

# Interferometric measurement of the optical phase distribution for Fresnel diffraction by a straightedge

Robert W. Boyd and Duncan T. Moore

The phase structure of the optical field diffracted from a straightedge has been measured using an ac interferometer. The measured phase distribution is in good agreement with the predictions of scalar diffraction theory.

## Introduction

Although diffraction phenomena have been observed for several centuries and are qualitatively well understood, careful measurements of the diffracted field at optical wavelengths are rather lacking. Diffraction phenomena have been studied in detail at microwave frequencies by Farnell<sup>1</sup> and by others,<sup>2</sup> and good agreement with theory is found. At optical wavelengths, diffraction theory becomes more complicated because of the difficulty in determining the proper boundary conditions for the optical field. Many practical problems involve diffraction from apertures in blackened screens, but the concept of blackness is not readily described in terms of conventional boundary conditions. Furthermore, most careful studies of diffraction effects at optical wavelengths<sup>3</sup> have been confined to measurements of the field intensity, whereas a complete experimental study of diffraction effects requires the study of both the phase and amplitude of the field.

Detailed measurements of the optical field for various diffraction geometries would be useful in choosing between different theories of diffraction effects. Within the framework of scalar diffraction theory, for example, the Kirchhoff and Rayleigh-Sommerfeld theories predict substantially different angular dependences to the diffracted light intensity at large diffraction angles. Although the Kirchhoff theory has been criticized by some as being mathematically inconsistent,<sup>4</sup> others

claim that in fact the Kirchhoff theory more accurately predicts the field diffracted from a black screen.<sup>5</sup> No experimental studies have been performed which would discriminate between these theories for diffraction by black objects. Electromagnetic diffraction theories predict even more complex diffraction effects than either the Kirchhoff or the Rayleigh-Sommerfeld theories and at optical wavelengths have been inadequately tested by experiment.

We present here a technique for studying diffraction effects by the measurement of the phase of the optical field by ac interferometry, and we demonstrate this technique by measuring the field resulting from Fresnel diffraction by a straightedge. Our measurement thus complements the more usual measurements of the intensity of the diffracted light. The measurement was made under conditions whereby no discrimination between competing theories was expected, and in fact our results are consistent with a simple theoretical model based on scalar diffraction theory.

Previous studies of the phase distribution of diffracted fields include a study using the qualitative coherent background method described by Zernike<sup>6</sup> and a study by Harris and Givens<sup>7</sup> using a holographic technique. Our results are similar to those of the latter authors but extend further into the geometrical shadow region.

The technique we present here is applicable to a number of related questions, such as the measurement of the phase distribution in a focal region, a problem of considerable historical interest.<sup>8</sup> Measurements of the phase distribution in the focal region of an aberrated optical system could prove useful in the field of optical aberration testing. Finally, it should be possible to measure the amplitude and phase of the diffracted light at large diffraction angles and thereby to test for differences between various formulations of diffraction theory.

The authors are with University of Rochester, Institute of Optics, Rochester, New York 14627.

Received 11 January 1979.

0003-6935/79/122013-04\$00.50/0.

© 1979 Optical Society of America.

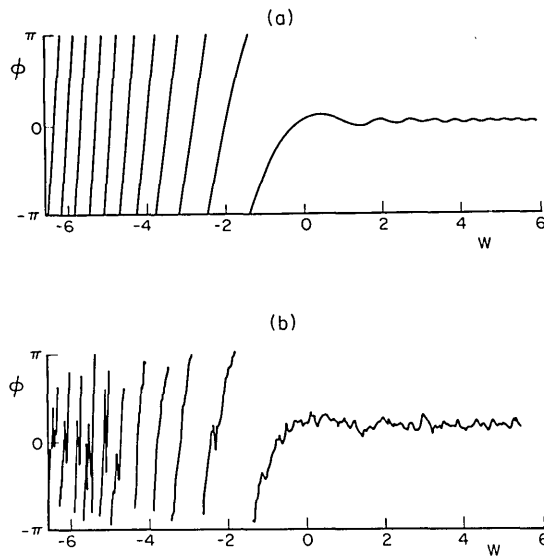


Fig. 1. (a) Theoretical phase distribution of a wave diffracted from a straightedge as a function of the normalized coordinate  $w$ . (b) Experimental phase distribution as measured by the ac interferometer.

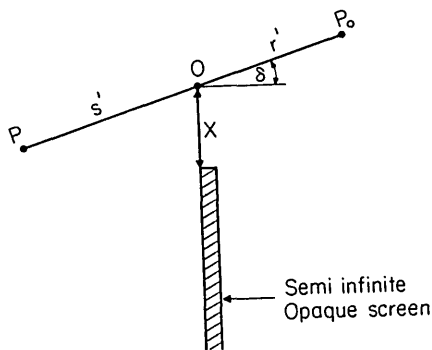


Fig. 2. Definition of the distances  $r'$ ,  $s'$ , and  $x$  and the angle  $\delta$  used in the theory of Fresnel diffraction by a straightedge.  $P_0$  denotes a point source, and  $P$  denotes a point of observation.

### Theoretical Phase Distribution

Figure 1(a) shows the predicted phase distribution for the field diffracted from a straightedge for the case of small diffraction angles. In this limit the predictions of the Kirchhoff and Rayleigh-Sommerfeld theories are identical. For an incident field  $U_0(r) = A \exp(ikr)/r$  emanating from a point source  $P_0$ , the diffracted wave at point  $P$  (see Fig. 2) is given by<sup>9</sup>

$$U(P) = \frac{A \exp(ik)(r' + s')}{2(r' + s')} \{ [C(w) + 1/2] + [S(w) + 1/2] - i\{ [C(w) + 1/2] - [S(w) + 1/2] \} \} \quad (1)$$

Here  $C(w)$  and  $S(w)$  are the Fresnel integrals expressed as functions of the dimensionless parameter  $w$  given by

$$w = \left[ \frac{2}{\lambda} \left( \frac{1}{r'} + \frac{1}{s'} \right) \right]^{1/2} x \cos \delta. \quad (2)$$

The phase of the field given by Eq. (1) is thus

$$\phi = \arctan \frac{S(w) - C(w)}{1 + S(w) + C(w)}, \quad (3)$$

and it is this function which is plotted as the theoretical curve in Fig. 1(a). The argument  $w$  is given by Eq. (2) with  $\delta = 0$  and in the limit of  $r' \rightarrow \infty$  corresponding to our well-collimated laser beam.

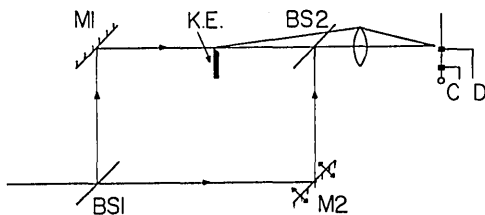


Fig. 3. Mach-Zehnder interferometer with a knife-edge in one arm. The irradiance from the other arm has been reduced to enhance the visibility of the figures in the dark region of the diffraction pattern. Mirror  $M2$  is mounted on a piezoelectric ceramic. Two detectors are placed in the fringe pattern. One detector scans across the fringe pattern, while the other serves as a fixed reference.

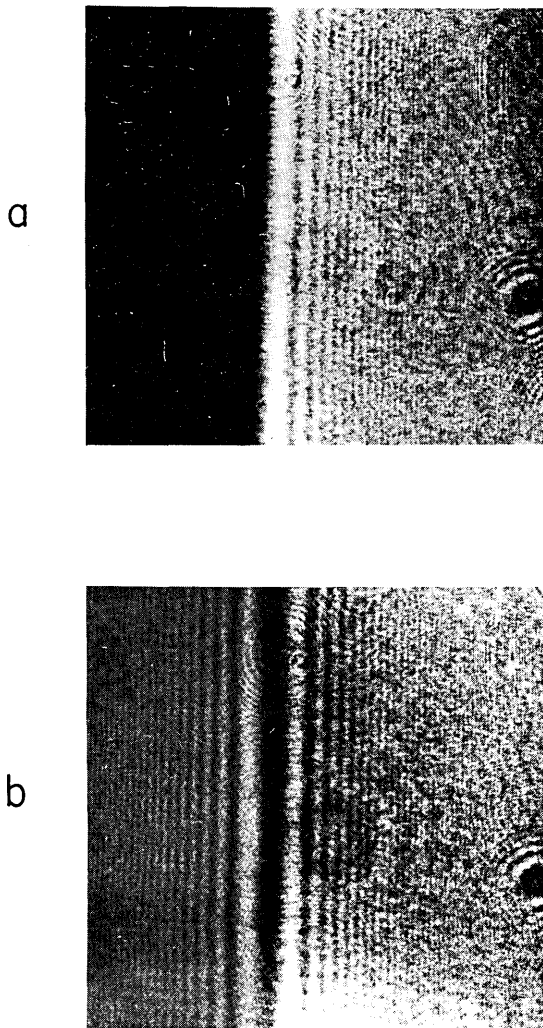


Fig. 4. (a) Photograph of the diffraction pattern due to Fresnel diffraction by a straightedge. (b) Interferogram formed by the interference of the diffraction pattern of (a) with a reference plane wave. The rapidly changing phase in the geometrical shadow region appears as a series of fringes.

### Instrumentation

The phase structure of any diffraction pattern can be made visible by combining the diffracted field with a reference field whose phase fronts are plane. If these two fields are coherent with respect to each other, the phase of the diffracted wave will appear as fringes in the resulting interference pattern. In Fig. 3, for example, a straightedge (K.E.) is placed in one arm of a Mach-Zehnder interferometer. The diffracted wave front is combined with a reference wave front at the second beam splitter *BS2*. If the resultant interference pattern is magnified, fringes are seen in the geometric shadow of the knife-edge pattern, as shown in Fig. 4. Each fringe represents a  $2\pi$  phase change. In this system over twenty fringes are seen. In the bright section of the diffraction pattern, the phase changes are approximately  $15^\circ$  ( $\lambda/20$ ), and, therefore, no interference

fringes are observed. (Of course, the intensity modulation of the diffraction pattern is observed.)

In order to measure accurately small phase variations say smaller than  $\lambda/10$ , it is useful to employ the technique of ac interferometry.<sup>10</sup> In this method the position of mirror *M1* is modulated by applying a voltage to the piezoelectric ceramic on which the mirror is mounted. The phase of the reference beam is thus alternately decreased and increased (by  $8\pi$  rad total change) as the mirror moves in and out. The ceramic is driven at 100 Hz by the triangular waveform seen in Fig. 5(a). If a detector is placed at reference point *C* in the observation plane, the irradiance onto the detector and, therefore, the voltage response of the detector varies as shown in Fig. 5(b). If a second detector *D* is placed a small distance from *C*, the electrical signal from *C* will have the same functional form as that from *D* but will, in general, be shifted in phase relative to that from *D*. This electrical phase difference is equal to the optical phase difference of the diffracted wave front at points *C* and *D*. The phase structure of the diffraction pattern can thus be determined as *D* is scanned across the interference pattern. Locally, the measured phase of the optical field is a number between 0 and  $2\pi$ , but the electronics possess the ability to count fringes and thus to measure the optical phase without this  $2\pi$  ambiguity.

The interferometer uses the 1.0-W, 0.5145- $\mu\text{m}$  line of an Ar-ion laser as a light source. This beam passes through a spatial filter to eliminate beam irregularities

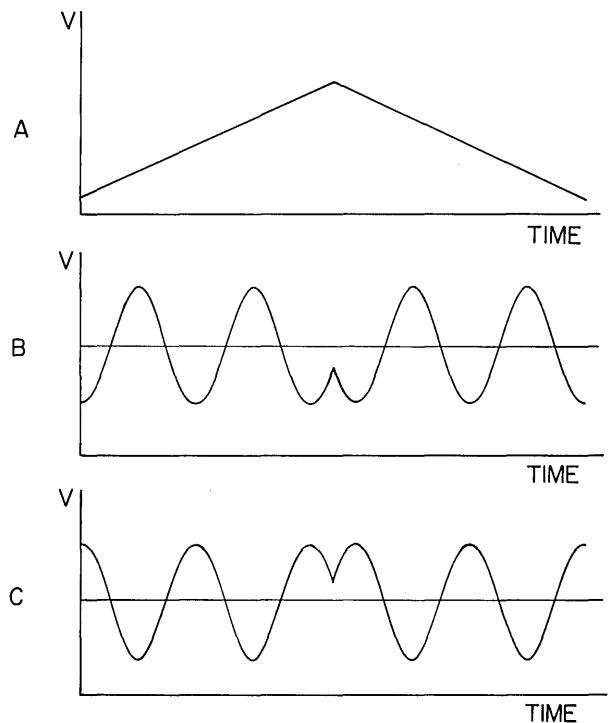


Fig. 5. Three electrical signals of ac interferometry: (a) voltage on piezoelectric ceramic; (b) output of detector at *C*; and (c) output of detector at *D*.

and is collimated at a diameter of 50 mm to approximate the infinite plane wave assumed by our theoretical model. The diffracting screen is a razor blade selected to produce a diffraction pattern appearing clean on visual inspection. The diffracted field is examined at a distance of 75 mm from the screen by a 40× microscope objective, which produces an enlarged image on our detection plane.

### Limitations to Measurement Sensitivity

The finite size of the detector limits the spatial and, therefore, the phase resolution. The effect is to convolve the detector area with the fringe pattern. We have enlarged our interference pattern so that the finite size of our detector has only a minimal effect ( $\lambda/50$ ) on our phase resolution.

A serious problem is the vibration of the interferometer components. Accurate measurements are possible only when the interferometer is vibrationally isolated from its environment. An additional problem results from thermal instabilities in the interferometer from air currents. This is reduced by shielding the interferometer. The deleterious effect of these fluctuations is minimized by continuously subtracting the phase measured by the reference detector from that measured by the scanning detector. Localized fluctuations are not eliminated by this technique and typically produce phase errors of the order of  $\lambda/100$ .

A number of tests have been performed to determine the reproducibility of the total apparatus. The results of one such test are shown in Fig. 6 where two scans of the same wave front are overlaid. The maximum difference of the scans is  $\lambda/128$ . This deviation is believed to be due to the thermal fluctuations within the interferometer. These two scans were separated by a time interval of about 2 min.

### Results

Using the apparatus shown in Fig. 3 we have been able to measure the phase distribution of the optical field diffracted from a straightedge. The resulting curve is shown in Fig. 1(b) and is compared with the theoretical curve shown above it. Within the limitations imposed by the noise in the experimental curve,

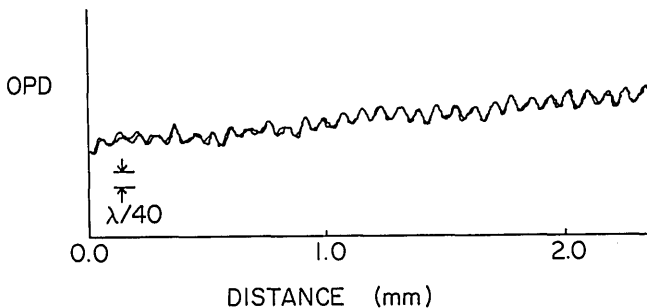


Fig. 6. Stability of the ac interferometer is illustrated by a double trace of the same wave front.

there is good agreement between the experimental curve and the theoretical curve based on scalar diffraction theory. The comparison is limited primarily by our uncertainty in locating the origin ( $w = 0$ ) of the measured curve. We have thus aligned the theoretical and experimental curves by eye for the best qualitative agreement.

In the illuminated region ( $w > 0$ ), the rms phase noise (including random and statistical errors) is  $10^\circ$  or  $0.03$  wavelength. Here one can nearly resolve the weak phase oscillations (of maximum amplitude  $20^\circ$ ) seen in the theoretical curve. In the geometrical shadow region ( $w < 0$ ), scalar diffraction theory predicts that the phase is a monotonically increasing function of  $w$ . With the exception of several noise spikes, the data show this predicted monotonic behavior. We note that for ease in display the continuously varying phase has been plotted as a broken curve defined only in the range of  $-\pi$  to  $\pi$ . The rms phase uncertainty of  $0.03\lambda$  is due primarily to systematic errors caused by such effects as stray reflections and ripples or scratches on the optical components of the interferometer. The random error in these measurements is approximately  $0.01\lambda$  as discussed previously.

At the level of sensitivity which we were able to obtain, no discrimination between various formulations of scalar diffraction theory is possible. More exacting measurements should be able to show this difference.

We acknowledge the useful discussion of diffraction effects with E. Wolf. RWB acknowledges support through NSF grant SER 77-06914.

### References

1. G. W. Farnell, *Can. J. Phys.* **35**, 780 (1957); **36**, 935 (1958); *J. Opt. Soc. Am.* **48**, 643 (1958); L. Lee and G. W. Farnell, *J. Opt. Soc. Am.* **51**, 273 (1961).
2. M. J. Ehrlich, S. Silver, and G. Held, *J. Appl. Phys.* **26**, 336 (1955); J. Buchsbaum, A. R. Milne, D. C. Hogg, G. Bekefi, and G. A. Woonton, *J. Appl. Phys.* **26**, 706 (1955); C. L. Andrews, *Phys. Rev.* **71**, 777 (1947); *J. Appl. Phys.* **21**, 761 (1950).
3. See, for example, C. A. Taylor and B. J. Thompson, *J. Opt. Soc. Am.* **48**, 844 (1958).
4. H. Poincaré, *Theorie Mathematique de la Lumiere* (George Carre, Paris, 1892), pp. 187, 188; B. B. Baker and E. T. Copson, *The Mathematical Theory of Huygens Principle* (Clarendon Press, Oxford, 1950), pp. 71, 72.
5. E. W. Marchand and E. Wolf, *J. Opt. Soc. Am.* **56**, 1712 (1966); F. J. Kottler, in *Progress in Optics*, E. Wolf, Ed. [North-Holland, Amsterdam, **4**, 281 (1965); **6**, 331 (1967)].
6. F. Zernike, *Proc. Phys. Soc. London* **61**, 158 (1948).
7. J. S. Harris and M. P. Givens, *J. Opt. Soc. Am.* **56**, 862 (1966).
8. M. Gouy, *C. R. Acad. Sci.* **110**, 1251 (1890); A. Rubinowicz, *Phys. Rev.* **54**, 931 (1938).
9. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, (1975), Chap. 8.7.
10. D. T. Moore and D. P. Ryan, *J. Opt. Soc. Am.* **68**, 1157 (1978).