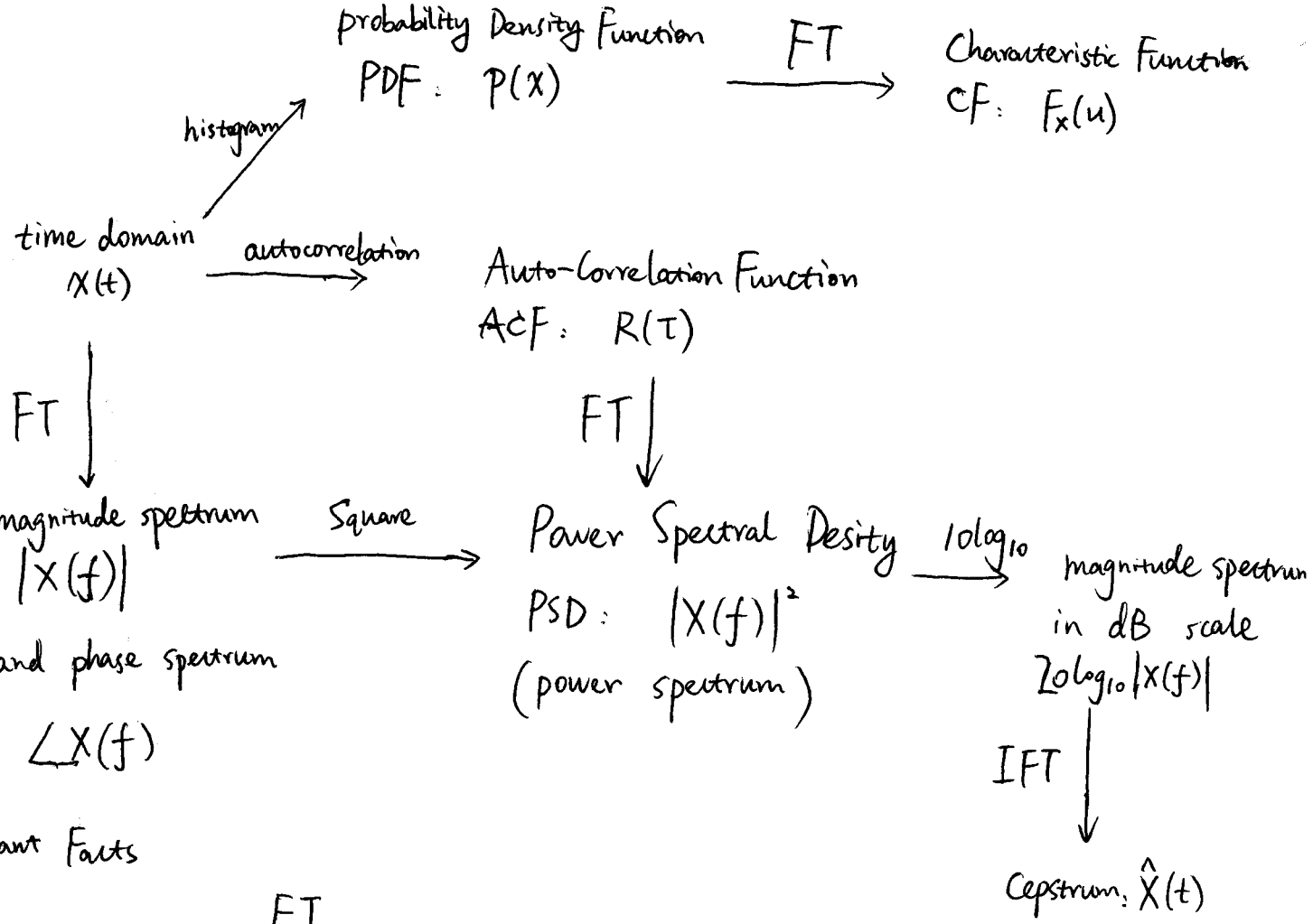
# ECE 272/472: Midterm Review

## Basics: Relation between different representations




## Important Facts

Multiplication $\overset{FT}{\Longleftrightarrow}$ Convolution

periodic impulse train $\overset{FT}{\Longleftrightarrow}$ periodic impulse train

periodic impulse train = sum of infinitely many sine waves with equal weight

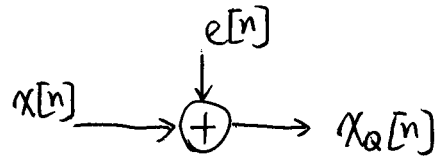
Function × impulse train, Function * impulse train

X, Y independent R.V. $Z = X + Y$, then $p(z) = p(X) * p(Y)$

$\therefore F_Z(u) = F_X(u) \cdot F_Y(u)$

Quantization

Additive noise model

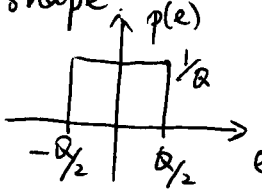



Quantization error/noise $e[n] = x_Q[n] - x[n]$

Quantify quantization noise by

① SNR. ② noise spectral shape.

- If noise PDF is uniform :




$\sigma_e^2 = \frac{Q^2}{12}$

$SNR = 10\log_{10}\frac{\sigma_x^2}{\sigma_e^2}$ can be calculated when $\sigma_x^2$ is known.

Calculate SNR for simple input, e.g., sine wave,

- If noise at different times are independent, then ACF of noise is an impulse, then PSD is flat, i.e., noise is white!

These nice properties are hard to achieve in practice, because $e[n]$ depends on input $x[n]$. Can we find some way to achieve these properties?

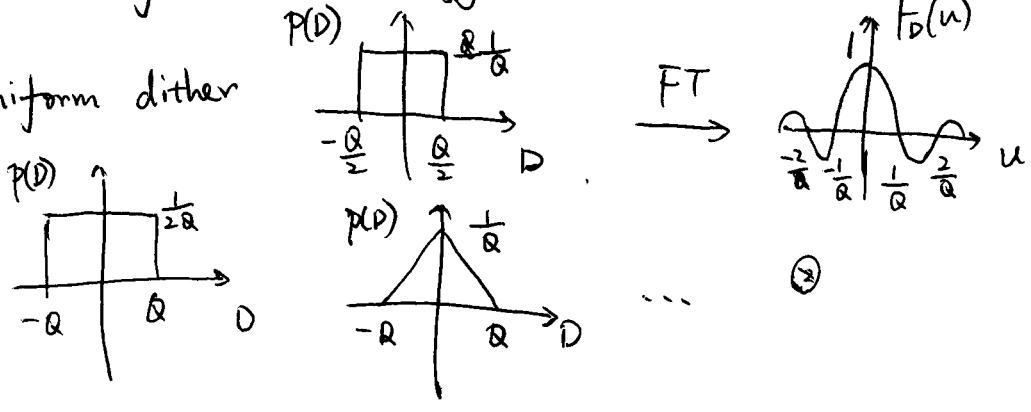
Dither: add independent noise $d[n]$ to input $x[n]$

- If CF $F_D\left(\frac{k}{Q}\right) = 0$ for all $k \in \mathbb{Z}, k \neq 0$, then the PDF of $e[n]$ becomes uniform between $\left[-\frac{Q}{2}, \frac{Q}{2}\right]$

- If CF $F_{D_n, D_m}\left(\frac{k}{Q}, \frac{l}{Q}\right) = 0$ for all $k, l \in \mathbb{Z}, k, l \neq 0$, then PSD of $e[n]$ becomes flat.

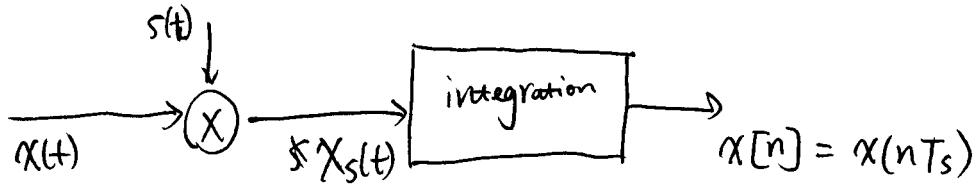
Dither signals that satisfy these conditions

① uniform dither



Sampling: $x[n] = x(nT_s)$, $T_s = \frac{1}{f_s}$

Two stage process for mathematical convenience




$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

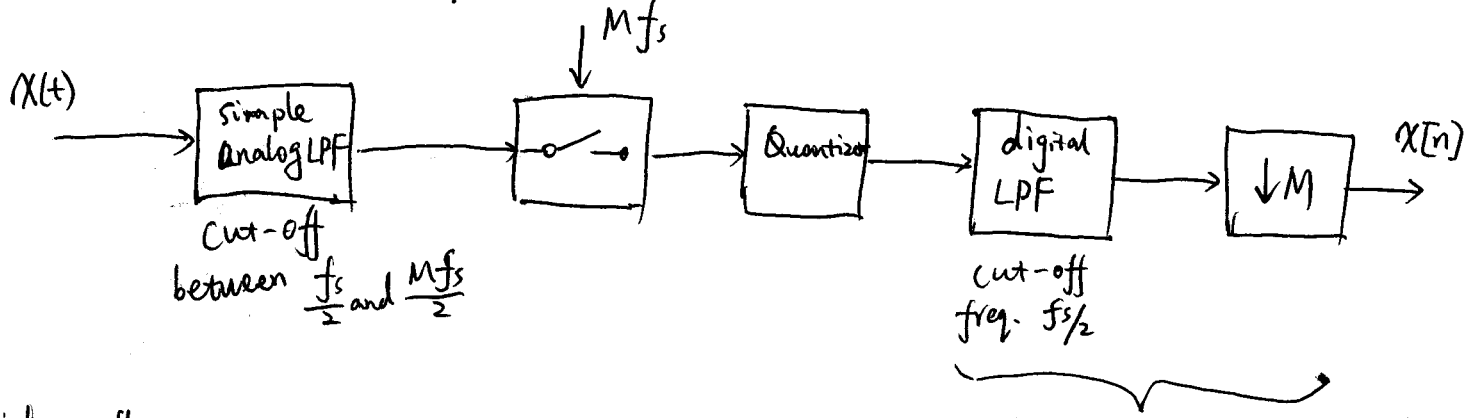
$FT\{x_s(t)\}$ is a periodic version of $FT\{x(t)\}$, with period $f_s$.

Aliasing happens when $x(t)$ contains frequencies higher than $\frac{f_s}{2}$.

To prevent aliasing: two ways: ① Analog LPF before sampling.
② sample faster.

Analog LPF with sharp cut-off freq. is hard to make, so let's sample faster.

Oversampling: Use $Mf_s$ to sample, where we only care about frequencies below $f_s/2$.




Side effect: quantization noise is reduced by factor of M; i.e., SNR increase $10\log_{10} M$

∴ Doubling M, SNR will increase for $10\log_{10} 2 = 3\text{dB} \Leftrightarrow 0.5$ bit depth increase.

Use $\Sigma$-$\delta$ modulator to achieve more quantization noise reduction.

idea: high-pass quantization noise, while all-pass signal.

L order modulator, ~~SNR in~~ Doubling M will give SNR increase of $(2L+1)\cdot 3$ dB $\Leftrightarrow$ ~~6L+5~~ L+0.5 bit depth increase.

Fourier Analysis

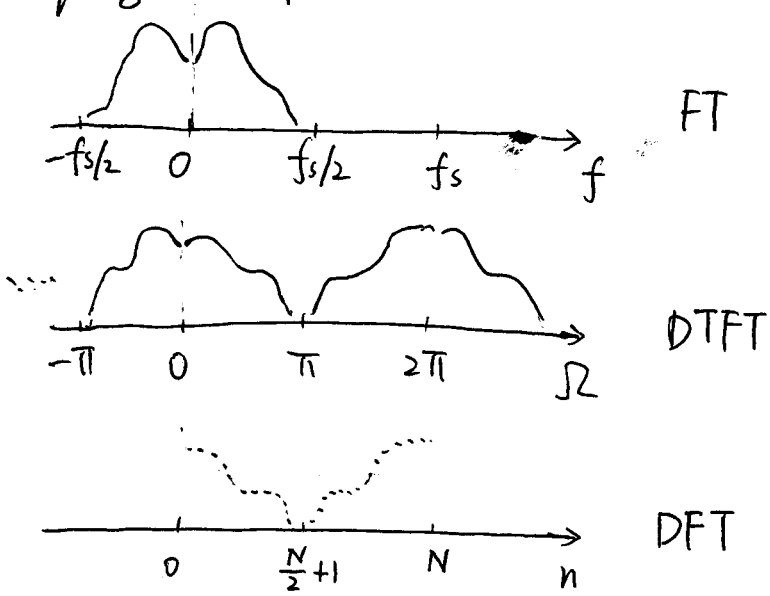
| | 1st Domain | | 2nd Domain |
|---|---|---|---|
| Four FTs. | continuous | $\Longleftrightarrow$ | aperiodic |
| | discrete | $\Longleftrightarrow$ | periodic |

Relation between FT spectrum and DTFT spectrum.

DTFT spectrum is periodic with period of $2\pi$.

Relation between DTFT spectrum and DFT spectrum

Frequency correspondence between FTs.




STFT: $\Delta t \cdot \Delta f = 1$.

| windows: | Rectangular window : | Hamm | Hamming |
|---|---|---|---|
| main lobe width | $2\Omega_M = \frac{4\pi}{M}$ | $4\Omega_M$ | $4\Omega_M$ |
| side lobe hight | -13dB | -31dB | -41dB |
| ----- decay | -6dB/Oct | -18dB/Oct | -6dB/Oct |

$\frac{2\pi}{M}$ angular frequency $\Longleftrightarrow$ $\frac{fs}{M}$ Hz frequency.

Frequency resolution decreased due to windowing effect.

Zero-padding : ideal spectral interpolation, doesn't increase frequency resolution

STFT:

Spectrogram calculation: window size, hop size, zero padding

Signal resynthesis / reconstruction:

Overlap-add technique. (OLA)

Constant OLA (COLA): $\sum_{m=-\infty}^{\infty} W[n-mR] = 1$, $R$: hop size

Weighted OLA (WOLA): apply synthesis window to remove click ~~sound~~ sounds.

$$\sum_m W[n-mR]\, V[n-mR] = 1$$

$W$: analysis window, $V$: synthesis window

phase Vocoder: magnitude spectrogram interpolation.
phase spectrogram reconstruction.

Source Filter Models: basic concept. (the best auto-regressive model).

LPC: $x[n] = \sum_{k=1}^{p} a_k\, x[n-k] + e[n]$

$a_k$: filter, $e[n]$: excitation

filter transfer function: $H(z) = \frac{X(z)}{E(z)} = \frac{1}{1 - \sum_{k=1}^{p} a_k z^{-k}}$

How to calculate $a_k$ ?

$\min \sum_n e^2[n]$ : calculate derivatives v.s. $a_k$.

Cross synthesis:

Cepstrum: $\mathrm{IFFT}\{\log|\mathrm{FFT}[x[n]]|\}$

low coefficients: filter.
high - - - - - : excitation.

Another explanation of Cepstrum: least-square solution of approximating log-amp spectrum with linear combination of sinusoids.