"Two-Photon" Coincidence Imaging with a Classical Source

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Coincidence imaging is a technique that extracts an image of a test system from the statistics of photons transmitted by a reference system when the two systems are illuminated by a source possessing appropriate correlations. It has recently been argued that quantum entangled sources are necessary for the implementation of this technique. We show that this technique does not require entanglement, and we provide an experimental demonstration of coincidence imaging using a classical source. We further find that any kind of coincidence imaging technique which uses a "bucket" detector in the test arm is incapable of imaging phase-only objects, whether a classical or quantum source is employed.

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Light has long been a convenient subject for the study and observation of nonclassical phenomena. Over the past decade, the ready availability of materials with strong optical nonlinearities has enabled significant advances in the practical use of such phenomena. Quantum cryptography [1], quantum teleportation [2,3], quantum lithography [4,5], and precision measurements below the standard quantum limit [6–9] have all been demonstrated experimentally. Even so, the use of nonclassical states in conjunction with the highly parallel nature of the optical field is still just beginning to be understood and exploited [10]. In particular, the technique of two-photon coincidence imaging [11] has recently emerged as a new way to extract information about an optical system. In this technique, a source which produces pairs of photons is required. One photon in each pair travels through a known (reference) optical system, while the other travels through an unknown (test) optical system (Fig. 1). The location of the reference photon is recorded on a detector array, while a second detector merely registers whether or not a test photon has been detected. The seemingly counterintuitive finding is that by placing the reference detector array in the appropriate plane in the reference arm, the image of the *test* system appears in certain coincidence statistics at the *reference* detector [13]. These coincidence statistics are obtained by gating the reference detector by the test detector. The first demonstration of this technique [11] used a source which produced quantum entangled photons [14,15]. At the time, the authors of [11] surmised that this imaging technique could also be implemented using a classical source with the proper statistical properties. Since then, some confusion has arisen whether or not entanglement is truly necessary for this technique. A recent Letter [12] has presented theoretical arguments for the case that entanglement is intrinsic to two-photon coincidence imaging. Although we agree that entangled sources possess some statistical properties that cannot by mimicked by classical sources, we believe that these properties are not required for coincidence imaging. In this Letter, we present theoretical arguments and provide experimental demonstration that coincidence imaging can be performed with a classical source, making it an even more practical technique.

Two-photon coincidence imaging relies on the fact that the single-photon detection probability distribution $p_r(x_r)$ can be different from the marginal distribution $\bar{p}_r(x_r) = \int p(x_r, x_t)dx_t$. Here $p_r(x_r)$ is the probability of detecting a photon at position x_r in the reference detector, and $p(x_r, x_t)$ is the probability of detecting a photon at x_r in the reference detector in coincidence with a photon at x_t in the test detector. Note that the marginal distribution is just the image collected by the reference detector when it is gated by the test (bucket) detector. According to classical probability theory, $\bar{p}_r(x_r) = p_r(x_r)$ provided that the integration over x_t (bucket detection) covers all possible outcomes. Thus, on first inspection it would seem that a difference between these two distributions is a violation of classical probability and hence is an intrinsically quantum phenomenon. This is the conclusion reached in [12].

One might suppose that the origin of this difference is the well-known quantum mechanical effect that measurement of one observable can alter the statistics of another observable. That cannot be the case here, however, as the photon counts at the two detectors are compatible

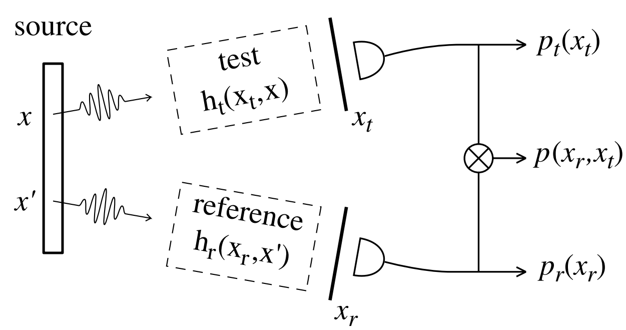

FIG. 1. (adapted from [12]) Two-photon coincidence imaging. The transfer function of the test system is to be obtained from the joint detection statistics using knowledge of the reference system.

observables. The only way the total number of photons detected at x_r can differ from the number of photons detected at x_r in conjunction with photons anywhere on the test detector is if some photons are lost en route to the test detector—that is, if the test region is lossy. In this case, integration of the joint detection probability over x_t does not cover all possible outcomes, and it is entirely consistent from a classical perspective that $p_r(x_r)$ and $\bar{p}_r(x_r)$ would differ.

The link between the presence of loss in the test arm and inequality of $p_r(x_r)$ and $\bar{p}_r(x_r)$ can be established rigorously. A quantum source in a general two-photon state is described by the state function $|\Psi\rangle = \int dx dx' \varphi(x, x')|1_x, 1_{x'}\rangle$, where 1_x denotes a single quantum of excitation at position x on the source. The projections of this state onto localized excitations at the test and/or reference detector are

$$\langle 1_{x_t} \mid \Psi\rangle = \int dx dx' h_t(x_t, x)\varphi(x, x')|1_{x'}\rangle, \tag{1}$$

$$\langle 1_{x_r} \mid \Psi\rangle = \int dx dx' h_r(x_r, x')\varphi(x, x')|1_{x}\rangle, \tag{2}$$

$$\langle 1_{x_t}, 1_{x_r} \mid \Psi\rangle = \int dx dx' h_t(x_t, x) h_r(x_r, x')\varphi(x, x'). \tag{3}$$

The single and joint photon detection probability distributions are then [12]

$$p_t(x_t) = \int dx' \left| \int dx h_t(x_t, x)\varphi(x, x') \right|^2, \tag{4}$$

$$p_r(x_r) = \int dx \left| \int dx' h_r(x_r, x')\varphi(x, x') \right|^2, \tag{5}$$

$$p(x_t, x_r) = \left| \int dx dx' h_t(x_t, x) h_r(x_r, x')\varphi(x, x') \right|^2, \tag{6}$$

where h_r and h_t are the impulse response functions of the reference and test systems, respectively. If the test system is without loss or gain, the kernel h_t is unitary and (by definition of unitarity) $\int dx_t h_t(x_t, x) h_t^*(x_t, y) = \delta(x - y)$. Integrating Eq. (6) over x_t and substituting this identity yields

$$\begin{aligned}\bar{p}_r(x_r) &= \int dx dx' dy dy' h_r(x_r, x') h_r^*(x_r, y')\delta(x - y) \\ &\quad \times \varphi(x, x')\varphi^*(y, y') \qquad (7)\\ &= \int dx dx' dy' h_r(x_r, x') h_r^*(x_r, y') \times \varphi(x, x') \\ &\quad \times \varphi^*(x, y') \qquad (8)\\ &= p_r(x_r). \qquad (9)\end{aligned}$$

If $p_r(x_r)$ and $\bar{p}_r(x_r)$ are to be unequal, as is required for coincidence imaging, h_t cannot be unitary, meaning that the test system must possess loss or gain. Thus the method of two-photon coincidence imaging with a bucket detector cannot be used to image phase-only objects.

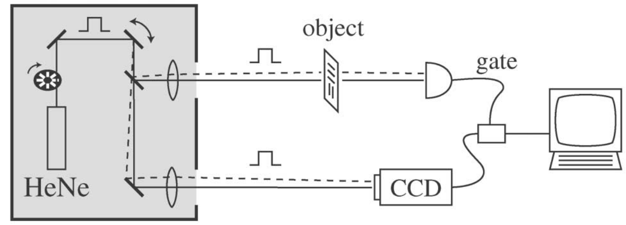

FIG. 2. The experimental setup used to perform coincidence imaging with a classically correlated source (shaded box).

A classical system which made use of the difference between these distributions to form an image is shown in Fig. 2. A classical source (the shaded box) produced pairs of angularly correlated pulses, with one member of each pair propagating through an amplitude mask to a bucket detector (the test arm) and the other propagating to a CCD camera (the reference arm). These pairs of pulses served as classical analogs of momentum-correlated photons produced by an entangled source. The frame grabber for the CCD camera could be gated by the bucket detector so that a frame would be recorded only if the test beam was not obstructed by the test pattern. The ordinary image was obtained by averaging all frames (no gating) and showed no pattern. The coincidence image was obtained by averaging only the gated frames and showed the test pattern (Fig. 3). This experiment is completely analogous to that of [11], the only significant difference being that a classical source was used rather than an entangled source. Our classical source was made by chopping a laser beam, deflecting it by a variable amount, then passing it through a beam splitter. The motion of the deflector was asynchronous with the period of the chopper, so that the deflection of the beam on any given pulse was a pseudorandom quantity. Although this quantity was in principle known, making the test arm unnecessary, we emphasize that the position of the deflector was *not* measured nor made

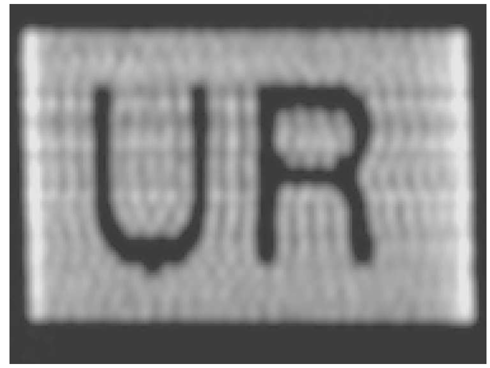

FIG. 3. The image formed in the reference arm when gated by the detector in the test arm. Such an image corresponds to the marginal probability distribution.

available to the imaging system, in order to demonstrate the principle of coincidence imaging.

We now turn to the broader question of whether any classical source can produce the distributions $p_t(x_t)$, $p_r(x_r)$, and $p(x_r, x_t)$ associated with a quantum source in an arbitrary pure state. A classical source may be modeled by a stochastic process which excites field distributions $E_n^{(t)}(x)$ and $E_n^{(r)}(x')$ in the test and reference arms, respectively, with probability P_n. The detection probabilities corresponding to Eqs. (4)–(6) are proportional to the expected values of the intensity and are

$$p_t(x_t) = \sum_n P_n \left| \int dx h_t(x_t, x) E_n^{(t)}(x) \right|^2, \tag{10}$$

$$p_r(x_r) = \sum_n P_n \left| \int dx' h_r(x_r, x') E_n^{(r)}(x') \right|^2, \tag{11}$$

$$p(x_t, x_r) = \sum_n P_n \left| \int dx h_t(x_t, x) E_n^{(t)}(x) \right|^2 \times \left| \int dx' h_r(x_r, x') E_n^{(r)}(x') \right|^2. \tag{12}$$

If the probability distributions Eqs. (10)–(12) are to equal Eqs. (4)–(6) for arbitrary test and reference systems, one must have

$$\varphi(x, x')\varphi^*(y, y') = \sum_n P_n E_n^{(t)}(x) E_n^{(r)}(x') E_n^{(t)*}(y) E_n^{(r)*}(y'). \tag{13}$$

When the quantum source is not entangled, $\varphi(x, x')$ is factorable and Eq. (13) is satisfied by a single classical state of unit probability with $E_1^{(t)}(x)E_1^{(r)}(x') = \varphi(x, x')$. When the source is entangled, then Eq. (13) cannot be satisfied, as there is no way for the right-hand side to factor into a function of (x, x') and a function of (y, y') while being unfactorable in x and x'. Thus, a classical source cannot mimic a quantum source in a pure state for *all* test and reference systems unless that state is nonentangled.

A classical source can be used for coincidence imaging, however, because the requirements are less stringent than those for mimicking a quantum source under general circumstances. For one, the classical source need only produce the same joint probability distribution as the quantum source, as the singles distributions are not used in forming the image. Second, the reference system is not arbitrary; its transfer function may be taken as known. Supposing the reference system to be lossless (or at least invertible), one may choose $E_s^{(r)}(x')$ to produce a diffraction-limited spot at the point s on the reference detector. Then $|\int dx' h_r(x_r, x') E_s^{(r)}(x')|^2 \approx \delta(x_r - s)$. Now, with the choice $E_s^{(t)}(x) = \alpha \int dx' h_r(s, x') \varphi(x, x')$ where $\alpha^{-2} \int ds = 1$, Eq. (12) becomes

$$p(x_t, x_r) = \sum_s P_s \alpha^2 \delta(x_r - s) \times \left| \int dx dx' h_t(x_t, x) h_r(s, x') \varphi(x, x') \right|^2. \tag{14}$$

In the continuum limit $\sum_s P_s \rightarrow \int ds P(s)$ with $P(s) = \alpha^{-2}$, Eq. (14) reproduces Eq. (6). Just as with a quantum source, $p(x_t, x_r)$ may be thought of as the intensity distribution of a point source at x_t which diffracts backward through the test system, is modulated by $\varphi(x, x')$, and diffracts through the reference system [16]. We emphasize that this (classical) joint distribution is *identical* to that obtained with the quantum source. Therefore, all techniques and phenomena which depend only on this distribution (and involve a test arm with a known, invertible transfer function), including coincidence imaging, coincidence "holography" [16], and ghost diffraction [13], can be demonstrated using a classical source. (We note that ghost diffraction uses a pinhole detector rather than a bucket detector in the test arm and has not been demonstrated by the experiment presented here.) In the case of a perfectly and uniformly entangled source $[\varphi(x, x') = \delta(x - x')]$, the entangled photons in each pair emitted by the quantum source are conjugates of one another and the classical fields likewise reduce to $E_s^{(t)}(x) = h_r(s, x) = [E_s^{(r)}(x)]^*$. In this case, each pair of fields in the classical ensemble corresponds directly to a pair of entangled photons.

In conclusion, we have shown that coincidence imaging can be performed using a classical source, even though it was previously thought that entanglement was necessary for implementing this technique. We have presented theory showing that, although classical sources cannot mimic the global statistics of entangled sources, the joint detection statistics which occur in the context of coincidence imaging can be produced using a classical source with the appropriate correlations. We further find that any type of coincidence imaging technique which uses a bucket detector cannot be used to image phase-only objects. Finally, we have confirmed the theory by forming a coincidence image of a test object without using a quantum (entangled) source.

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