can be generated using reverberant chamber techniques. The autocorrelation function of normal velocity at an aperture on the reverberant chamber is described by a Bessel function of order 3/2, and the autocorrelation function for pressure is given by the "sinc" function. The farfield directivity pattern for this type of secondary source is therefore given by a cos $\psi$ dependence, and is directly analogous to thermal or Lambertian sources in optics.

The result is significant since it provides a straightforward means of producing an ultrasonic source characterized by a small coherence length. This source is relatively incoherent, compared to radiation from commonly used piezoelectric transducers, and this incoherence may be advantageous in experimental measurements of material properties such as attenuation or reflection coefficients. In addition, an improvement may be expected in certain imaging techniques which suffer from severe diffraction "ringing" effects[2,9] under coherent radiation. These effects can be dramatically reduced by using an incoherent or partially coherent sound source, as demonstrated by the Fresnel diffraction patterns shown in Figs. 5 and 6.


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