



## *Principal Investigators:*

**Steve Barnett**, Strathclyde, overall system theory

**Robert Boyd**, Ottawa, Rochester, mode sorting, turbulence mitigation

**Daniel Gauthier**, Duke, QKD system, detectors, entangled source

**Paul Kwiat**, Illinois, QKD system, detectors, entangled source, theory

**David Miller**, Stanford, fundamental limits mode sorters

**Miles Padgett**, Glasgow, mode sorters, detectors

**Glenn Tyler**, tOSC, turbulence mitigation

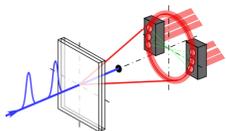
## *Partners:*

**Robert Calderbank**, Duke, quantum error correction theory

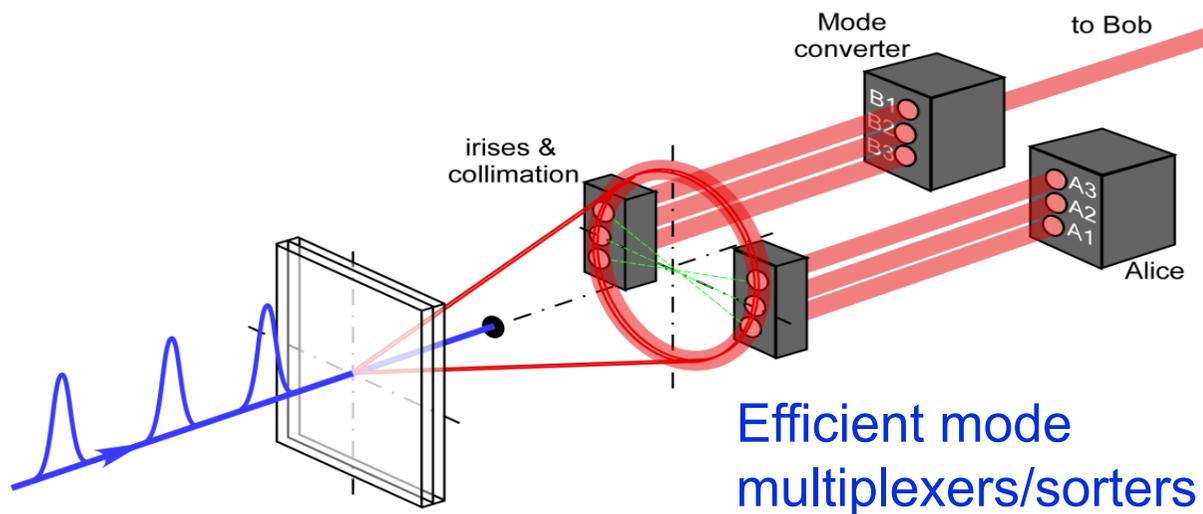
**Andrew White**, Queensland, mode sorters, detectors, QKD system

**Norbert Lutkenhaus**, Waterloo, quantum security

**Gerard Milburn**, Queensland, overall system theory



**Goal:** Develop a discrete, entanglement-based quantum key distribution (QKD) system to meet program objectives

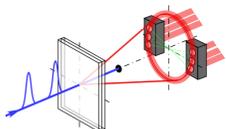


Turbulence mitigation

Low deadtime,  
efficient, high-speed  
single-photon-  
counting detectors

Trade space of  
data rate/security

Bright polarization-entangled source



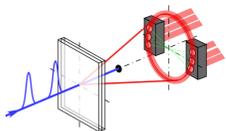
Use hyperentanglement to achieve many bits/photon (bpp)  
and high secure-key rate: **QUANTUM DATA HYPERCUBE**

Use polarization, time-bin, spatial mode degrees-of-freedom

Multiplex many independent channels

➔ 1 spatial mode per channel (e.g., orbital angular  
momentum states (OAM))

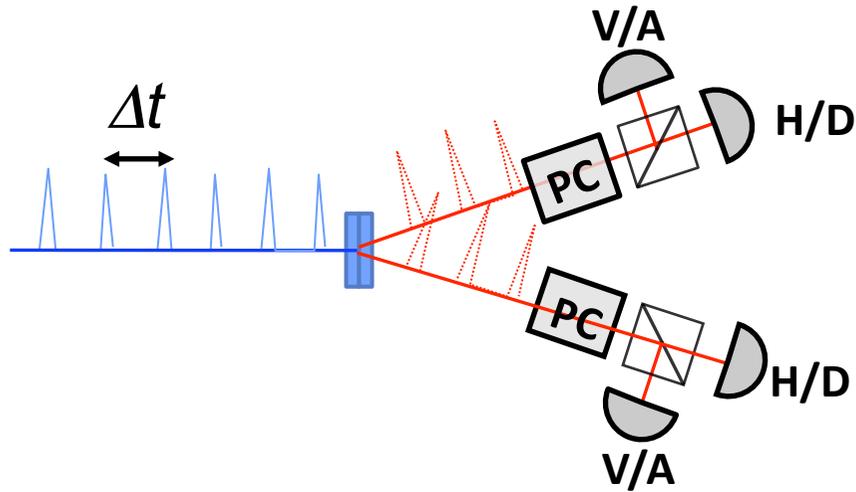
**Single channel:** create mutually-unbiased bases (MUBs) only  
in polarization, use time bins to achieve bpp



# Single Channel



Encode in time, verify security in polarization



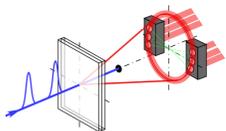
Classical comm channel  
2 bits per code length  
(+error correction  
+ privacy amplification)

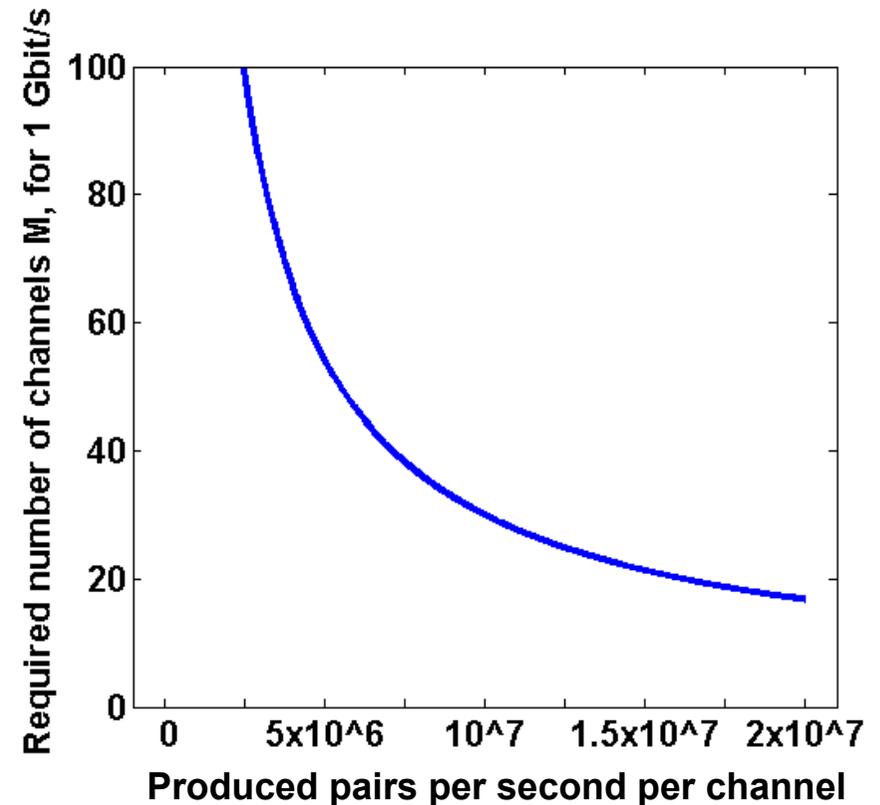
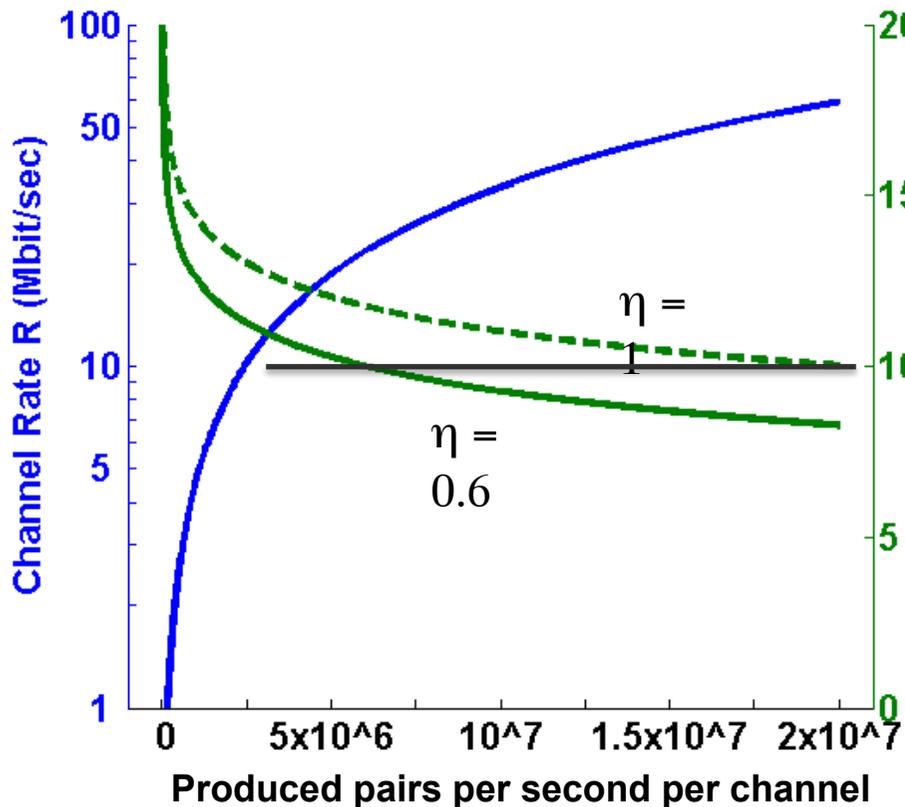
$$|\psi\rangle \propto (|HH\rangle + |VV\rangle) \otimes (|t_0t_0\rangle + |t_1t_1\rangle + |t_2t_2\rangle + \dots + |t_Nt_N\rangle)$$

Bin spacing:  $\Delta t$                        $\Delta t \sim 130$  ps

Code "length":  $\sim N\Delta t$

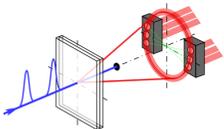
# bits/photon  $\sim \log_2 N$                        $\Rightarrow$                        $N \sim 1,024$



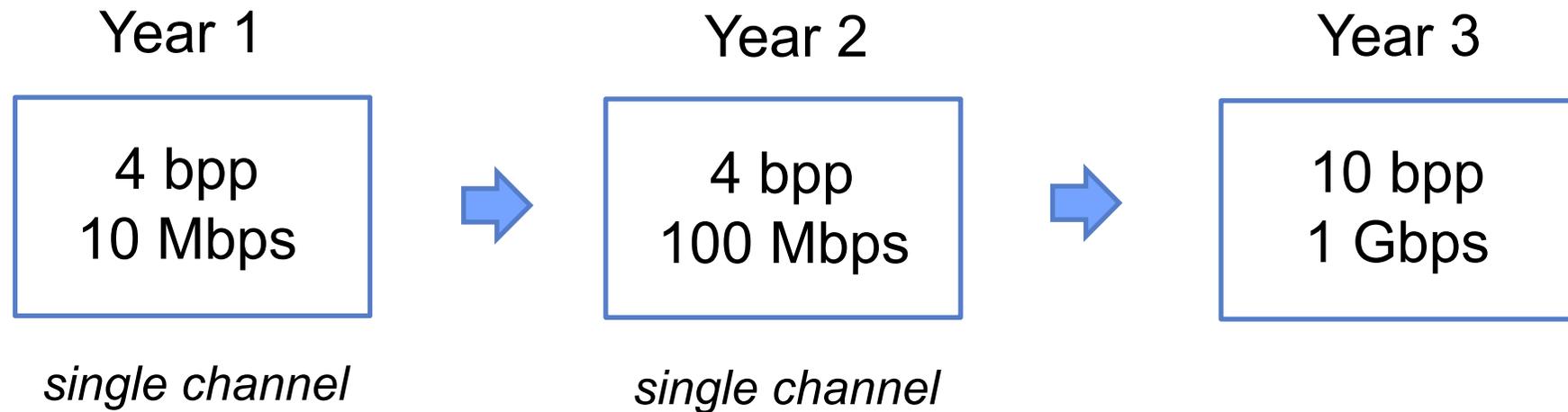


Trade-off between high count rate and high bits per photon (bpp).

- highest bpp: only send 1 photon per day  $\rightarrow$   $\sim 49$  bits/click (but only 1 click/day!)
- highest rate: send at near maximum detector saturation rate  $\rightarrow$  only  $\sim 1$  bpp
- we can simultaneously satisfy 1Gb/s and 10 bpp, by using multiple channels (10-30, depending on SPDC rate, efficiency, and BER)



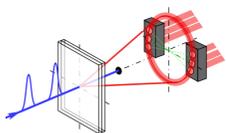
# Task 1: QKD Milestones



Book keeping of classical channel?

Need to divide bits per photon by ~100?

~1.1 using InPho Classical Com results?



**Kwiat** and everyone else!

# Task 2: Source Development



Year 1

4 bpp  
10 Mbps



BiBO  
High P  
Rate multiplier  
 $\Delta t \sim 1$  ns  
 $10^7$  pairs/s

Year 2

4 bpp  
100 Mbps



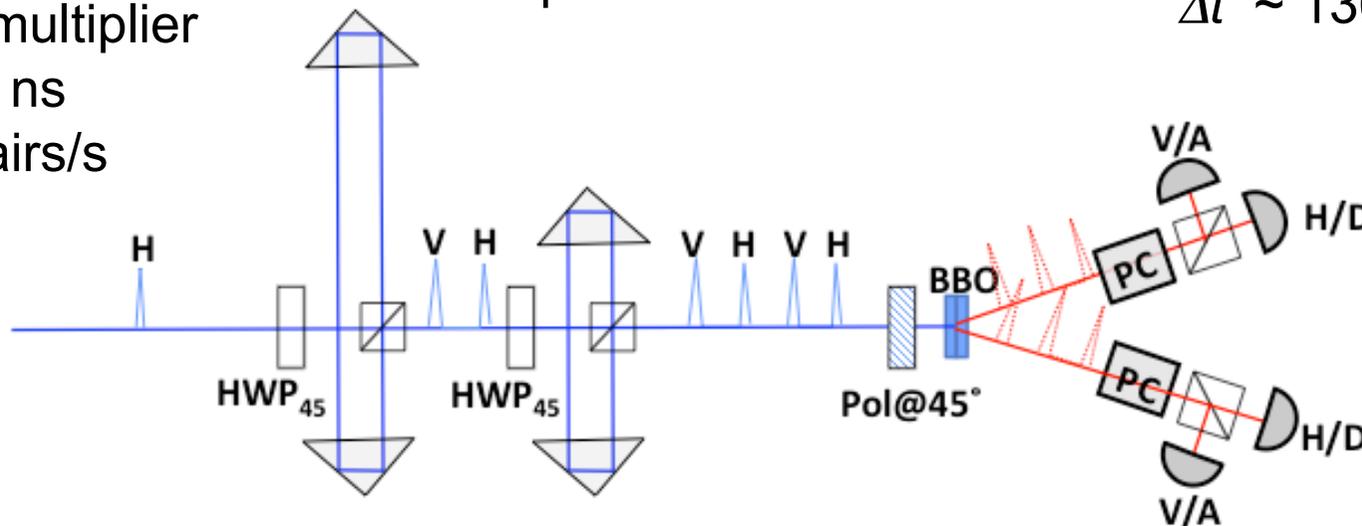
More rate multiplication  
 $10^8$  pairs/s

Year 3

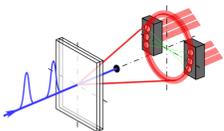
10 bpp  
1 Gbps



$2 \times 10^9$  pairs/s  
20 channels  
 $\Delta t \sim 130$  ps



Boyd, Gauthier, Kwiat, Padgett



# Task 3: Mode Multiplexers/Sorters



Year 1

4 bpp  
10 Mbps

Year 2

4 bpp  
100 Mbps

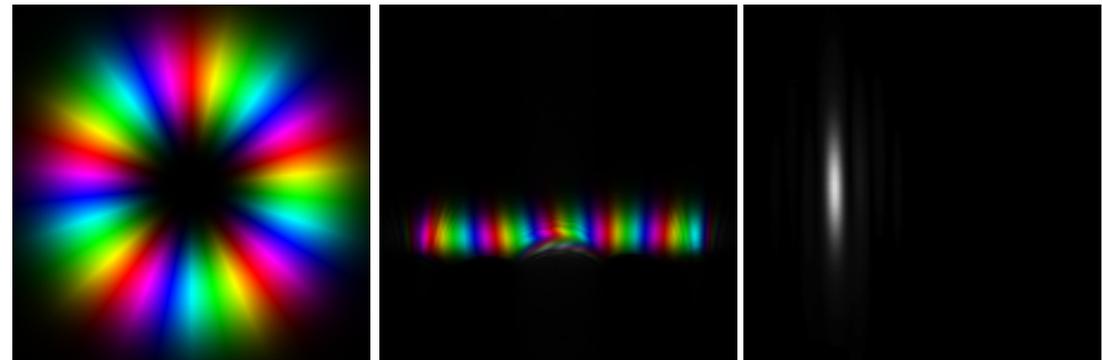
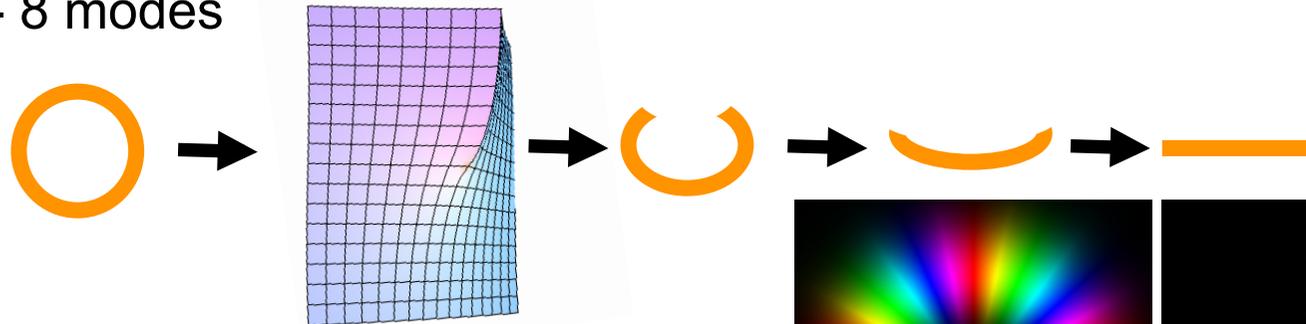
Year 3

10 bpp  
1 Gbps

- Demonstrate sorting low-rate QKD testbed
- Thick holograms, other approaches
- 8 modes

- High efficiency sorter
- Fundamental limits
- 32 modes, >80% eff.

- Integrate into high bit rate QKD system
- 64 modes, 70% eff.



Boyd, Miller, Padgett, White

# Task 4: Detector Development



Year 1

4 bpp  
10 Mbps



- switching fabric w/ high QE detectors
- < 20 ns deadtime
- 4-8 detectors
- SiPMTs
- < 250 ps jitter,
- 10 element arrays
- > 15% QE



Year 2

4 bpp  
100 Mbps



- 100 element array
- > 35% QE
- < 250 ps jitter

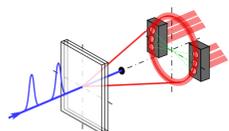


Year 3

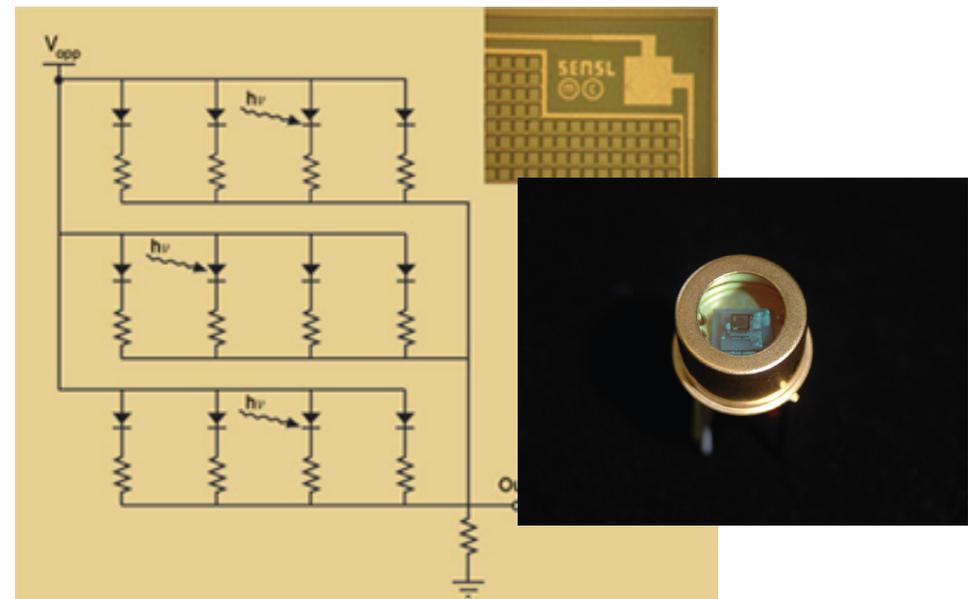
10 bpp  
1 Gbps



- < 130 ps jitter
- additional arrays



Gauthier, Kwiat



# Task 5: Turbulence Mitigation



Year 1

4 bpp  
10 Mbps

- Identify minimum energy loss states
- Generate minimum energy loss states

Year 2

4 bpp  
100 Mbps

- Investigate spatial entanglement
- Predistorted MUBs
- Optimum aperture sizes
- State-dependent loss
- Test in low-rate QKD testbed, > 5 bpp

Year 3

10 bpp  
1 Gbps

- Test in low-rate QKD testbed w/ turbulence cells

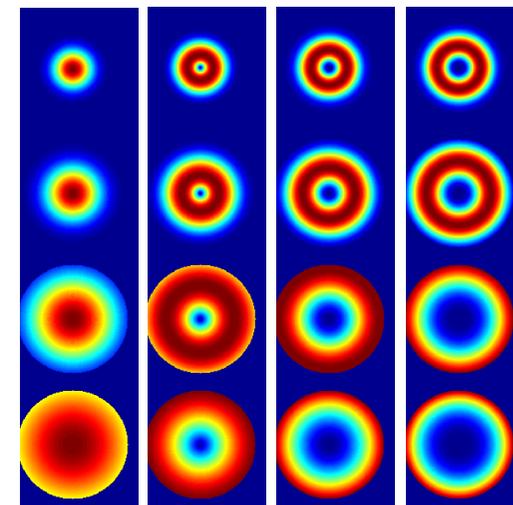


$N_f=10$

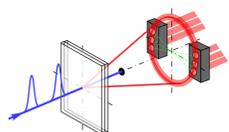
$N_f=5$

$N_f=2$

$N_f=1$



*minimum energy loss states*



Boyd, Tyler

# Task 6: Theoretical analysis of security



Year 1

4 bpp  
10 Mbps



- Optimum error correction method for large Hilbert space
- What attacks will break us?



Year 2

4 bpp  
100 Mbps



- Optimum Hilbert space dimension, # MUBs
- Trade space of security, bbb, bps
- Security compromised by state-dependent loss?

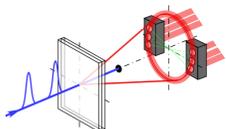


Year 3

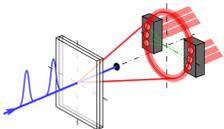
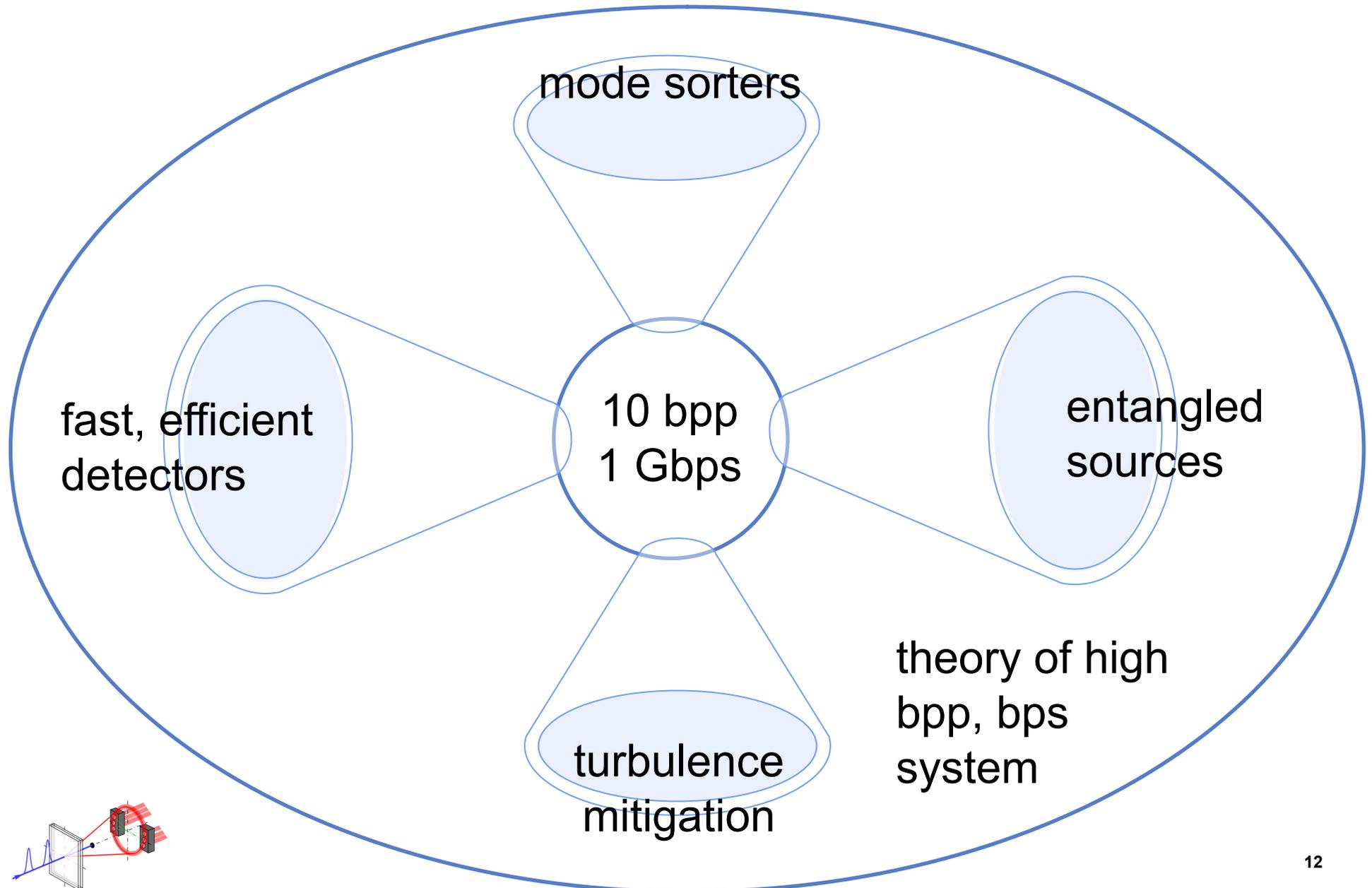
10 bpp  
1 Gbps



- Identify system with absolute security
- Decoy states to improve security



**Barnett, Calderbank, Kwiat, Lutkenhaus, Milburn, Tyler**



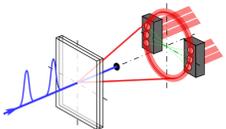


## Interactions via Social Media

Hourly  
Tweets



Daily  
Videos





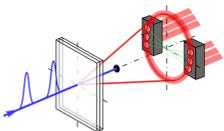
1. Central concept, expanded
2. Laser and pulse multiplexer
3. Down-conversion source
4. Detectors
  - switched, optimized APDs
  - array detectors

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5. Multi-channels → spatial multiplexing
6. AOM mode sorters, turbulence

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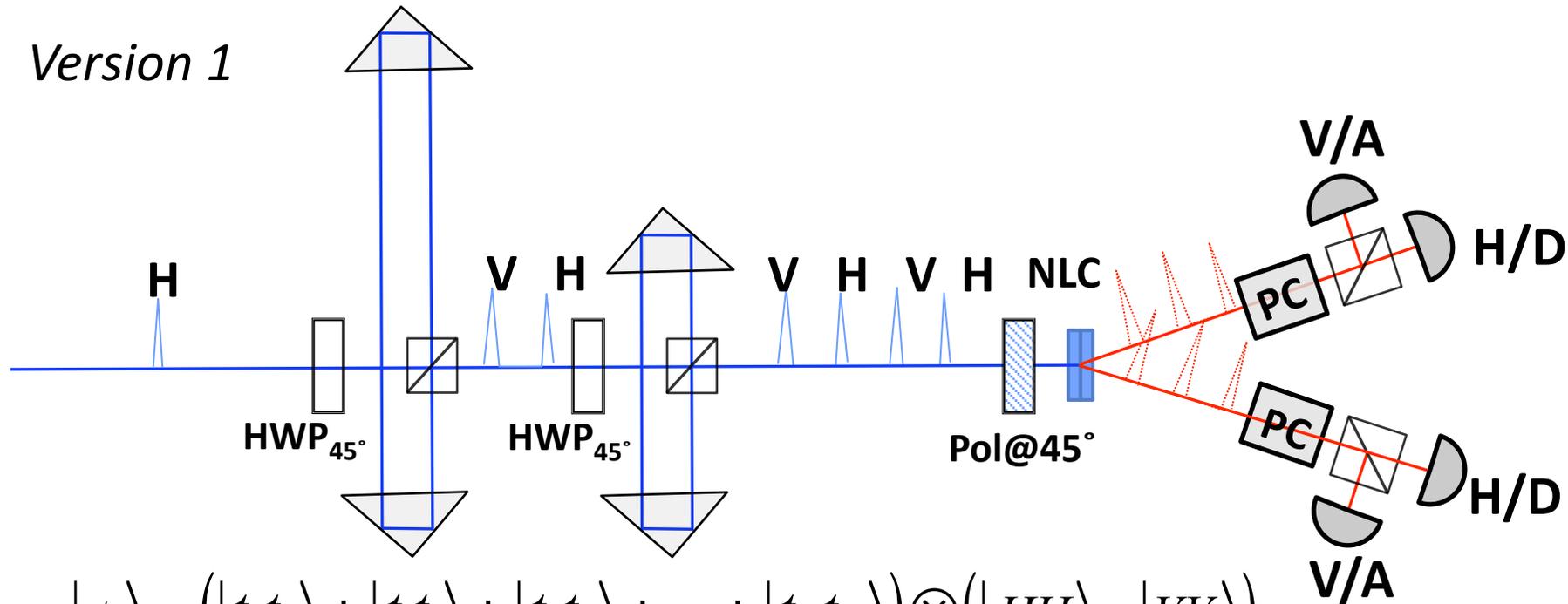
7. Eavesdropping, security
8. Open theoretical questions



# Central Concept: Encode in time, verify in polarization



Version 1

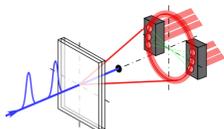


$$|\psi\rangle \propto \left( |t_0 t_0\rangle + |t_1 t_1\rangle + |t_2 t_2\rangle + \dots + |t_N t_N\rangle \right) \otimes \left( |HH\rangle + |VV\rangle \right)$$

Alice and Bob use which time bin they detect a photon in to generate multiple bits per click.\* Get extra bpp from BB84 with polarization.

They can constantly check for an eavesdropper using the D/A polarization basis (assuming no QND capability for Eve).

Perform standard error detection/correction† and privacy amp.



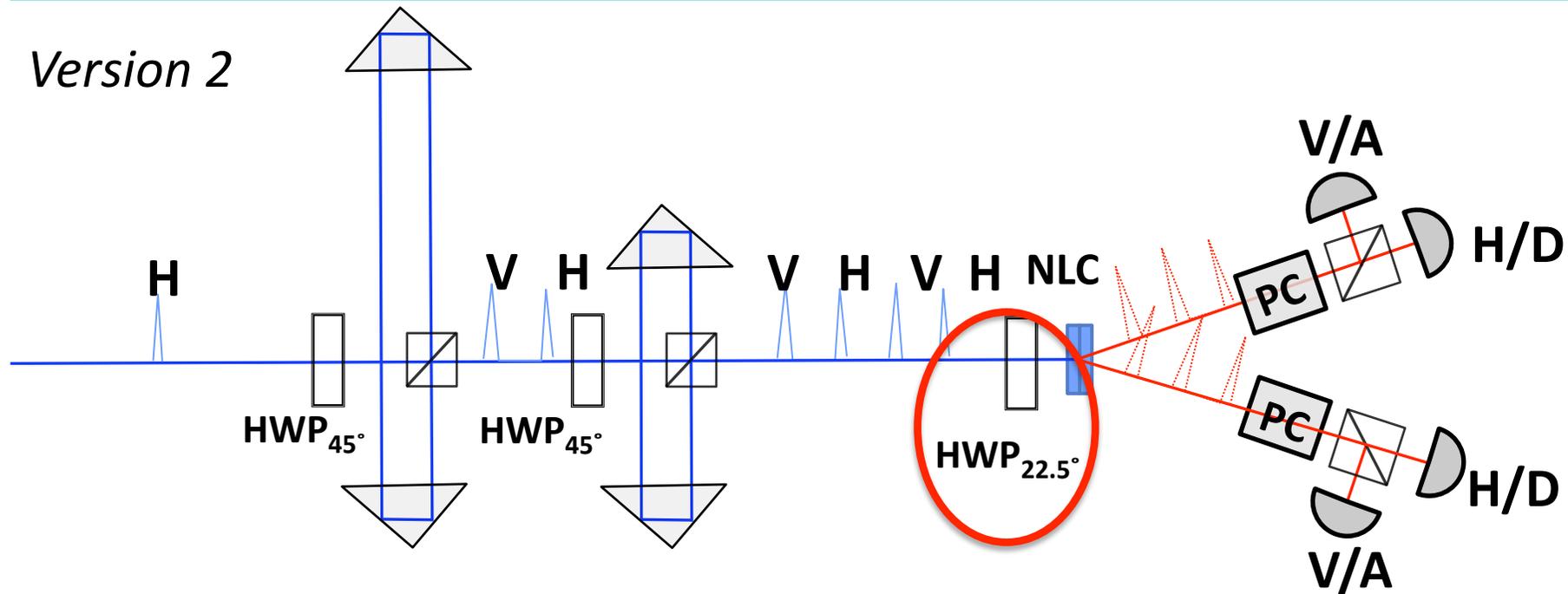
\*Ali-Khan, Broadbent, Howell, Phys. Rev. Lett. **98**, 060503 (2007)

† Modified CASCADE

# Central Concept: Encode in time, verify in polarization



Version 2

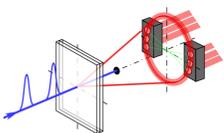


$$\begin{aligned}
 |\psi\rangle \propto & \left( |t_0 t_0\rangle + |t_2 t_2\rangle + \dots + |t_{N-1} t_{N-1}\rangle \right) \otimes \left( |HH\rangle + |VV\rangle \right) \\
 & + \left( |t_1 t_1\rangle + |t_3 t_3\rangle + \dots + |t_N t_N\rangle \right) \otimes \left( |HH\rangle - |VV\rangle \right)
 \end{aligned}$$

*Advantages: No pump power lost, ?harder? to eavesdrop*

*Disadvantages: Error checking depends on time bin, ???*

*NOTE: Active basis choice (PC) can be replaced by BS and twice as many detectors.*



# Pump Laser Source

*Paladin Compact 355-4000 by Coherent*

- *4W @ 355 nm (x10 over our past pump,  $10^{20}$  photons/sec)*
- *120 MHz mode-locked laser*
- *15 ps pulse width\**

*$\Delta t = 8.3$  ns between pulses  
Min detection interval  $\sim 50$  ns*

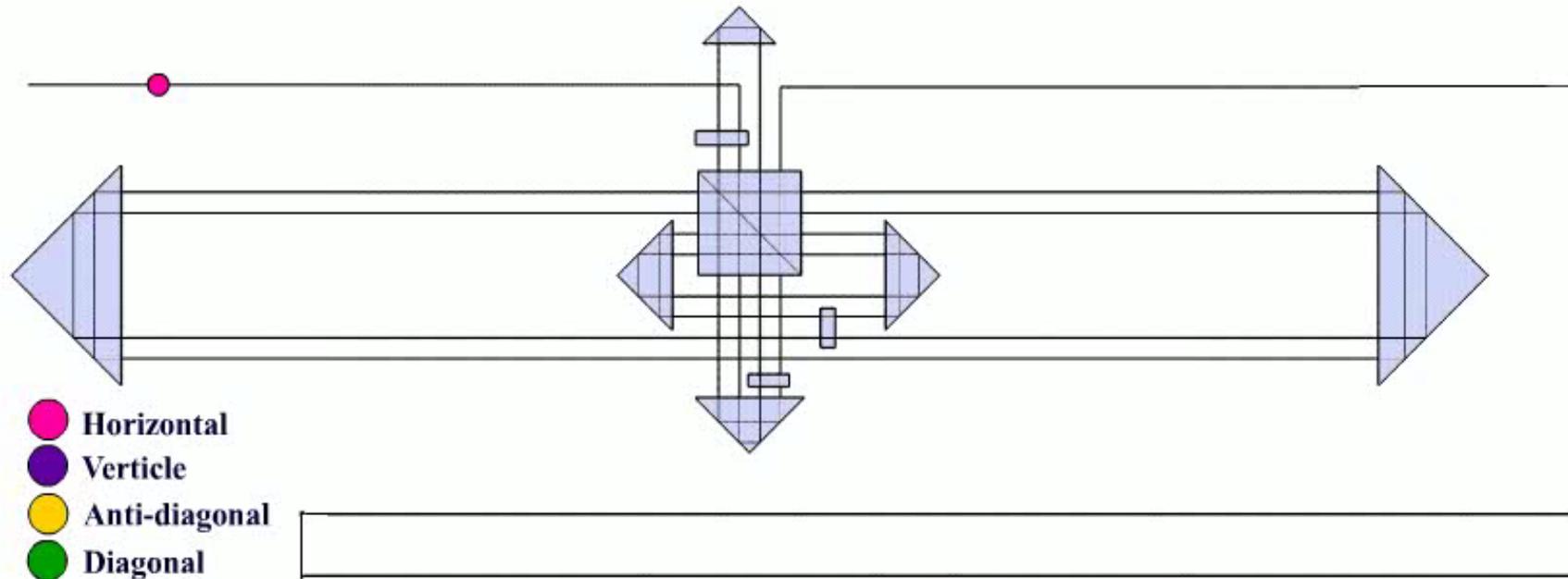
*$\rightarrow \log_2(50/8.3) < 3$  bpp*

*Need more time bins...*

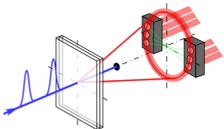


*\*In principle, if our detectors could resolve this, we could get up to 6 more bits/photon*

# Pump Pulse Multiplexer

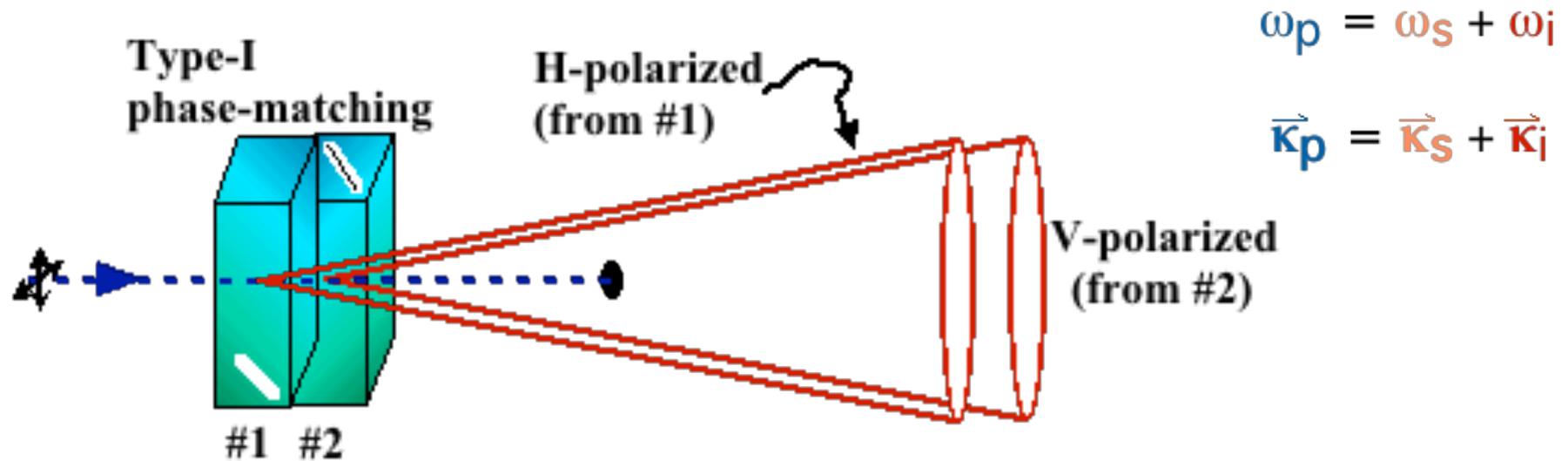


$2^4 = 16$  system shown\*; cycles 1-2 and 3-4 perfect “doubles”  
120 MHz (Pump rep rate)  $\times 2^6 = 7.7$  GHz (130-ps time bins)



\*Phase 1-2 implementations; Phase 3: add two more cycles 18

# Two-crystal Polarization-Entangled Source



PGK et al., PRA **60**, R773 (1999)

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |H\rangle_2 + e^{i\phi} |V\rangle_1 |V\rangle_2)$$

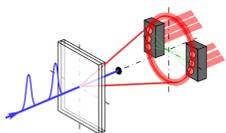
*Maximally entangled state*

*Spatial-compensation: all pairs have same phase  $\phi$*

*We detected  $2 \times 10^6$  pairs/s, with >99%-fidelity entanglement.*

*→ implied production rate  $> 2 \times 10^7 s^{-1}$*

*Now: BiBO (3x BBO), up to 10x power →  $> 10^8$  production rate*





What we want/need:

- High efficiency (coincidence rate  $\propto \eta^2$ )
- Excellent timing jitter ( $< 130$  ps, ideally  $< 15$  ps)
- Low deadtime/high saturation rate ( $< 10$  ns/ $> 50$  MHz)



What we (traditionally) get:



SPCM

Jitter: 250 ps FWHM  
(long tail)

Deadtime: 45 ns

Max count rate  $> 5$  MHz

Efficiency  $< 65\%$



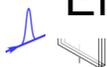
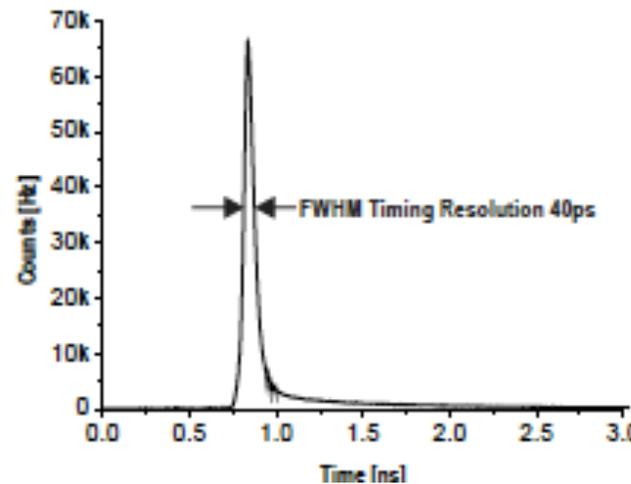
8-channel SPAD module

Jitter: 70 ps FWHM

Deadtime: 50 ns

Max count rate  $> 5$  MHz

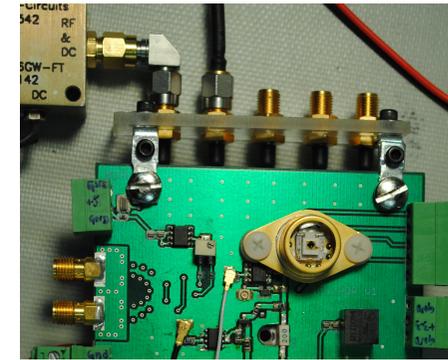
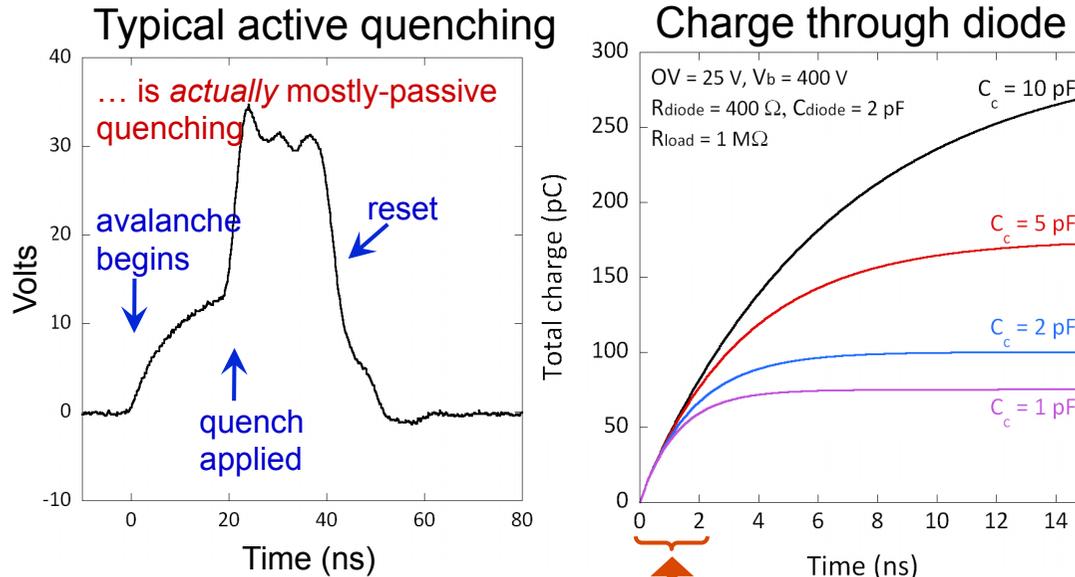
Efficiency  $< 35\%$



# Increasing Count Rates in Thick Si SPADs

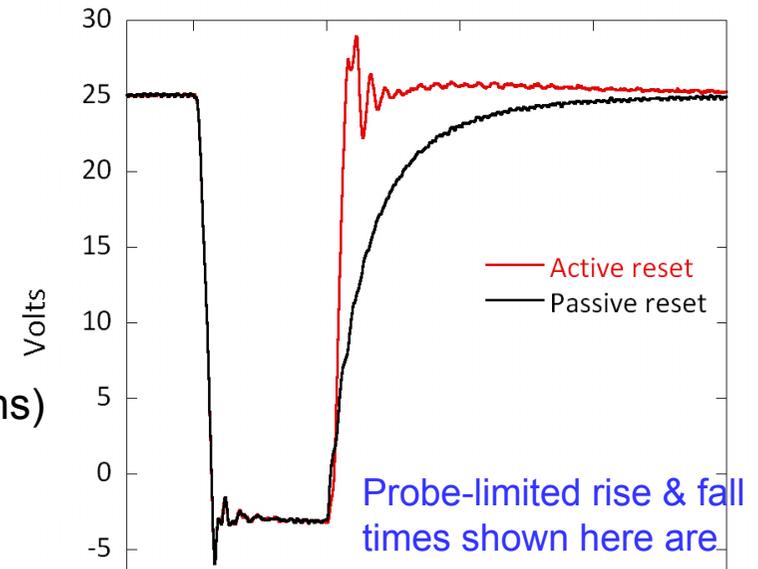


- Afterpulsing is reduced by minimizing the total avalanche charge through diode.
- High-speed electronics to promptly quench & reset Si SPADs.



Want to apply quench here

- Requires both short *signal delays* and *fast edges*
  - Gb/s electronics provide  $< 200$  ps delays
  - RF power amplifiers provide large slew rates ( $> 40$  V/ns)
- Virtex-V FPGA provides nanosecond pulse control
- Afterpulse experiment underway

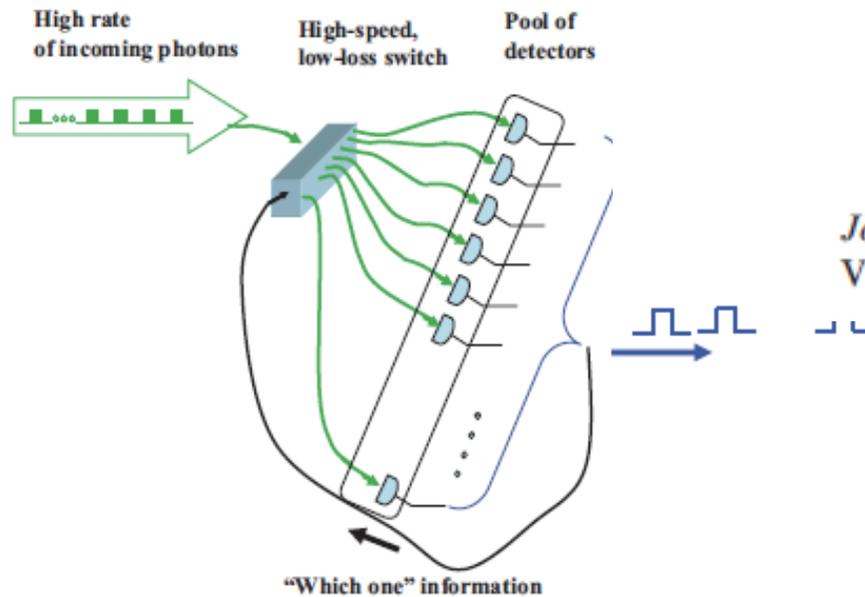


Goal: Reduce 40-ns deadtime  $\rightarrow$   $< 20$  ns

# Detectors: How to run APDs faster...



## Sequential detection scheme:



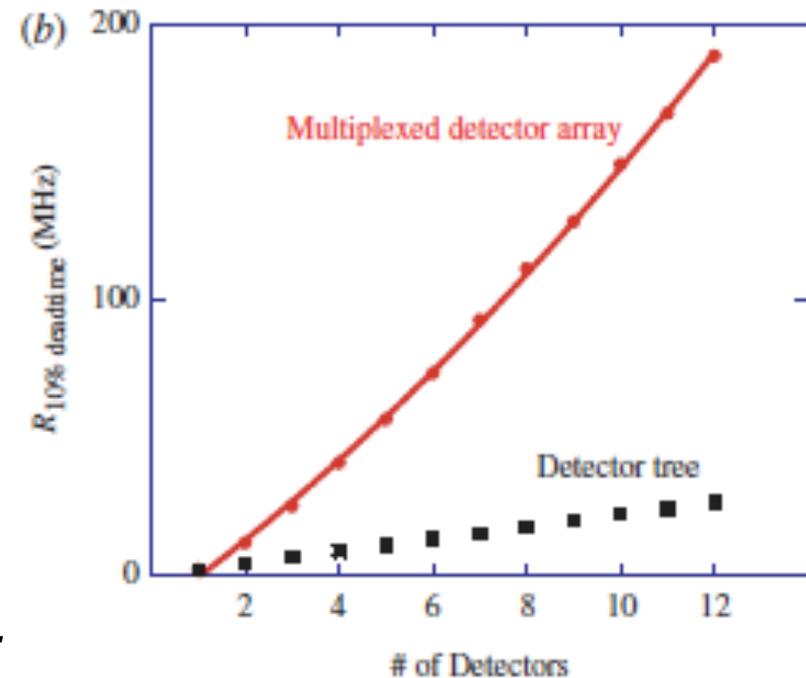
## Reduced deadtime and higher rate photon-counting detection using a multiplexed detector array

S. A. CASTELLETTO<sup>†</sup>, I. P. DEGIOVANNI<sup>†</sup>,  
V. SCHETTINI<sup>†</sup> and A. L. MIGDALL<sup>\*‡</sup>

*Journal of Modern Optics*

Vol. 54, Nos. 2-3, 20 January-15 February 2007, 337-352

Do much better because no photons go to detector  $k$  while it is recovering.  
Hard to implement with low loss (need fast "FSO" switch network)...



# Detectors: How to run APDs faster...



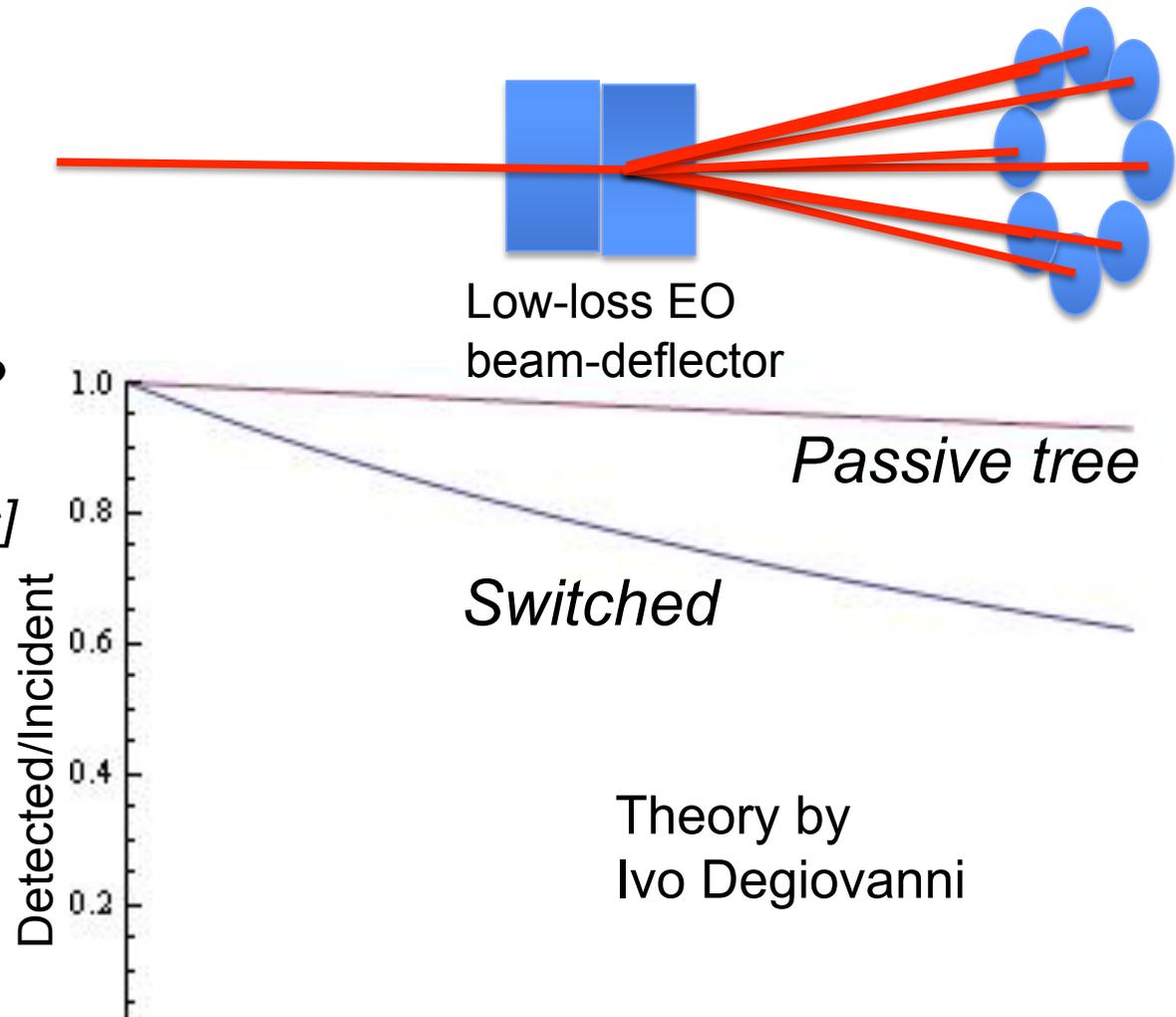
*Solution: Periodic sequential detection scheme*

*Periodic switching is much easier to implement (i.e., no logic).*

*(How much) does it help?  
[assuming 8 detectors, 10 bpp,  
 $T_{dead}=40\text{ ns}$ , and  $\Delta t = 130\text{ps}$ ]*

*It **hurts!** Why?*

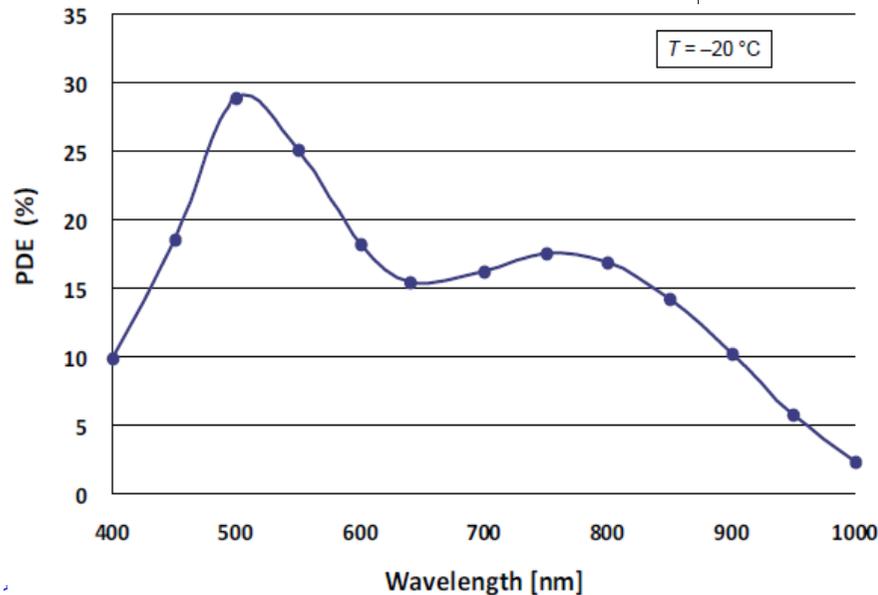
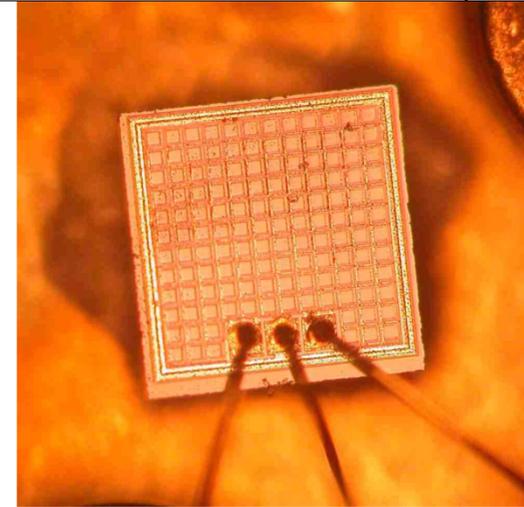
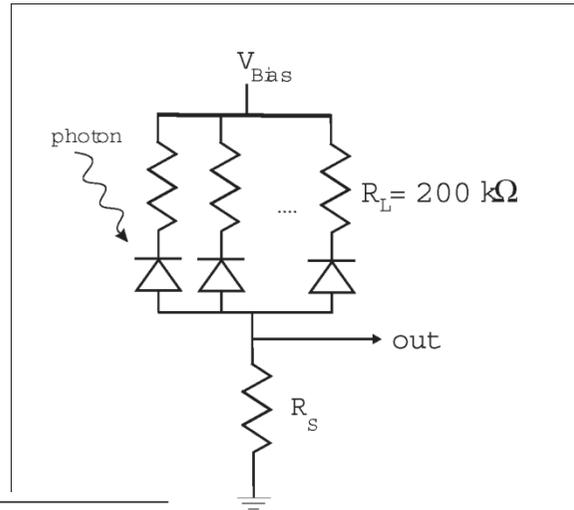
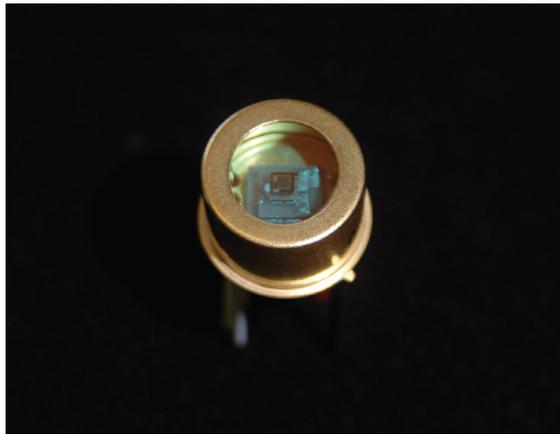
*Because to get 10 bpp,  
average time  
between detections =  
 $2^{10} \times 130\text{ ps} = 133\text{ ns}$   
> 3 x 40-ns deadtime*



**Preliminary conclusion: Passive 'beamsplitter tree' is optimal...**

## Voxel "Silicon Photomultiplier" TE Cooled: SQBF-EK0A (comes in chip form too)

Eraerds et al. (2007)



### Photon # resolving

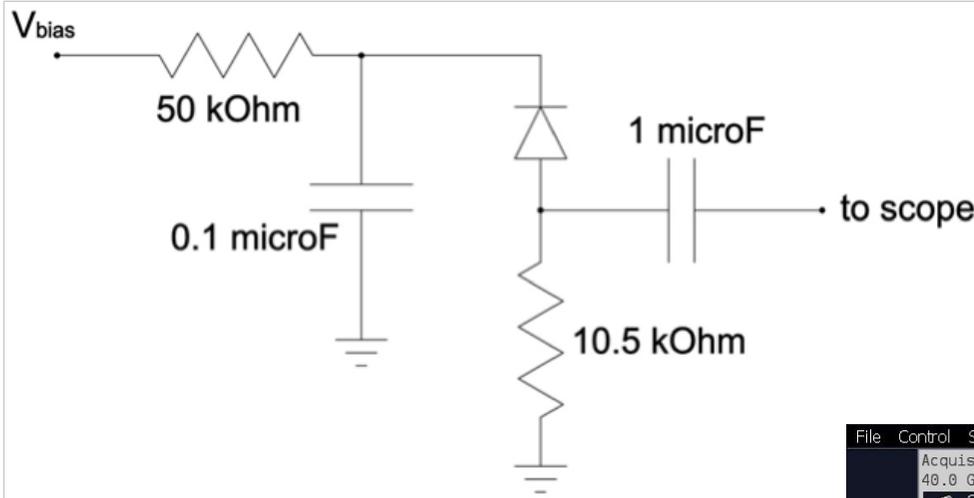
~10<sup>6</sup> dcps room temp

~1,000 dcps TEC cooled

~0.1 dcps LN2 cooled

>50% QE soon available (?)

# Initial Results with Voxtel Detectors

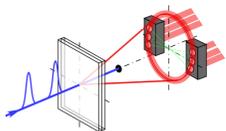


(high-speed layout not used)

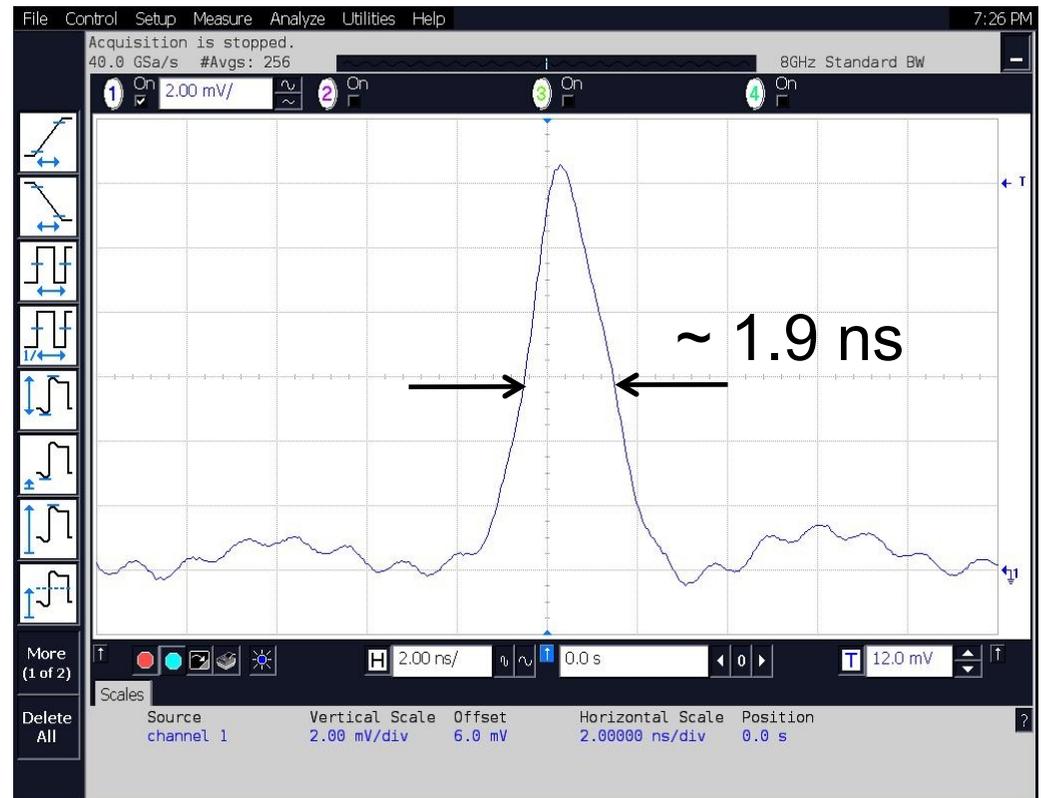
single-photon event

Cooled  $-90^{\circ}\text{C}$  below ambient

guesstimate 5,000 dcps



2 ns/div, 2 mV/div

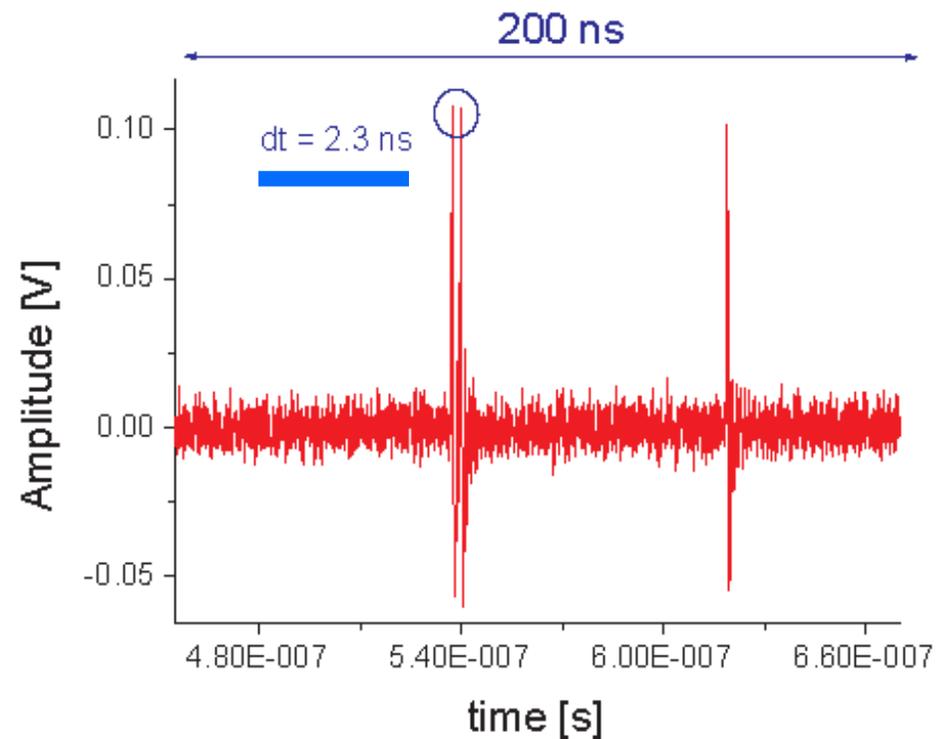
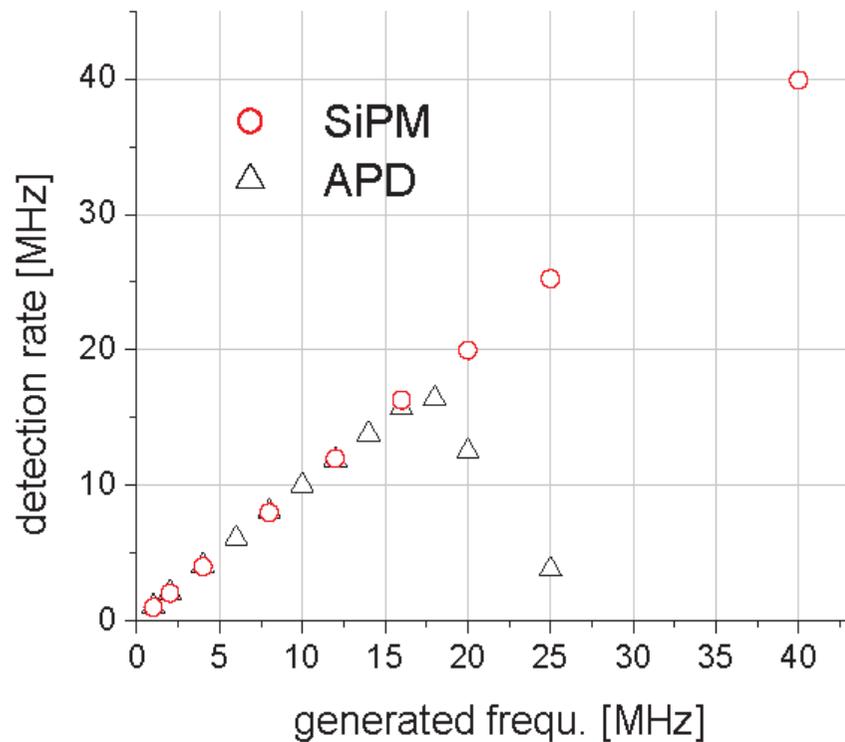


# SiPMs: Demonstrated Characteristics (by different groups)

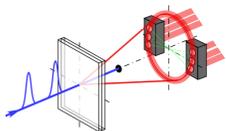


~ 30-ps timing resolution

(Buzhan, Dolgoshein *et al.* ICFA Instrumentation Bulletin)



weak saturation starts ~150 Mcps





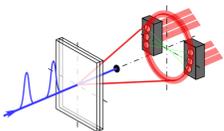
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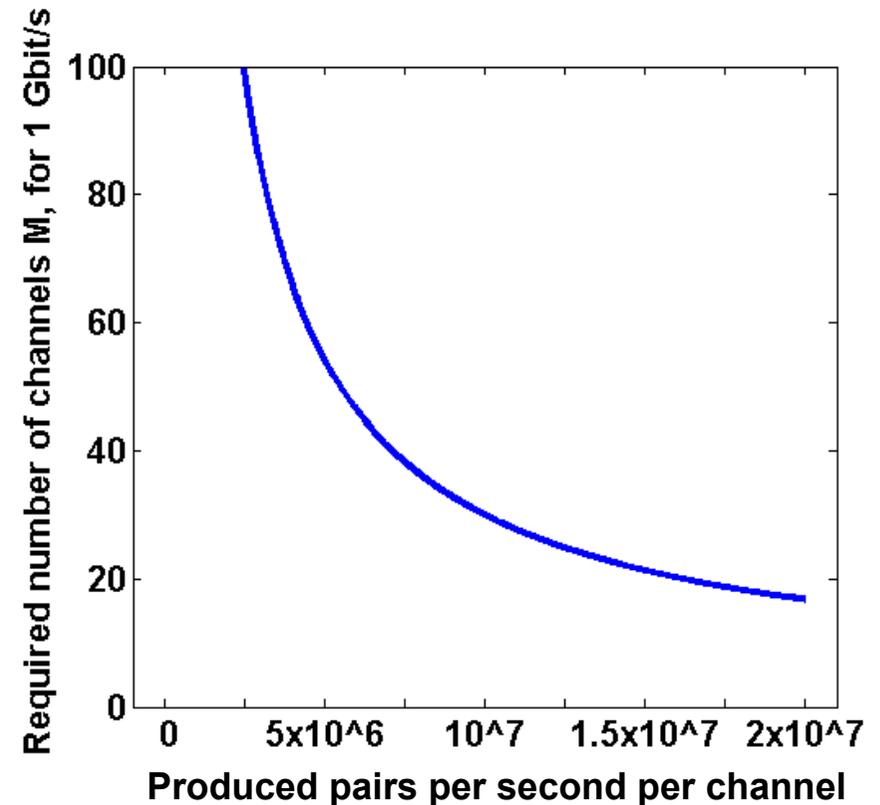
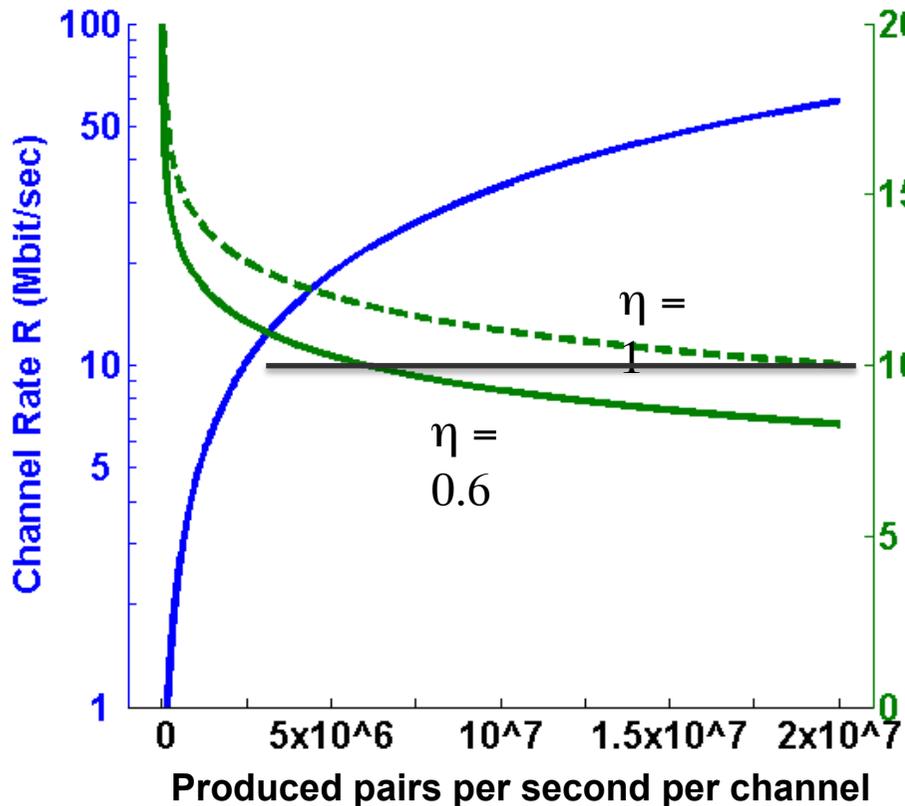
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5. Multi-channels → spatial multiplexing
6. AOM mode sorters, turbulence

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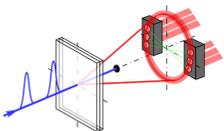
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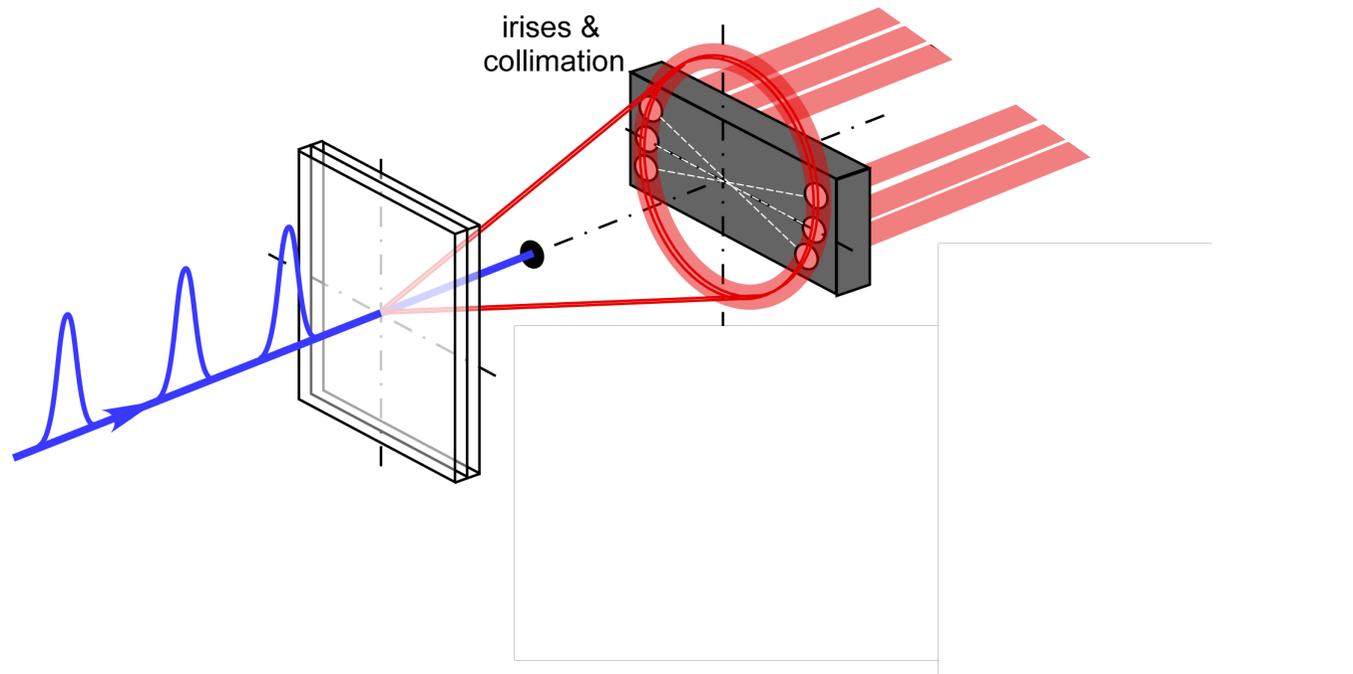


Trade-off between high count rate and high bits per photon (bpp).

- highest bpp: only send 1 photon per day  $\rightarrow$   $\sim 49$  bits/click (but only 1 click/day!)
- highest rate: send at near maximum detector saturation rate  $\rightarrow$  only  $\sim 1$  bpp
- **we can simultaneously satisfy 1Gb/s and 10 bpp, by using multiple channels (10-30, depending on SPDC rate, efficiency, and BER)**



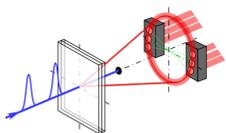
# Use spatial modes as independent channels



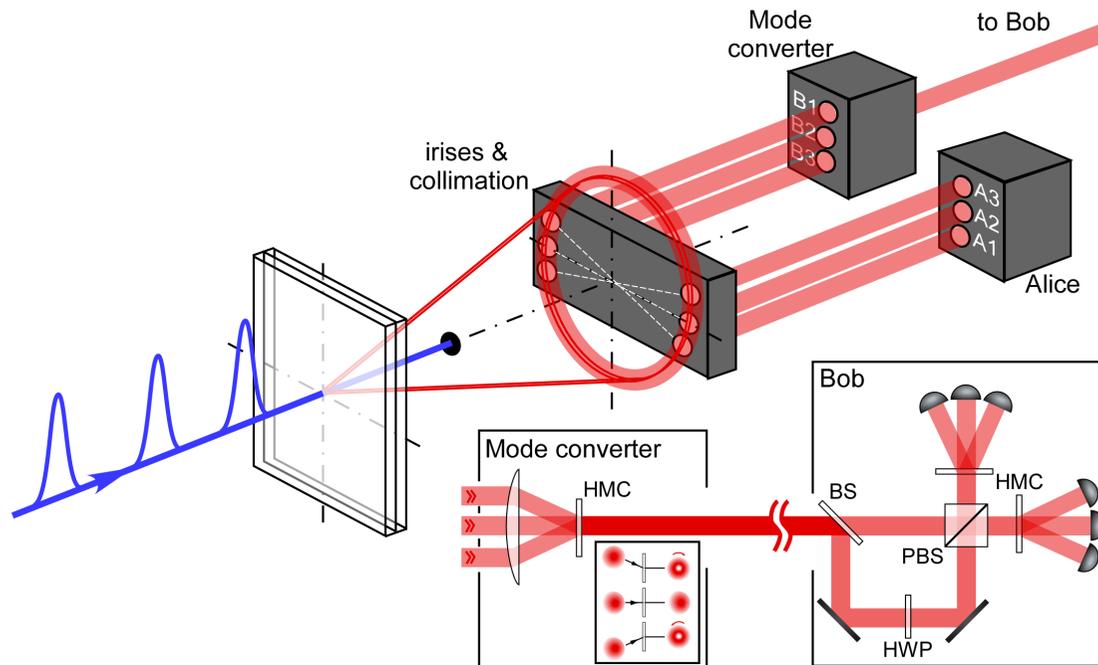
*Convert different spatial (gaussian) modes into overlapping OAM-type modes (optimized for turbulence robustness).*

*Advantages:*

- Potentially simpler optical transmission system*
- Bob can polarization analyze them all using same setup*



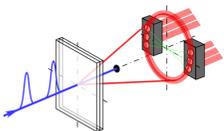
# Use spatial modes as independent channels



*Potential problem: turbulence may 'mix' the channels, i.e., causing crosstalk (?less problematic than if info was encoded using the spatial modes?)*

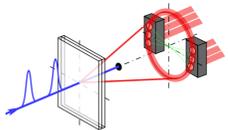
*Solutions:*

- choose crosstalk-robust states
- use non-degenerate frequencies for 'adjacent' spatial modes, to further reduce state overlap



# Bob Boyd on AOM sorters, turbulence...

InPho: FSQC



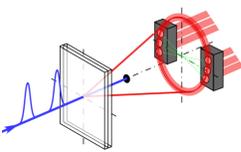
## Need for quantum state sorters and spatial mode converters

One approach to high-capacity QKD is to encode in the transverse degree of freedom (DoF) of the photon, using, for example states that carry orbital-angular momentum (OAM) such as the Laguerre-Gauss (LG) states.

Crucial Comment: This approach is NOT the baseline approach for our InPho team.

Nonetheless, transverse DOF relevant in two ways:

- (1) Use to transmit many quantum channels through the same aperture
- (2) Constitutes an alternative approach that might be exploited in future.

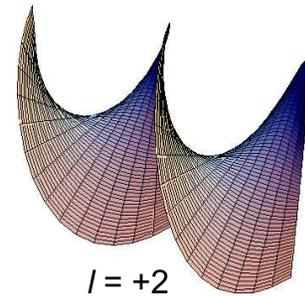
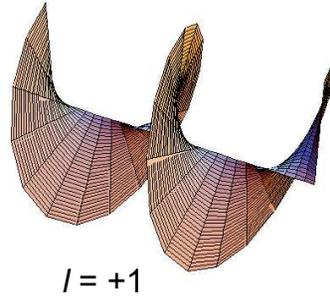
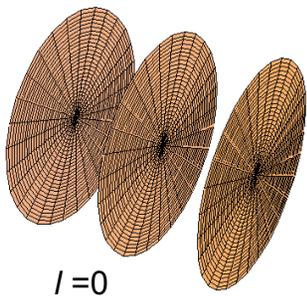


# What Are the OAM States of Light?

---

- Light can carry spin angular momentum (SAM) by means of its circular polarization.
- Light can also carry orbital angular momentum (OAM) by means of the phase winding of the optical wavefront.
- A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of  $\exp(il\phi)$  and carry angular momentum of  $l\hbar$  per photon. (Here  $\phi$  is the azimuthal coordinate.)

Phase-front structure of some OAM states



# Laguerre-Gauss Modes

The paraxial approximation to the Helmholtz equation  $(\nabla^2 + k^2)E(\mathbf{k}) = 0$  gives the paraxial wave equation which is written in the cartesian coordinate system as

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z} \right) E(x, y, z) = 0. \quad (1)$$

The paraxial wave equation is satisfied by the Laguerre-Gaussian modes, a family of orthogonal modes that have a well defined orbital angular momentum. The field amplitude  $LG_p^l(\rho, \phi, z)$  of a normalized Laguerre-Gaussian modes is given by

$$LG_p^l(\rho, \phi, z) = \sqrt{\frac{2p!}{\pi(|l| + p)! w(z)}} \frac{1}{w(z)} \left[ \frac{\sqrt{2}\rho}{w(z)} \right]^{|l|} L_p^l \left[ \frac{2\rho^2}{w^2(z)} \right] \\ \times \exp \left[ -\frac{\rho^2}{w^2(z)} \right] \exp \left[ -\frac{ik^2 \rho^2 z}{2(z^2 + z_R^2)} \right] \exp \left[ i(2p + |l| + 1) \tan^{-1} \left( \frac{z}{z_R} \right) \right] e^{-il\phi}, \quad (2)$$

where  $k$  is the wave-vector magnitude of the field,  $z_R$  the Rayleigh range,  $w(z)$  the radius of the beam at  $z$ ,  $l$  is the azimuthal quantum number, and  $p$  is the radial quantum number.  $L_p^l$  is the associated Laguerre polynomial.

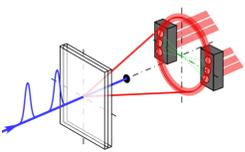
Alice's encoding makes use of OAM

(Generating single photons in an OAM state, if not easy, is at least straightforward)

Bob needs a sorter to separate each input OAM mode into a different output channel

Because Bob has only one photon, he can perform only one measurement in determining what state he has.

How to do this?

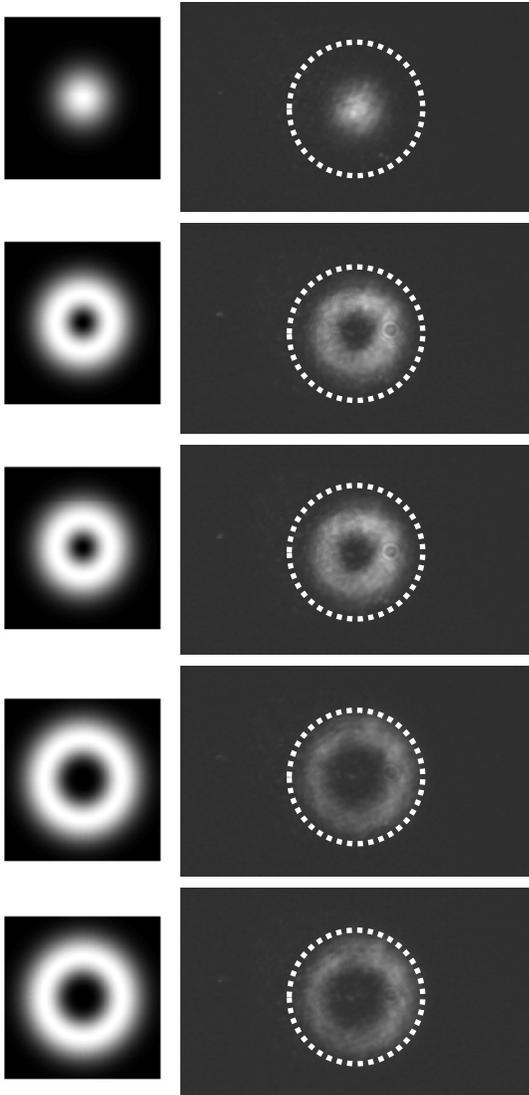


# State Generation ( $d=5$ )

## Basis 1 (LGs)

Theory

Experiment



$$LG_{0,0}$$

$$LG_{1,0}$$

$$LG_{-1,0}$$

$$LG_{2,0}$$

$$LG_{-2,0}$$

$$\frac{1}{\sqrt{5}} \sum_{l=-2}^2 LG_{l,0} e^{i2\pi l/5}$$

$$\frac{1}{\sqrt{5}} \sum_{l=-2}^2 LG_{l,0} e^{i4\pi l/5}$$

$$\frac{1}{\sqrt{5}} \sum_{l=-2}^2 LG_{l,0} e^{i6\pi l/5}$$

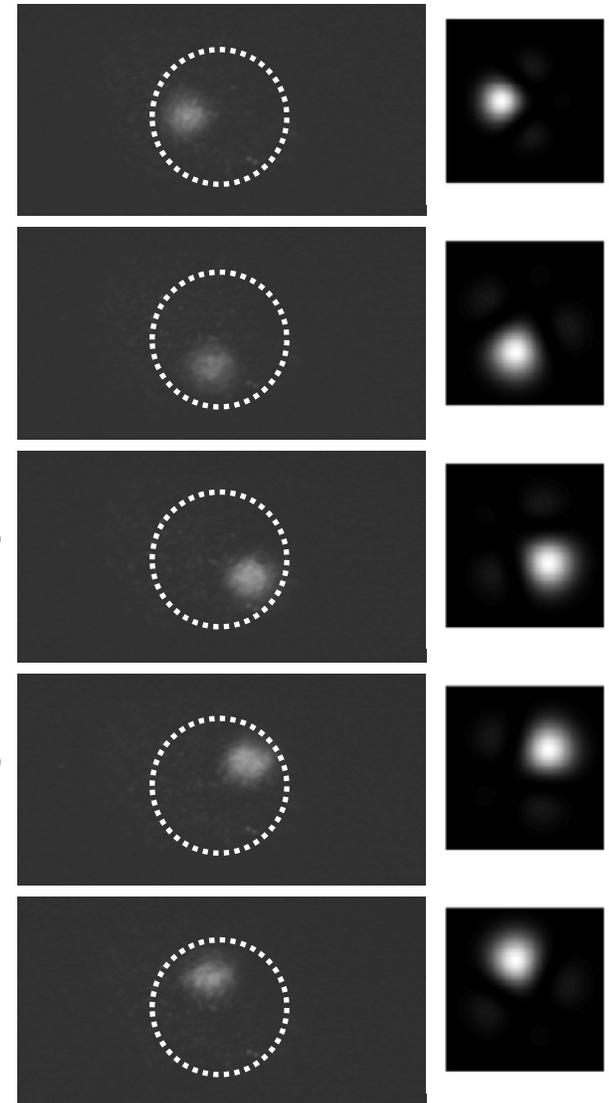
$$\frac{1}{\sqrt{5}} \sum_{l=-2}^2 LG_{l,0} e^{i8\pi l/5}$$

$$\frac{1}{\sqrt{5}} \sum_{l=-2}^2 LG_{l,0}$$

## Basis 2

Experiment

Theory



# Basic idea (assuming OAM modes for definiteness)

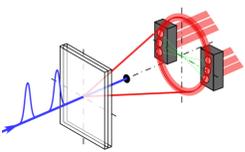
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(Generating single photons in an OAM state, if not easy, is at least straightforward)

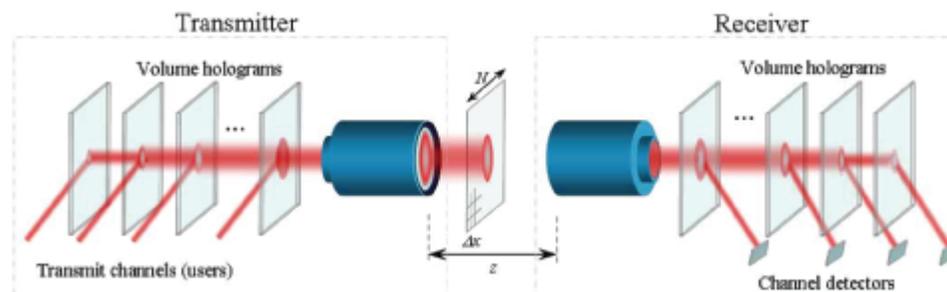
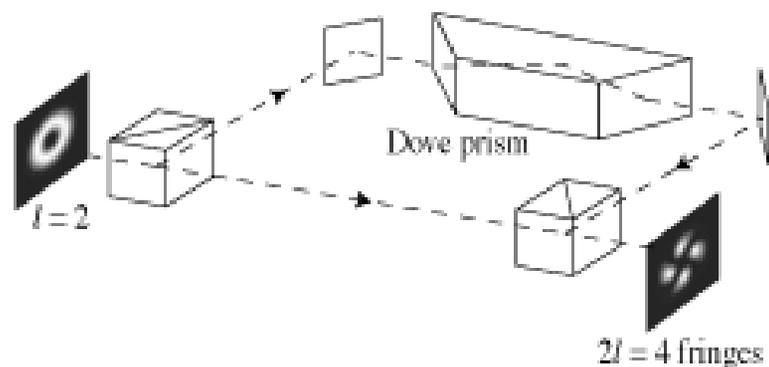
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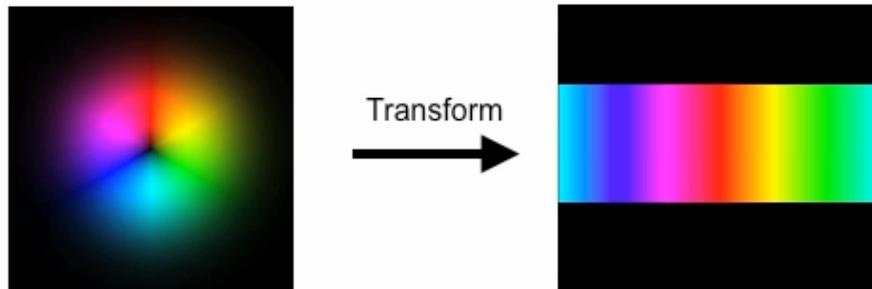


1. Use a thin hologram (widely believed that this will not work)
2. Use a thick hologram (seems plausible, but no one has yet made this work)
3. Use a cascade of  $(d-1)$  interferometers, each containing a Dove prism
4. Use a diffractive optical element (DOE); inverse problem; what structure?
5. Mode reformatter (Padgett group, PRL in press)  
It works! What are its limitations?



Turbulence-induced channel crosstalk in an orbital angular momentum-multiplexed free-space optical link, J. A. Anguita, M. A. Neifeld, and B. V. Vasic, *Applied Optics* 47, 2414 (2008).

## Sorting States    Sorting Light's Orbital Angular Momentum

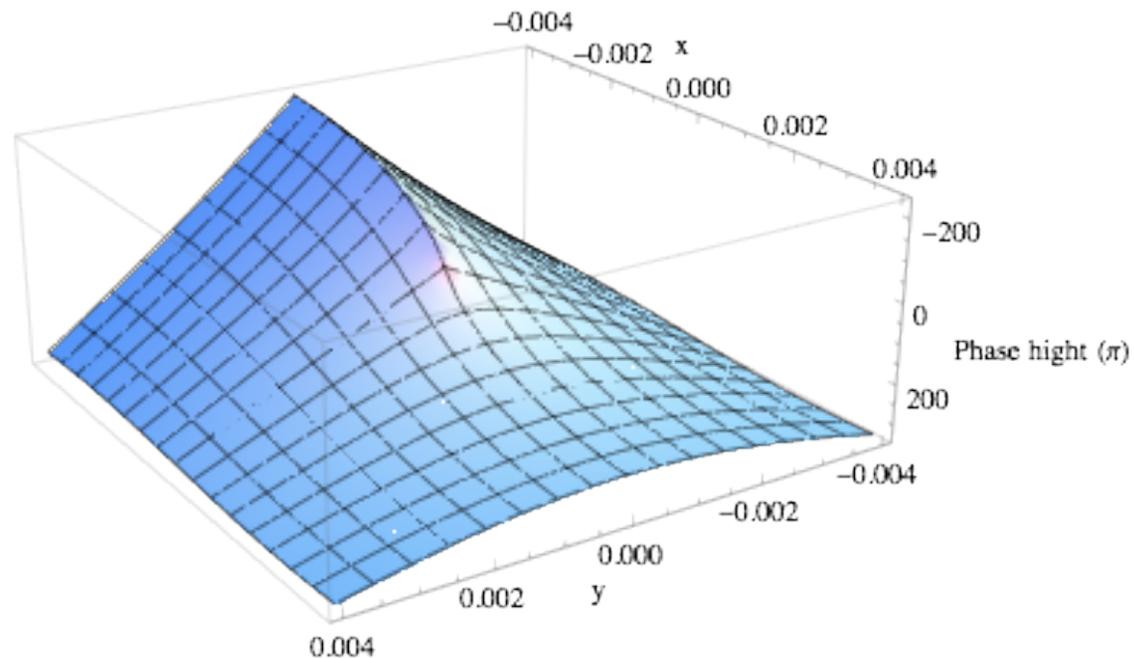


We “unwrap” the azimuthal phase distribution to form a linear mapping.

Linear maps are easily sorted just using an ordinary lens.

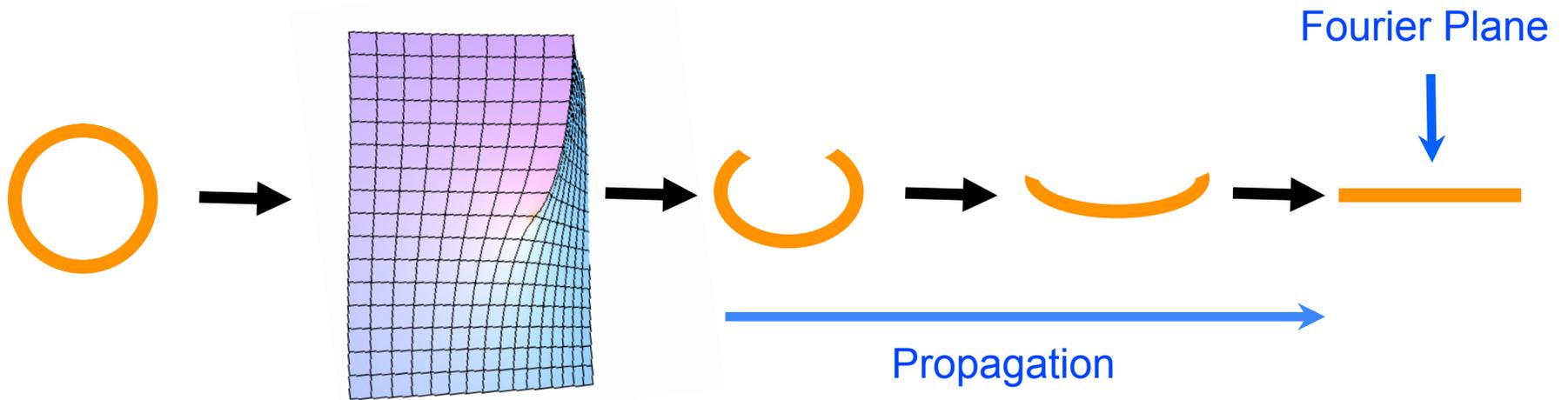
## Sorting States Geometric Transformations in Optics

The surface to do this transformation looks like:



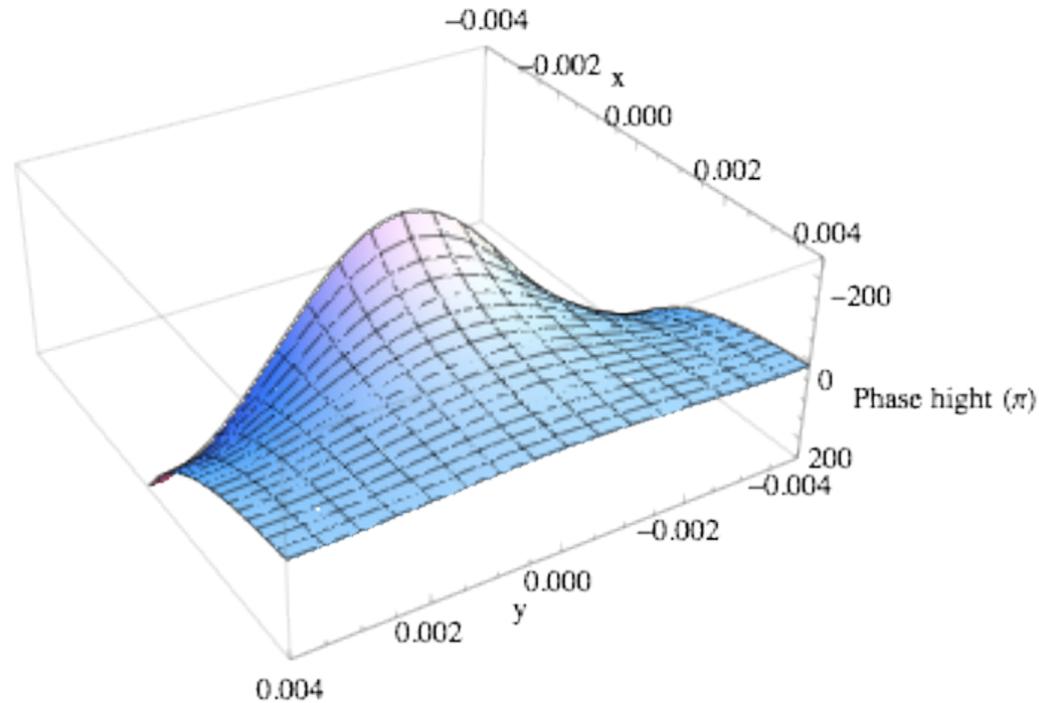
$$\phi_1(x, y) = \frac{2\pi a}{\lambda f} \left[ y \arctan \left( \frac{y}{x} \right) - x \ln \left( \frac{\sqrt{x^2 + y^2}}{b} \right) + x \right]$$

# Sorting States Geometric Transformations in Optics



## Sorting States Geometric Transformations in Optics

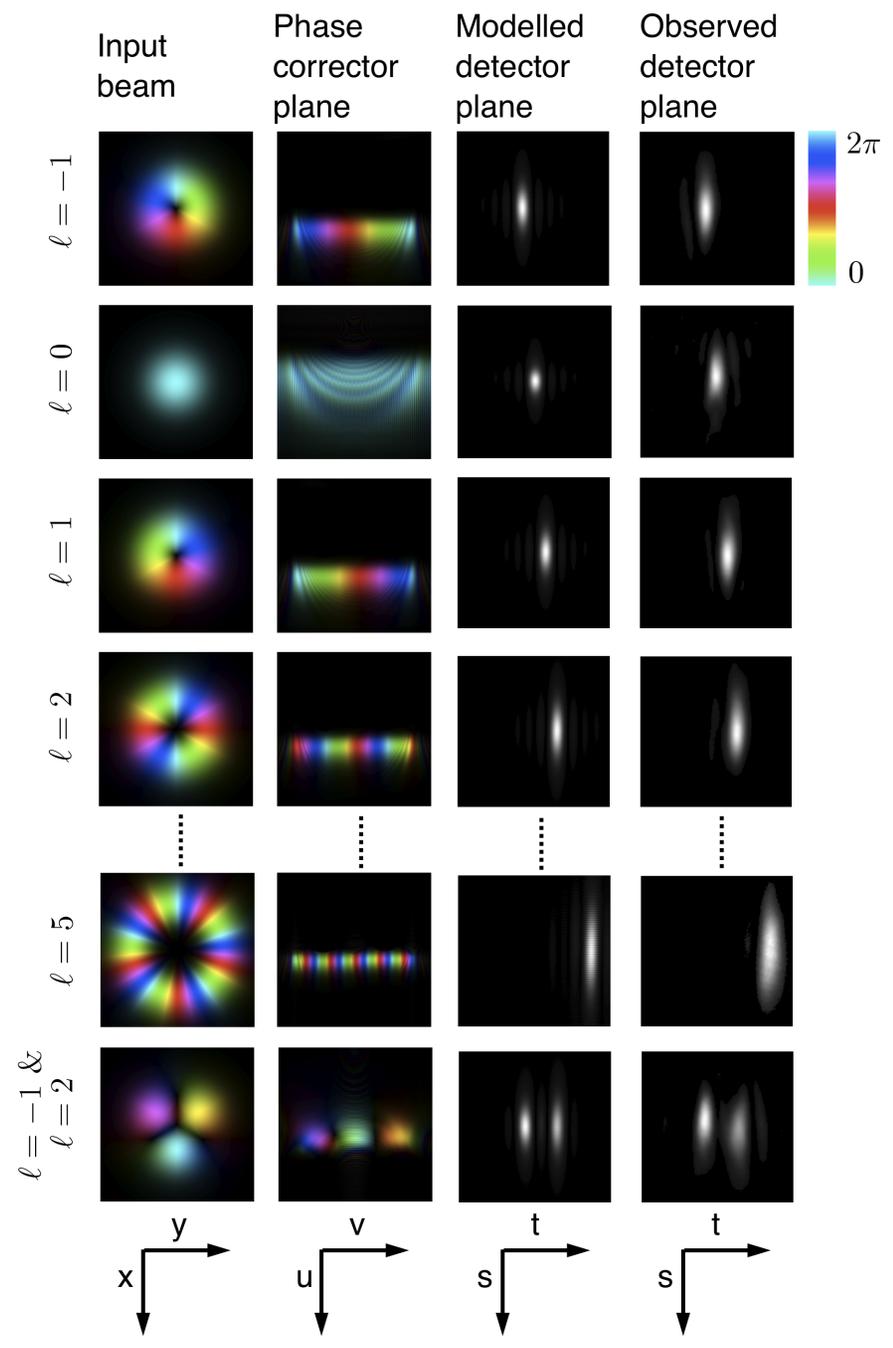
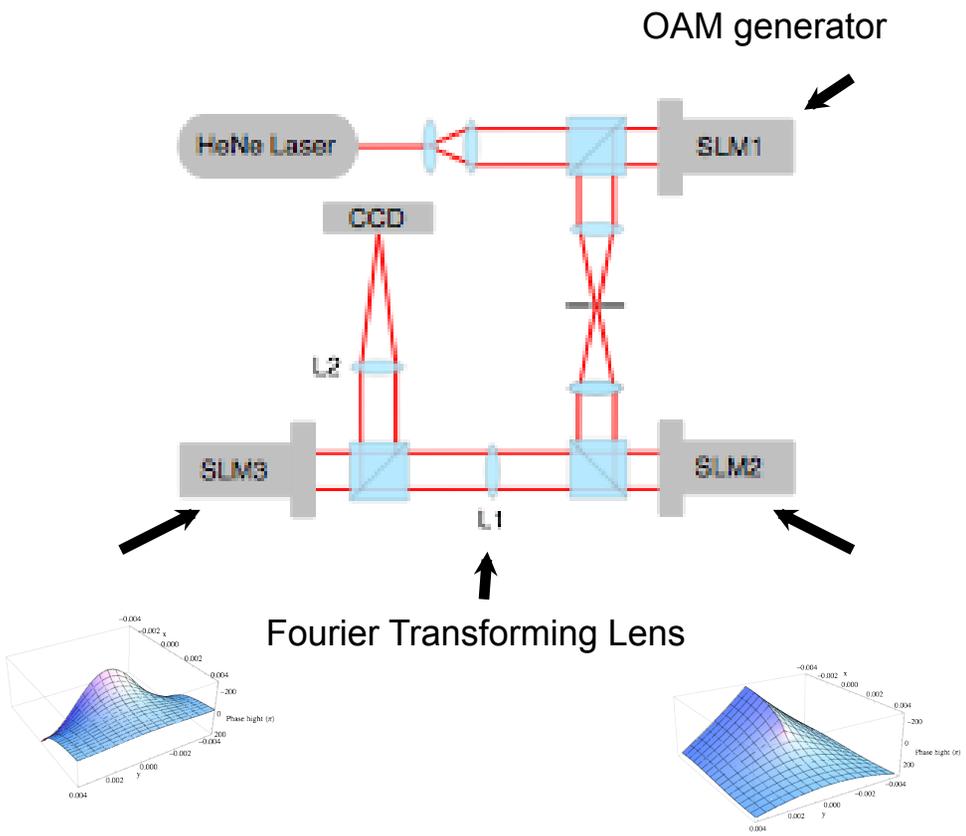
The surface to correct the phase looks like this:



$$\phi_2(u, v) = -\frac{2\pi ab}{\lambda f} \exp\left(-\frac{u}{a}\right) \cos\left(\frac{v}{a}\right).$$

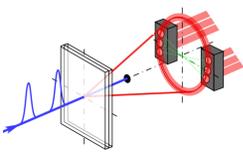
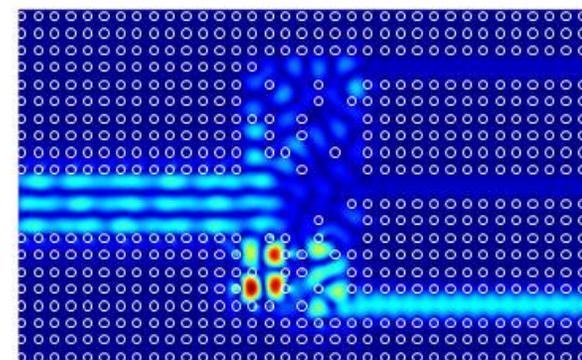
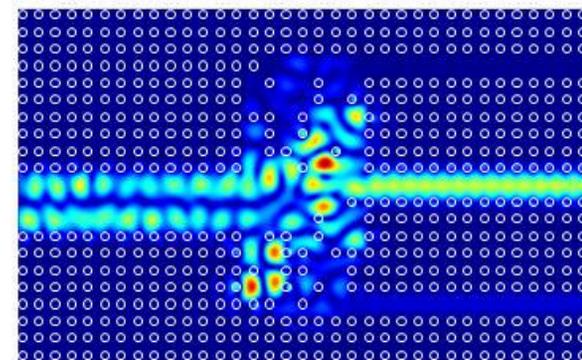
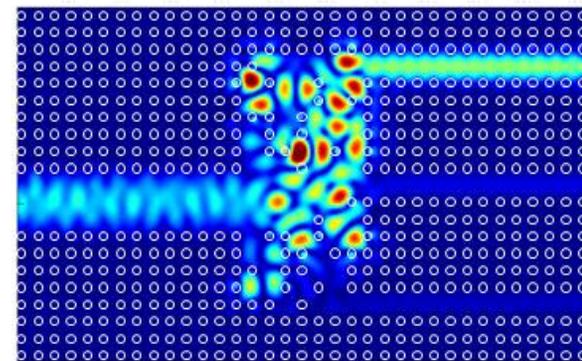
# Proof of concept experiment

It works!:



# Fundamental limits to mode converters

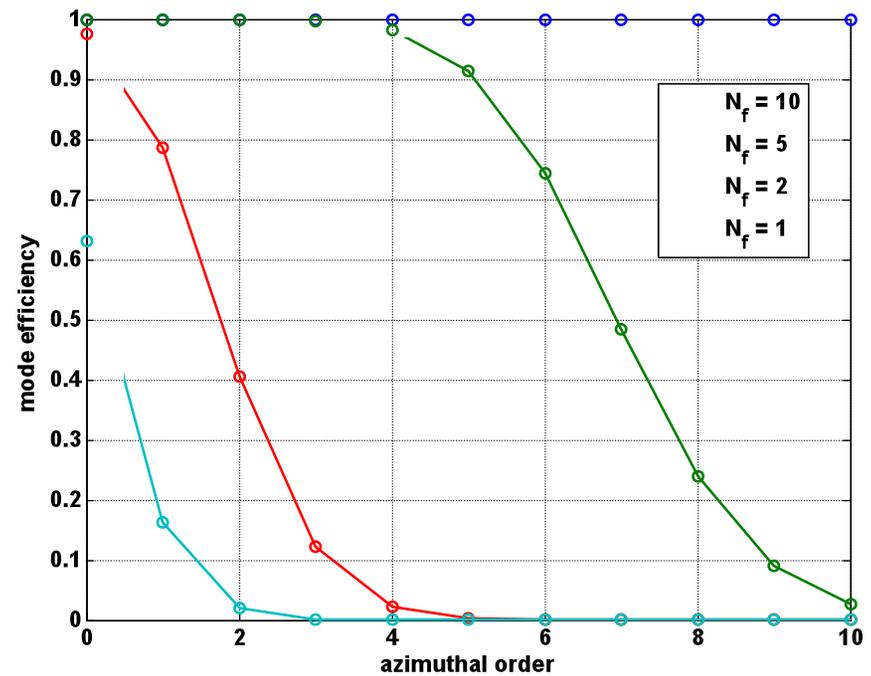
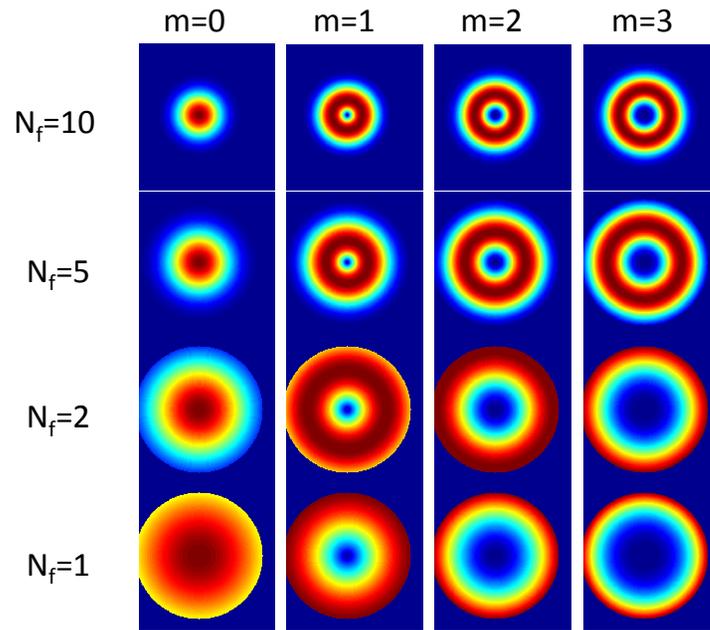
- **Miller's limit theorem**
  - provides general limits to the performance of linear optical components based on modal analysis (JOSA B 24, A1 (2007))
- **Previous work**
  - Explicit limits for 1D optical structures for
    - Pulse dispersive devices, Slow light
  - Existence proof design of compact mode converters
- **Overall work planned on this program includes**
  - extending previous work to get explicit results for 2D and 3D structures
    - explicit limits for monochromatic mode converters
      - e.g., limits to thin and thick “holograms”
- **Future directions include**
  - Multiple wavelength systems 2D and 3D systems, including
    - More restrictive limits for materials that are not themselves dispersive
    - Possible pulsed field mode and pulse converters



# Influence of Atmospheric Turbulence on Quantum Communications: Project Overview

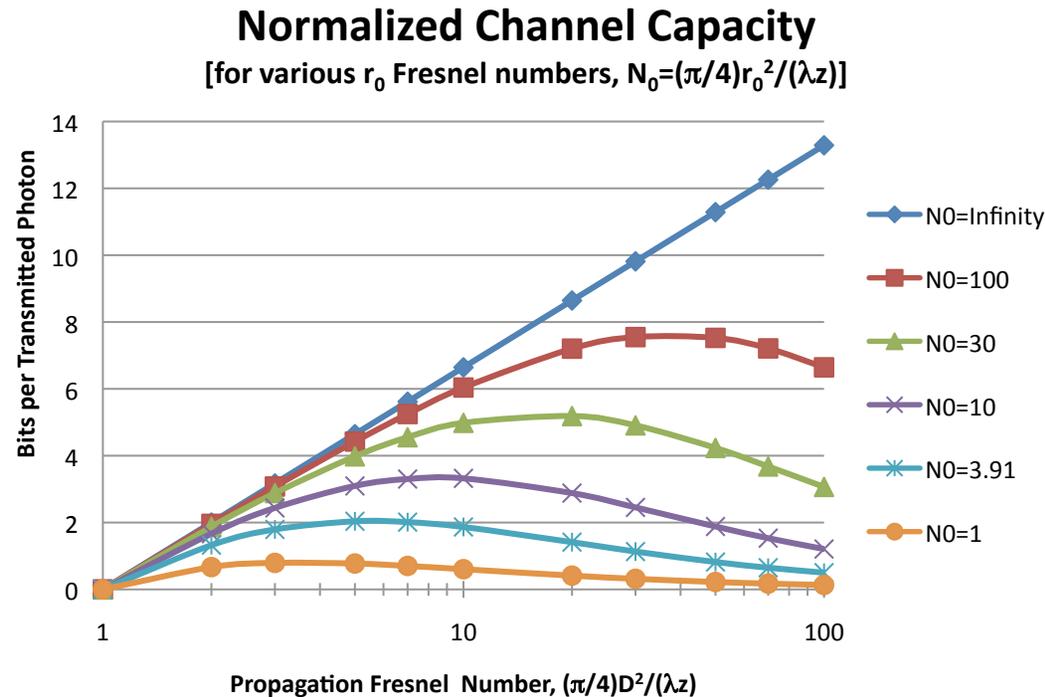
- Near term research concerned with:
  - Baseline Protocol (elementary propagation effects – polarization, dispersion, etc.)
  - Optimum transmission efficiency with minimum energy loss states
  - Entropy and information content associated with a free space propagation link
  - Reduced information content in presence of turbulence
  - Turbulence characteristics associated with horizontal path (SOR 2 mile site)
  - Scintillation and fading probability
- Advanced concepts
  - Preconditioned MUB states with minimum energy loss bases
  - Adaptive Optics (used only if required to sustain link channel capacity)
  - Filament exploitation in deep turbulence
- Experimental considerations
  - Laboratory experiments at U of R, Duke with support from tOSC
  - Field experiments when program is sufficiently mature (tOSC with support from team)
- Our present understanding supports conclusion that 256 time bins and 6 spatial parallel channels (through a single aperture) leads to more than 10 bits per photon in even presence of horizontal path turbulence (without AO)
- Continuing research will address added margin required for security and additional turbulence variability (fading, time varying statistics)

# Protocol: Transmit Minimum Energy Loss States



- The minimum energy loss states have the functional form,  $F_{nm}(r,\varphi)=f_{nm}(r) \exp(im\varphi)$ , where the functional form of  $f_{nm}$  is controlled to minimize the energy loss for a propagation link defined by Fresnel number  $N_f=(\pi/4) D_1 D_2 /(\lambda z)$
- The left hand figure illustrates the amplitudes associated with the lowest loss states for each designated  $m$  and  $N_f$
- The propagation efficiencies are illustrated in the right figure
- These states have interesting properties
  - They automatically self image as they propagate from transmitter to receiver
  - They are also eigenmodes of a resonator with phase conjugate mirrors
  - In the limit of a large Fresnel number they asymptote to Laguerre Gauss functions
  - In the limit of a small Fresnel number they asymptote to Prolate Spheroidal wave functions
- We also have developed Preconditioned MUB States that use these states as their basis

# Normalized Transverse Channel Capacity Significantly Reduced by Atmospheric Turbulence



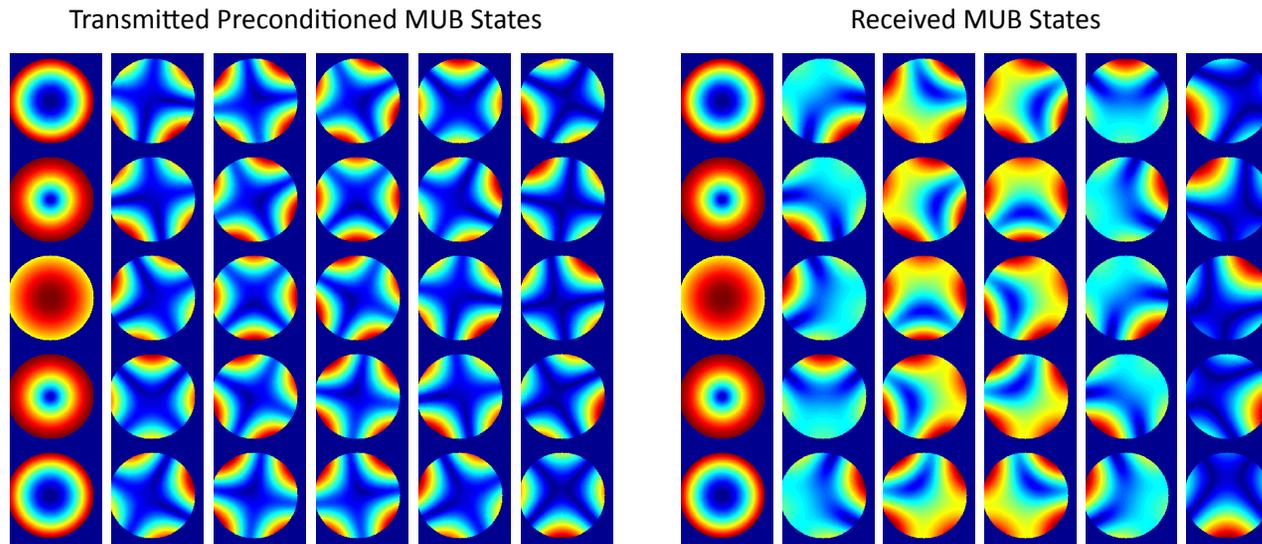
- For vacuum propagation the normalized channel capacity is (approximately) equal to  $\log_2(N_f^2)$
- The number of parallel channels supported by a propagation link is  $N_f^2$
- In the presence of turbulence the effective diameter is  $r_0$  which can significantly limit the information content of the propagation link
- If one attempts to increase the diameter significantly beyond this value, turbulence induced aberrations dominate and further degrade the link
- For a case of interest, consider the SOR Two Mile Site ( $r_0=0.1\text{m}$ ,  $\lambda=0.8\mu\text{m}$ ,  $z=3200\text{m}$ ,  $N_0=3.91$ )
- The optimum occurs at  $N_f=5.5$  ( $D=0.12\text{m}$ ) resulting in a normalized channel capacity of 2.05 bits per transmitted photon or four parallel channels

# The SOR Two Mile Site Provides a Typical Example of a Horizontal Path with Well Known Characteristics

| $C_n^2$              | 1      | 2      | 5     | 10    | 20    | 50    | $\times 10^{15}$ |
|----------------------|--------|--------|-------|-------|-------|-------|------------------|
| $r_0$                | 12.7   | 8.35   | 4.82  | 3.18  | 2.10  | 1.21  | cm               |
| $\vartheta_0$        | 12.4   | 8.20   | 4.73  | 3.12  | 2.06  | 1.19  | $\mu\text{rad}$  |
| Rytov                | 0.0365 | 0.0731 | 0.183 | 0.365 | 0.731 | 1.83  |                  |
| jitter               | 2.59   | 3.77   | 6.11  | 8.73  | 12.4  | 19.7  | $\mu\text{rad}$  |
| $f_G$                | 30.4   | 46     | 80    | 121   | 183   | 318   | Hz               |
| $f_{TG}$             | 5.19   | 7.56   | 12.3  | 17.5  | 24.9  | 39.5  | Hz               |
| $D_{\text{opt}}$     | 0.132  | 0.111  | 0.095 | 0.089 | 0.086 | 0.084 | m                |
| $D_{\text{opt}}/r_0$ | 1.04   | 1.33   | 1.97  | 2.80  | 4.09  | 6.97  |                  |
| $C_{\text{opt}}$     | 2.66   | 1.64   | 0.744 | 0.369 | 0.172 | 0.059 | bits/photon      |

- DARPA has expressed an interest in assessing quantum communication over a horizontal path
- As an example we consider the SOR Two Mile Site
- The  $C_n^2$  varies from almost as low as  $10^{-16}$  to almost as high as  $10^{-13}$  depending upon time day and time of year
- The above table illustrates the important turbulence parameters for conditions spanning this range (assuming  $\lambda=0.8\mu\text{m}$ ,  $z=3200\text{m}$ )
- We note that as the Rytov number approaches 0.2|1 we experience Branch Points|Deep Turbulence
- We also illustrate the optimum diameter and channel capacity for these conditions

# Advanced Concepts: Preconditioned MUB States



- We are developing a new protocol involving Preconditioned MUB States that use the minimum energy loss states as their basis states
- The initial functional form of these state is chosen so that upon propagation the states have the desired MUB character in the receiver plane
- The above figures pertain to a Fresnel number of one and a dimension of five
- For high dimensions and low Fresnel numbers ( $N_f=1$ ,  $d=5$ ,  $b=6$ ), the preconditioned MUB states look quite similar except for the fundamental basis
- For low Fresnel Numbers the transmitted and received fields are quite different
- The basis vectors are unaffected because they are already a minimum energy loss state



*Eve does not have a QND measurement:*

- *Eve has to absorb the photon to measure its timeslot.*
- *She can send Bob a fresh photon in the correct timeslot\*, but she cannot mimic the QM correlations of the expected Bell state.*
- *Standard QKD error correction + priv. amplification should apply.‡*

*Eve does have a QND measurement†:*

- *Need to measure in an appropriate MUB*

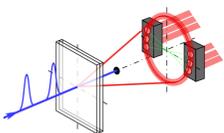
$$|\theta_m\rangle = N^{-1/2} \sum_{n=0}^N \exp(in\theta_m) |t_n\rangle \quad \theta_m = \theta_0 + \frac{2\pi m}{N} \quad \sum_{n=0}^N |t_n\rangle |t_n\rangle = \sum_{m=0}^N |\theta_m\rangle |-\theta_m\rangle$$

- *Optimal implementation still under consideration...*

\* This requires extremely fast processing on her end, to avoid delay...

‡ We will need to account for info. released to identify pol. correlations (~1 bit)

† Needs to be polarization-preserving QND!



# Temporal MUB measurement

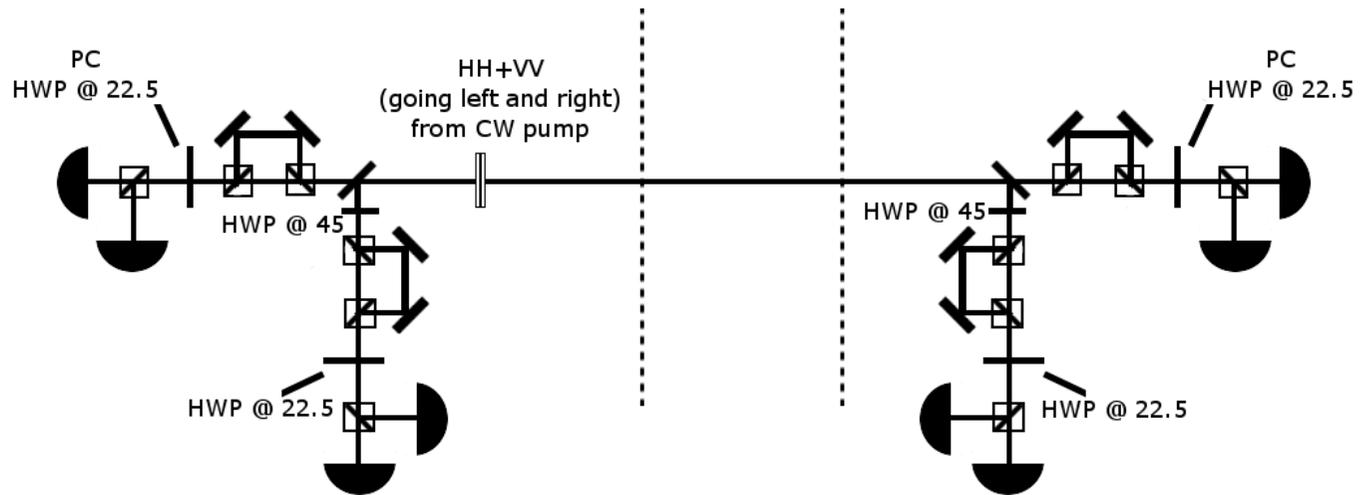


Version 1:

Measure superpositions of adjacent time bins:

$$(|\langle t_i | + \langle t_{i+1} | |)$$

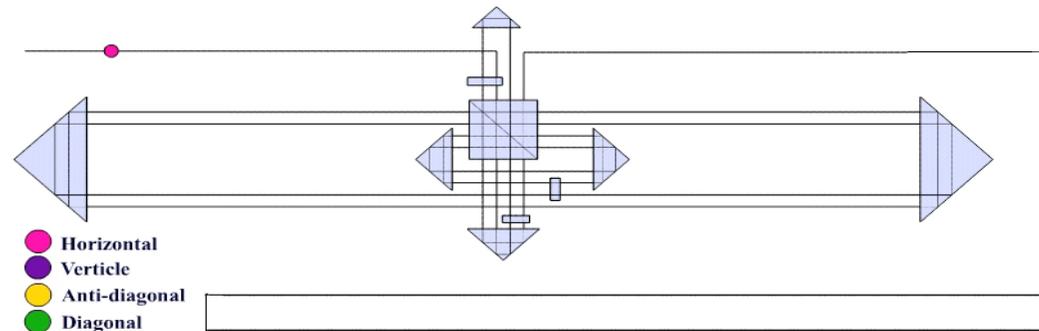
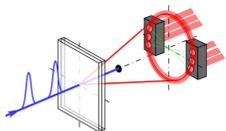
- "Franson" interference (PGK et al. PRA '90, '93, '96)
- ~easy to implement; by induction **all** time bins coherent
- probably not very eavesdropper sensitive



Version 2:

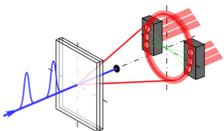
Measure superposition of all time bins:

$$|\theta_m\rangle = N^{-1/2} \sum_{n=0}^N \exp(in\theta_m) |t_n\rangle$$



## *Benefits of time-encoding/polarization checking*

- *Polarization checking is easy; can even check Bell inequalities; typical error rates quite low, e.g.,  $F > 99\%$ .*
- *Every photon can be used to check for Eve (cf. Ekert protocol)*
- *Every photon contributes to key (cf. BB84 [1/2] or SSP [1/3])*
- *Non-polarization sensitive QND far from realization (best QND measurements to date on microwave photons [Haroche, Martinis])*
- *Errors in time-bin not assumed to be from Eve—her measurements needn't disrupt timing at all [if she can implement an unnoticed delay; likely hard in FSO path (since system is timed to  $<150$  ps)]*
  - *Do not have to account for x5 overhead to account for Priv. Amp. (due to timing errors).*
  - *Just need usual classical error detection/correction*
- *But one pol. error means Eve could know all bits for that photon*
  - *e.g., 1% BER  $\rightarrow$  Eve looked at  $\sim 4\%$  of the photons (and knows their bits completely)  $\rightarrow$  input to Privacy Amplification.*



*If we only verify polarization, we may be susceptible to “bit-forcing”:*

*Eve blocks the channel for some time bins*

*→ eliminates possible bit choices, gains information*

*But the cost to Eve is high:*

*To force  $m$  of  $N$  possible bits ( $2^N$  bins), Eve must block  $N(1 - 0.5^m)$*

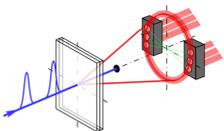
*E.g., to determine 1 bit out of 10, she must block half of the bins*

*to determine 5 bits out of 10, she must block 97%.*

*This intrusion can certainly be detected, e.g., using decoy\* pulses with different amplitudes.*

*Open questions:*

- What's the most efficient (per photon) decoy encoding?*
- What is the optimal implementation of temporal MUB?*
- What is the impact of hyper-entanglement on security?*
- What is the optimal encoding in DOFs ( $q$ -dits vs channels)?*



\*H.-K. Lo, X. Ma, and K. Chen. PRL **94**, 230504 (2005)