# CEREMONIAL MATH DEPARTMENT MACE

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## ABSTRACT

*For this project, the team was tasked with designing and manufacturing a ceremonial mace to be used by the Mathematics Department at the University of Rochester. This process began by researching the history of the University's mace. A visit to view and document the University Mace provided an excellent basis to build this design. A presentation for the Math Department allowed the team to share preliminary thoughts and ideas and address any feedback from the Department. To ensure the team was on the same page as the sponsor, a bi-weekly meeting occurred, where both designs and iconography to be featured on the mace were discussed. The team continued the research by inquiring about the thoughts and requests of the Department staff, and then considered this information for the next iteration of designs. After much deliberation, the team decided on a design consisting of a carbon-fiber tube with aluminum detailing. The bottom head of the mace features an aluminum Menger sponge, and the top head of the mace features a 3D printed dodecahedron sporting iconography representative of the Math Department. The shaft of the mace consists of carbon-fiber tubing, a pair of aluminum toruses, and an aluminum helicoid all held together by two threaded steel rods. A stand consisting of red oak wood and felt was also designed to support the mace and protect it from damage when not in use. The stand also serves as a testament to the Department's history, as it features all the names of past and present Department chairs dating back to the 1800s. As this project serves to display the university's manufacturing capabilities, this device was fabricated using a wide range of techniques, including machining by hand, 4-Axis CNC, CNC lathe, laser cutting and etching, and 3D printing. If the final product is approved, the mace will be used in the Department's commencement ceremony this year and for years to come.*

## PROBLEM DEFINITION

The Math Department does not currently have a mace for their ceremonial graduation that highlights the prestige and history of Mathematics at the University of Rochester. The tradition of a ceremonial mace dates back to medieval England, where a mace is held by a person of power; it represents unity and authority under a common goal. Our university's tradition of the ceremonial mace began in 1935, and in 2023 the Mechanical Engineering Department established their own tradition of using a custom-made mace representing the Department. Now, in 2024, the Math Department will be continuing this tradition by receiving their own custom mace, which represents the prestige of math at the University of Rochester. Additionally, a secondary goal of the construction of the mace is to highlight as many of the manufacturing capabilities on campus as feasibly possible.

## REQUIREMENTS, SPECIFICATIONS, DELIVERABLES

The design requirements of this project were used to guide the team's design choices. The manufacturing process is required to include as many manufacturing methods as possible and be built with as many materials as possible. The weight and balance of the mace should be designed to allow for easy transportation so that anyone can hold it comfortably. The Math Department's mace should also be shorter than the University Mace and not include the official university seal. The final requirement is that the mace should be aesthetically pleasing.

The design specifications allow for a precise description of items or tasks necessary to the design and allow the team to measure results. The specifications for the overall length is less than 48 inches top to bottom. The specification for overall weight is 6.4±2.5 lbf. The specifications for fatigue failure or the endurance limit for the mace is 85.99 ksi. The center of mass must be within 10% of the total length from the geometric center. The final specification is that the mace or stand must feature the names of all past and present Department chairs.

The deliverables for the project made it clear what was expected of the group and what would need to be accomplished throughout the semester. Deliverables for this project included; a prototype model of the mace, a bill of materials along with associated CAD drawings, an iconography pamphlet describing the symbols included, a manufactured Department mace, a stand for the mace, and a final report documenting the team's findings.

## CONCEPTS

The concepts presented for the head of the mace were one of the most significant parts of the concept design. Sketches of each design (Figure 1) can be seen in the Appendix along with the Pugh Matrix (Table 4) described below. In the matrix, there is a baseline design, by which each design is compared through a set of categories, being identified as better (1), on par with the baseline (0), or worse (-1). The baseline design is a sphere- a simple shape that is often associated with the head of medieval maces. For this design, there is an exception to the "baseline" values of 0: the sphere has mathematical significance, and thus was chosen to have a value of 1 in this category. Thus, some designs could end up being worse than the baseline solely due to having less mathematical significance, meaning the baseline is a valid option for the final concept. The four concepts outside of the baseline are three regular polyhedra: the tetrahedron, cube, and dodecahedron, and an additional shape, the octagonal prism.

Categories in the Pugh Matrix were chosen based on a variety of parameters, considering the design and manufacturing of the mace, as well as the requirements for the project. "Ease of Manufacturing" was considered to compare how difficult it would be to create these different designs. As the mace represents mathematical concepts, the significance of the shape ("Mathematical Significance") and its ability to display iconography were considered. The "Space for Concepts" category considers both the number of unique surfaces, as well as the total usable surface area. Finally, the "Uniqueness" category was used to ensure that the mace did not blend in with the other University of Rochester maces. In addition, a unique shape will draw attention to the mace, and make the design recognizable for the Department as more maces are developed.

The results in the Pugh Matrix show that the tetrahedron and dodecahedron were the two leading designs for this concept. The team decided to use the dodecahedron for the top head's design. This choice was made based on the number of unique faces that provided space for additional mathematical concepts. The final mace design should represent as many mathematical concepts as possible, as well as the Math Department at the University of Rochester. The tetrahedron having only three functional faces leaves little room for expression of a wide range of concepts. In addition, any dynamic design (such as an abacus) would require the use of an entire face, drastically reducing the amount of space for other concepts. Thus, the eleven functional faces in the dodecahedron design (the twelfth being used for connection to the shaft) provided more opportunities for representation of fields of math, and this was the chosen design for the top head of the mace. An additional calculation, the volume of the object, was completed (these calculations can be seen in the Appendix in Figure 2) and provides additional information for the consideration process, as outlined below.

One of the requirements for this design is for the total mace to be balanced in the user's hands. This was defined as having the center of gravity within 10% of the center (lengthwise) of the mace. For this calculation, a constant height for the head is assumed, so that the total mace length is consistent throughout. In addition, the material is assumed to be the same uniform density material, so that only the volume of the shape indicates the influence the head will have on shifting the center of gravity upwards. After calculating the volume of the concept designs assuming a height of 6 inches, the dodecahedron was found to have a volume of 149.86 cubic inches. This design has a large volume compared to the others, indicating the importance of modeling the mace's bottom head to ensure the center of gravity is balanced. While this calculation results in a large volume, the Klein bottle included in the interior means that the shape is hollow, greatly reducing the weight of the dodecahedron. The dodecahedron head design not only provides many faces to create individual designs, but has an important mathematical meaning of its own and makes for an aesthetically pleasing focal point of the Mathematics Department ceremonial mace.

In addition to this dodecahedron, the mace has a variety of mathematical concepts. These include a Menger sponge, two toruses, a helicoid, and a Klein bottle. These concepts were all chosen in collaboration with the Math Department. After interviewing different faculty, the most popular symbols, including the Department's old logo, the Klein bottle, were chosen to be represented on the mace. In addition, the designs for the faces of the dodecahedron were selected by a faculty vote for the top ten most popular symbols to include. Once created, these specific designs were approved by the sponsor and a team of math undergraduates. In addition, to represent a significant historical part of mathematics, a functional abacus was modeled for the top face of the dodecahedron. The significance of each aspect of the mace is detailed in the Iconography Pamphlet provided to the math team. An image of each face can be seen in the Appendix in Figure 5. Images of the assemblies in CAD can also be seen in the Appendix, in Figures 6 and 7.

## MECHANICAL ANALYSIS

This section includes a variety of analyses for different aspects of the mace. Some of these analyses were based on specifications and physical results of manufacturing, while others are based on equations for a numerical result. For this section, Shigley and Mitchell's *Mechanical Engineering Design, Fourth Edition* was used as a reference for equations and constants.

**Tolerance Analysis:**

The tolerance of the pins for the edge features of the dodecahedron was tested by the team to ensure easy assembly for the dodecahedron. When the first samples were printed, the edges could move around in the corner pieces, meaning that the structural integrity of the dodecahedron was compromised. Thus, a variety of pin sizes were printed against one control-sized corner hole, and these were tested for the best tolerance for the connection. Since this was only one pin in a hole, there was no stack-up tolerance to consider. In addition, due to the variation in the print quality of parts from the 3D printing process, a typical tolerance table would not necessarily provide the best results for this application, as the ±tolerance is dependent on the machine. The goal was to find a pin size that fit well and did not allow for movement in the hole, but also was not tight enough that the assembly could not be taken apart by hand easily. The end result for this was a hole diameter of 0.3" and a pin diameter of 0.29". This was verified with multiple print samples of both the corner and the edge, so that the variation in 3D printing was accounted for.

While the ±tolerance is based on the accuracy of the 3D printer, a tolerance grade based on ITS standards provides a reference point for what the pin size should be. Using the standard H7/h6, the hole basis has a IT7 tolerance grade of 0.0006". Equations 1 and 2 show the tolerance for a hole, where D is the nominal hole size, and $\Delta D$ is the tolerance grade for a hole.

$$D_{max} = D + \Delta D \quad (1)$$

$$D_{min} = D \quad (2)$$

Based on these equations, the hole size for the corner should be:

$$D_{max} = 0.3in + 0.0004in = 0.3004in$$

$$D_{min} = 0.3in$$

For the pin, the tolerance grade is based on IT6, and is 0.0004". In addition, there is a value called the fundamental deviation. Since this is an H deviation, this value is 0. Equations 3 and 4 show the tolerance for a pin, where d is the nominal pin size, $\Delta d$ is the tolerance grade for a shaft, and $\delta d_F$ is the fundamental deviation.

$$d_{max} = d + \delta_F \quad (3)$$

$$d_{min} = d + \delta_F + \Delta d \quad (4)$$

Based on these equations, the pin size for the edge should be:

$$d_{max} = 0.29in + 0in = 0.29in$$

$$d_{min} = 0.29in + 0in - 0.0004in = 0.2896in$$

These values are the tolerance that the edge shaft and corner hole should be designed to in a typical manufacturing setting. Due to the 3D printing used, these are a benchmark measurement, and the deviation of the printer may not accurately fall within the max and min values for each measurement.

**Fatigue Analysis:**

For ferrous materials, there is an endurance limit associated with fatigue failure. Since aluminum does not have an endurance limit, the team only considered the steel threaded rod for this calculation. Based on the manufacturer's information, the tensile strength of the threaded rod is 125 ksi. For common ferrous materials with a strength below 200 ksi, the endurance limit is half of the ultimate tensile strength, as shown in Equation 5:

$$S_e' = 0.5 \cdot S_{UT} \qquad S_{UT} \leq 200\, ksi \quad (5)$$

Therefore, the endurance limit of this threaded rod is 62.5 ksi. Under the normal operating conditions of the mace, this stress value will not be achieved, meaning the mace should not fail due to fatigue, despite the higher fatigue limit specification (See Test Plan and Results).

**Fastener Torque Analysis:**

To hold the dodecahedron in place, a ¼-20 nut was torqued down on the threaded rod above the interface. This connection is held from the other end by the circular face of the carbon-fiber tube against the aluminum face, with the interface end inside the carbon-fiber tube to align them concentrically. The equation for the required torque is shown in Equation 6:

$$T = K \cdot F_i \cdot d = K \cdot 0.75 F_p \cdot d = 0.75 \cdot K \cdot A_t \cdot S_p \cdot d \quad (6)$$

In this equation, T is the required torque, K is a material-dependent constant, $F_i$ is the preload, and d is the nominal diameter of the bolt (the threaded rod, in this case). The preload is equivalent to 0.75 times the proof load $F_p$ for a non-permanent connection, which in turn is the proof strength $S_p$ times the area of the threaded portion of the bolt, $A_t$. For this calculation, K is 0.3 for steel-steel fastening, $A_t$ is 0.0318 $in^2$ (Shigley's) $S_p$ is 125 ksi (Shigley's), and d is 0.25 inches. The resulting torque is shown below. This value is the torque needed to apply to the nut for a secure connection.

$$T = 0.75 \cdot 0.3 \cdot 0.0318\, in^2 \cdot 125\, ksi \cdot 0.25\, in = 223.59\, lbf - in$$

This calculation provides a value for necessary torque to ensure a good connection and prevent separation for the nut holding the dodecahedron in place. In addition, a torque requirement for the Menger sponge is also calculated. For this calculation, the same equation is used. In addition, as the threaded rod is the same, the values $A_t$, $S_p$, and d are all the same. Since the Menger sponge is made of aluminum, the K value for steel-aluminum is around 0.4. Therefore, a simple conversion can be done to calculate this torque.

$$T = \frac{0.4}{0.3} \cdot 223.59\, lbf - in = 298.12\, lbf - in$$

This value is the necessary torque for the bottom head connection, however, the shape of the Menger sponge means that this cannot be achieved with typical torque applications. Therefore, this provides only a reference for the assembly requirement.

**Material Selection:**

The abacus was chosen to be made from ABS in the 3D printer in Rettner shop. Due to the complex geometry, the two materials that were considered were ABS and wood. For the mace to have a long lifespan, the team was concerned about painting wood, as it eventually wears off, especially under exposure to the elements during graduation ceremonies. In addition, the abacus is designed to be touched and used, so it will experience wear. The team did not know whether the 3D printer could use multiple colors at once but wanted to make the beads and frame different colors. As a backup, if the 3D printed part was not satisfactory, wood could be used. This would provide a shorter lifespan, but a simpler manufacturing process.

Thus, a 50% scale abacus was tested for 3D printing. The 3D printing slicer was able to print the beads in black while printing the frame in white all as one piece. Since it was able to print multiple colors, no complex manufacturing was required, and the lifespan of the abacus could be longer. In addition, due to the positive feedback received from the project sponsor and team members, this manufacturing method, and thus the ABS material, was ultimately chosen for its simplicity of manufacturing and longevity.

**Structural Finite Element Analysis:**

Due to the selection of materials for each aspect of the mace, in the event of the mace being hit and potentially damaged, the most likely part to break would be the dodecahedron. To understand its response to an impact, a finite element analysis was conducted on the frame of the model. This model was a simplified version of the entire frame, but it was created conservatively. The model was created as one frame and was slightly larger due to the variability of the 3D printer- assuming all parts were made slightly larger meant that supports were further away, and the model was at its weakest. Then, the frame was made as all one connected system of beams. This meant that the corners did not have the connectors to hold the faces in place, so the FEA model had less support than the final product would. Only the frame was considered, meaning that the faces in the final model would only add additional support to the structure.

This model assumes that the bottom face is rigid and fixed: given that this face is made of aluminum, this assumption is valid since, compared to the rest of the material, this face is relatively rigid. Then, a force was applied to one of the top corners. This force was equivalent to three times the weight of the entire mace. In addition, it was applied to only one corner, to account for the worst-case scenario with the load not being distributed across the frame. The results of this test indicate that the structure as modeled will not fail either in a statics application or a bending application. The full analysis, details and results can be seen in the Appendix in Figure 3.

**Bending Stress Analysis:**

When the mace is held horizontally and supported at the middle of the helicoid, the dodecahedron and Menger sponge can apply bending moments to the two carbon fiber tubes. There is a Grade B7 Medium-Strength Steel Threaded Rod that is concentric with each carbon-fiber tube.

Based on the design, the carbon-fiber tube on the bottom is 8.750” in length and the carbon-fiber tube on the top is 8.625” in length. The outside diameter of the tube is 1.1280”, the inside diameter of the tube is 1”, and the nominal diameter of the steel rod is 0.25”. The weight of the bottom head (Menger sponge) is 1.9956 lbf, and the weight of the top head (dodecahedron) is 2.0837 lbf.

The carbon-fiber material purchased has a tensile strength of 125,000 – 175,000 psi according to the manufacturer. The steel rod’s strength is 125,000 psi, which is close to carbon-fiber tube’s strength.

Below is the calculation for the moment of inertia of the carbon-fiber tube and the steel rod together:

$$I = \frac{\pi}{64}(1.280^4 - 1^4 + 0.25^4) = 0.0306\, in^4 \qquad 8$$

For the carbon fiber tube on the bottom, the max bending stress occurs on the outside surface of the carbon fiber tube:

$$\sigma_{max-bottom} = \frac{M_{bottom}c}{I} = \frac{W_{bottom}L_{bottom}\frac{d_{max}}{2}}{I} \qquad 9a$$

$$= \frac{1.9956 \times 8.75 \times \frac{1.1280}{2}}{0.0306} = 321.8\, psi$$

For the carbon fiber tube on the top, the max bending stress can be calculated with the same method:

$$\sigma_{max-top} = \frac{M_{top}c}{I} = \frac{W_{top}L_{top}\frac{d_{max}}{2}}{I} \qquad 9b$$

$$= \frac{2.0837 \times 8.625 \times \frac{1.1280}{2}}{0.0306} = 331.2\, psi$$

Maximum bending stresses in carbon fiber tubes are individually 0.51% and 0.53% of the max acceptable stress when the factor of safety is 2. As a result, a failure caused by either bending moment will not occur.

## MANUFACTURING

Manufacturing methods are described from top to bottom of the model.

**Dodecahedron**

- Frame, Abacus, Klein Bottle: 3D printed by Dalton and Will, with the assistance of Jim Alkins, on the Stratasys 3D printer in Rettner Shop
- Acrylic Faces: Cut and laser-engraved by Chengxiao, with the assistance of Prof. Mohammad in Gavett

- Bottom face and Interface: Made in Rettner Shop by Dalton, Jack, and Will, with Jim Alkins's assistance, on the CNC mill and the manual lathe, respectively

**Shaft**

- Carbon-fiber tube and Threaded rod: cut in Taylor Shop and Rettner Shop by Will, with the assistance of Jim Alkins and Bill Mildenberger
- Torus: Made on the HAAS Machine in Taylor Shop by Professor Muir
- Helicoid: Made on the HAAS Machine in Taylor Shop by Professor Muir

**Menger Sponge and Interface**

- Interface: Made on the manual lathe in Rettner Shop by Jack with the assistance of Jim Alkins
- Menger Sponge (both halves): Made on the HAAS machine in Taylor by Yijun, with the assistance of Professor Muir and Bill Mildenberger.

**Assembly:**

- Dodecahedron Sub-Assembly: Dalton, Will, and Chengxiao
- Heat-set insert and Klein Bottle: Jack
- Entire Assembly: Math Mace Team

For each of these, the team discussed the materials based on the application of the part. The Klein bottle was 3D printed so that it could be made transparent. In addition, the colors and flexibility of the ABS plastic made 3D printing the optimal choice for the dodecahedron frame and abacus. The acrylic was chosen as a safe material that is transparent for laser-engraving. A steel rod was chosen due to its high strength to weight ratio. Surrounding this, the carbon-fiber tubing was selected as another lightweight yet strong material, with a clean finish that feels comfortable for handling. Finally, aluminum 6061 was chosen for the rest of the parts, as it is of high quality visually, and is also easy to work with, and relatively cheap to purchase.

| ITEM | COST ($) |
|---|---|
| Aluminum (total) | 116 |
| Carbon-Fiber Tube | 131.33 |
| Threaded Rod | 5.16 |
| ABS Plastic (Blue) | 26.01 |
| ABS Plastic (Yellow) | 31.77 |
| PolyJet Material (Clear/Support Plastic) | 173.11 |
| Heat-Set Insert | 10 |
| Acrylic | 188.05 |
| Red Oak Wood | 83.72 |
| TOTAL | 765.15 |

**Table 1: Material and Hardware Purchase Costs**

Table 1 shows the price of each part purchased. Additional details can be seen in the Bill of Materials. In addition, due to changes in manufacturing, some aluminum parts were exchanged with University stock, and the nut needed for assembly was also taken for stock. The prices for ABS and support material also include the price for the backup dodecahedron pieces. This table only serves as a measure of materials directly purchased by the team for use in this project.

| Team Member | Hours | Cost ($) |
|---|---|---|
| Dalton | 123 | 12,300 |
| Will | 108 | 10,800 |
| Jack | 102 | 10,200 |
| Yijun | 112 | 11,200 |
| Chengxiao | 104 | 10,400 |
| TOTAL | 549 | 54,900 |

**Table 2: Development Time**

| Team Member | Hours | Cost ($) |
|---|---|---|
| Dalton | 25 | 2,500 |
| Will | 25 | 2,500 |
| Jack | 23 | 2,300 |
| Yijun | 19 | 1,900 |
| Chengxiao | 18 | 1,800 |
| TOTAL | 110 | 11,000 |

**Table 3: Manufacturing Time**

Tables 2 and 3 show the total time each member spent on the project. Table 2 includes the time for development, including design, meetings, and other non-manufacturing tasks. Table 3 shows the time spent manufacturing and assembling parts. These times include a projected estimate for final steps after the completion of this document as well.

If this mace were to be manufactured 1000 times, a few things could change for this shift to mass production. First, all 3D printed parts could be replaced with injection-molded parts. This would require more design and setup time in order to make molds but would reduce the time to manufacture individual parts drastically. Additionally, the team could cast some of the aluminum parts to be made faster. Finally, instead of cutting the carbon-fiber tube and threaded rod to fit perfectly with the manufactured parts, a tolerance analysis could be conducted, so that multiple could all be cut at once fit within a 6-sigma standard for mass production.

Figure 5 in the Appendix shows the final iconography that was used. Figures 6 and 7 show the final CAD models that were referenced during manufacturing.

## TEST PLAN AND RESULTS

The specifications for the mace include a total length, weight, center of mass, and fatigue limit for ferrous materials. The mace must be less than 48 inches, the length of the University Mace, from end to end. To ensure this, the mace was designed in CAD with this in mind and was tested following the completion of manufacturing the mace. The CAD model satisfies this requirement ay 39.97". After the assembly was complete, the team measured the mace to be 40.01". Second, the weight of the mace must be 6.4 ± 2.5lbf. Again, this was verified in CAD while designing the mace at 7.1806 lbf (see Appendix Figure 4) and the final product was weighed at 6.51 lbf. Next, the center of gravity of the mace is required to be within 10% of its geometric center. This was considered while designing the mace and the center of mass was verified to be within 10% of the geometric center of the mace in CAD. Figure 4 in the Appendix shows the results of this analysis, with the center of mass lengthwise at 17.4168” from the bottom of the mace. With a length of 40 inches, the center of mass should be between 16” and 24”. In addition, this center of mass was verified with the final model, where the center was 18.75” from the bottom of the mace.

Finally, to determine the stress and fatigue limit of ferrous materials, a calculation (see mechanical analysis) was completed. The result of this calculation is a fatigue limit of 62.5 ksi. While this is less than the specification of 85.99 ksi, the specification value is the result of the Mechanical Engineering Mace’s fatigue limit, and thus is a good point of reference for this mace. While the result did not meet this specification, the fatigue limit is still much higher than the stress the threaded rod should experience, so the results do not indicate any issue with the design. Furthermore, they indicate that stresses will not induce fatigue. Additionally, an FEA was completed for the dodecahedron head of the mace, which can be seen in the Appendix in Figure 3.

## INTELLECTUAL PROPERTY

The ceremonial mace's design is not patentable by the Department, as the design utilizes symbols and equations that are not necessarily exclusive to the Department. It is possible the University could patent the assembly or some of the design aspects used.

## SOCIETAL AND ENVIRONMENTAL IMPLICATIONS

Since this device is symbolic in nature and only one unit will be produced, the scope of environmental implications from this project is limited. One of the team’s real concerns is that if the carbon-fiber tube is significantly damaged, it may emit toxic particles into the air that can be harmful to those in the immediate vicinity. The fumes generated by laser cutting and acrylic are harmful to inhale, so it is recommended to allow the acrylic to remain in the machine for a few minutes after cutting to avoid exposure. It is unsafe to use polycarbonate for laser cutting as it produces potentially deadly fumes, which is why acrylic was used instead of polycarbonate for the faces of the dodecahedron. For societal implications, this device is intended to represent the Department, however, as technology changes very rapidly, some symbols used may become outdated. In addition, as departments develop maces of their own, other departments may request their own maces.

## RECOMENDATIONS FOR FUTURE WORK

The design underwent multiple revisions considering both structural and aesthetic factors. The main part of the mace, the head, utilizes 3D printing and laser cutting techniques to facilitate future replication and avoid aesthetic degradation due to wear. Additionally, its design allows for easy assembly and disassembly, accommodating potential future changes. The base supporting the mace bears the names of mathematics professors, with the convenience of the material allowing for the addition of new names in the future. The shaft of the mace is made from carbon-fiber and aluminum. The aluminum used in the base ensures stability, serving as support whether the mace is being held or not in use. The production of the mace includes CAD models and assemblies for components such as the dodecahedron, abacus, helicoid, torus, Menger sponge, and more. Specific dimensions are provided for the carbon-fiber tubing, along with vectorized designs for panel images. All related files will be sent to Prof. Funkenbusch for future reference. With these files, any necessary changes can be easily implemented for future designs and rebuilds, and for adding additional names to the mace stand.

## ACKNOWLEDGMENTS

We acknowledge the help of Professor Paul Funkenbusch, along with the mathematics faculty and undergraduates, in their assistance with the development of this project. We also acknowledge the assistance of Professor Christopher Muir, Chris Pratt, Jim Alkins, Alexander Prideaux, Bill Mildenberger, Professor Ed Herger, Melissa Mead, and the TAs (Sanjeev, Dominique, Rebeca, and Robert) for their help in the development, design, and manufacturing of the mace.

## REFERENCES

McGraw-Hill Series in Mechanical Engineering. Mechanical Engineering Design, by Joseph Edward Shigley and Larry D. Mitchell

## APPENDIX A: TABLES AND FIGURES

| PUGH MATRIX | A | B | C | D | E |
|---|---|---|---|---|---|
| Ease of Manufacturing | 0 | 1 | 1 | 0 | -1 |
| Mathematical Significance | 1 | 1 | 1 | 0 | 1 |
| Space for Concepts | 0 | -1 | 0 | 1 | 1 |
| Uniqueness | 0 | 1 | -1 | -1 | 1 |
| **Total** | **1** | **2** | **1** | **0** | **2** |

**Table 4: PUGH Matrix**

Concepts (Columns):

- A- Baseline (Sphere)
- B- Tetrahedron
- C- Cube
- D- Octagonal Prism
- E- Dodecahedron

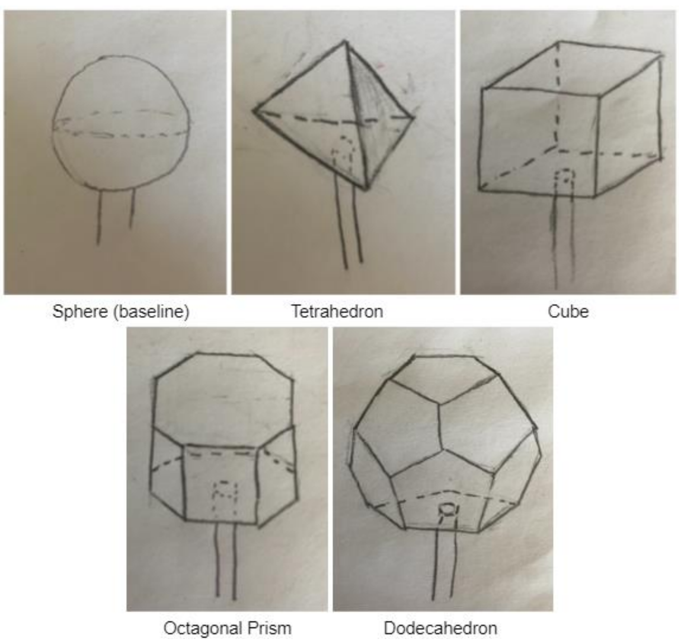



**Figure 1: Concept Drawings**

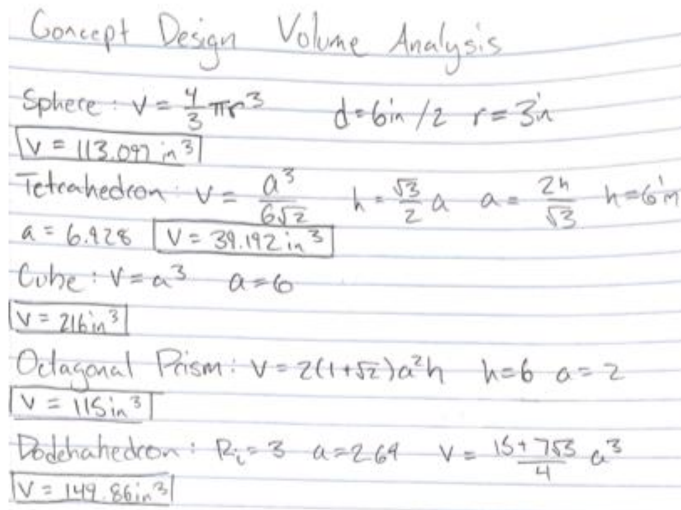



**Figure 2: Volume Analysis**

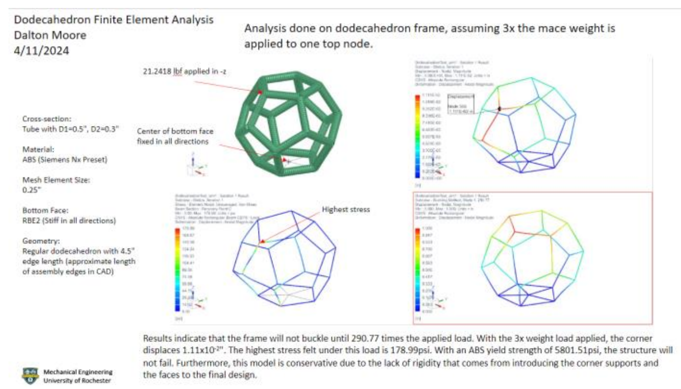
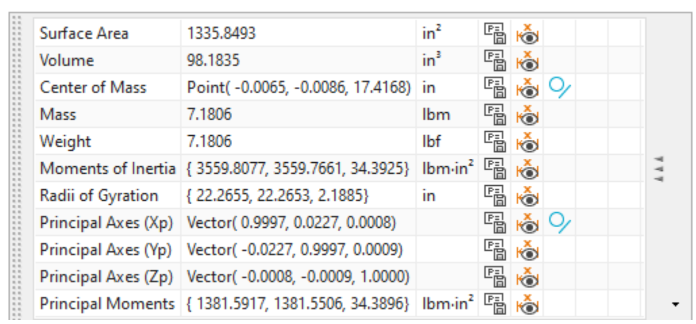



**Figure 3: Dodecahedron FEA**
**(For clearer text, please see additional files)**



| | | |
|---|---|---|
| Surface Area | 1335.8493 | $in^2$ |
| Volume | 98.1835 | $in^3$ |
| Center of Mass | Point( -0.0065, -0.0086, 17.4168) | in |
| Mass | 7.1806 | lbm |
| Weight | 7.1806 | lbf |
| Moments of Inertia | { 3559.8077, 3559.7661, 34.3925} | $lbm \cdot in^2$ |
| Radii of Gyration | { 22.2655, 22.2653, 2.1885} | in |
| Principal Axes (Xp) | Vector( 0.9997, 0.0227, 0.0008) | |
| Principal Axes (Yp) | Vector( -0.0227, 0.9997, 0.0009) | |
| Principal Axes (Zp) | Vector( -0.0008, -0.0009, 1.0000) | |
| Principal Moments | { 1381.5917, 1381.5506, 34.3896} | $lbm \cdot in^2$ |

**Figure 4: Assembly Analysis**

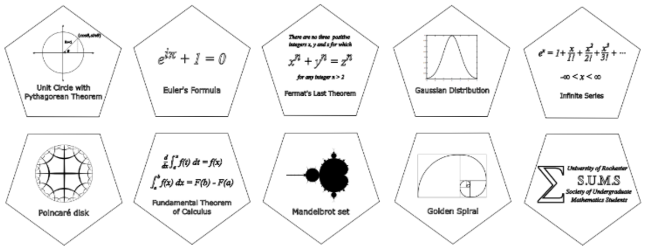



**Figure 5: Face Iconography**

## APPENDIX B: CAD MODELS

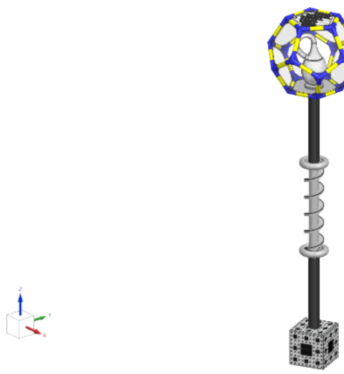


**Figure 6: Mace CAD Assembly**

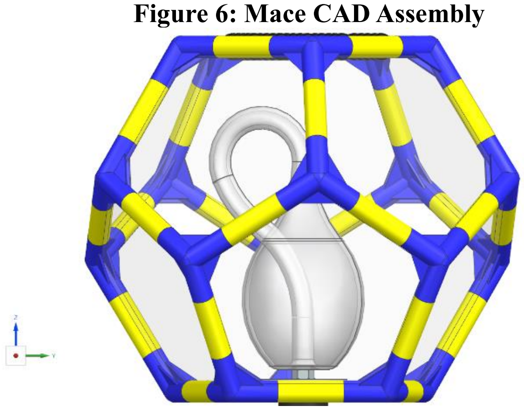


**Figure 7: Dodecahedron and Klein Bottle CAD Sub-Assembly**

## APPENDIX C: REVISIONS

The team created a first draft of this FDR as a single file. After completion, this document was duplicated, and each member added comments to sections written by others in the duplicate copy. Then, a copy of the version with comments was created. Each member edited their section based on the comments added, along with a discussion with the commenter. This resulting edit was version 1. After completion, this review process was repeated, resulting in the final document, version 2. Thus, 5 individual documents (draft, r1, v1, r2, v2) were created as part of this process.